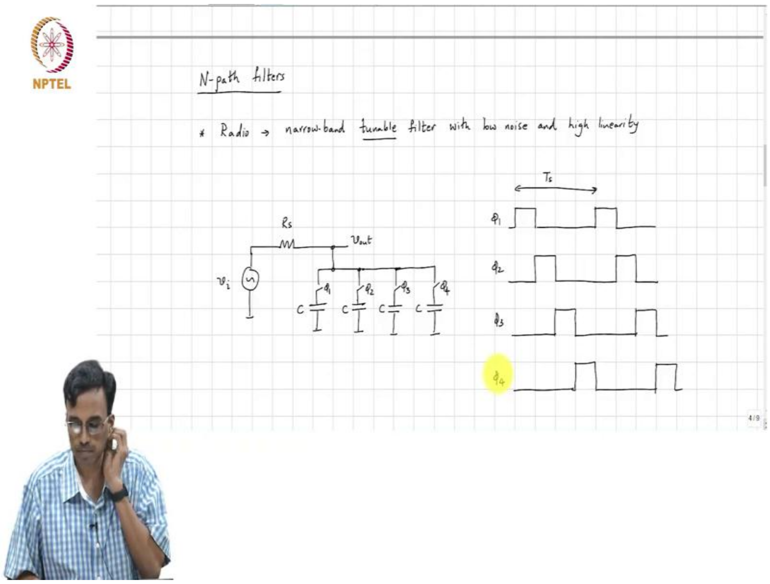
Introduction to Time - Varying Electrical Networks
Professor Shanthi Pavan
Department of Electrical Engineering
Indian Institute of Technology Madras
Lecture 50
The N-path filter

(Refer Slide Time: 0:22)



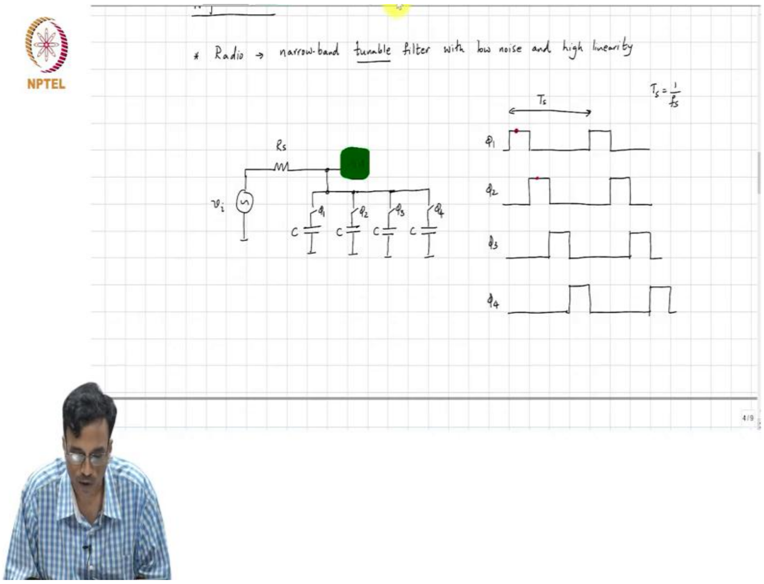
N-path filters

* Radio → narrow-band tunable filter with low noise and high linearity

Draw your attention to is the N-path filter as used in RF transceivers and the idea is the following. Remember, a radio is simply a narrow band filter is nothing but a narrow band tunable filter with low noise and high linearity. And well, it is, as we have discussed earlier, the use of inductors and capacitors to make tune circuits was done in the past and is still done of course, but they have difficulties making a filter that is tunable because easily changing an inductor or a capacitor is physically difficult.

So, what one, I mean another way of doing this is to use time varying circuits. And I am not going to derive this topology today. But maybe a few classes down the line, we will see how this comes about. But here is an example of a bandpass filter. I am going to take a 4 path example today. And what would you do? It turns out that so let us call this phi 1. This is Ts, this is phi 1, this is phi 2, this is phi 3, this is phi 4.

(Refer Slide Time: 4:17)



* Radio → narrow-band tunable filter with low noise and high linearity

So, first question that I have for you is, is this a periodically time varying network? Yeah, well of course, it is periodically time varying because all the switches have been turned on and off periodically with a period Ts, which is basically equal to 1 over fs. Now, look carefully. And based on what we discussed yesterday. Can you tell me that even though it is, technically it is periodic with Ts, do you notice something else? What comment can we make about the harmonic transfer functions for instance, at that node? Do you see a periodicity which is shorter than Ts?

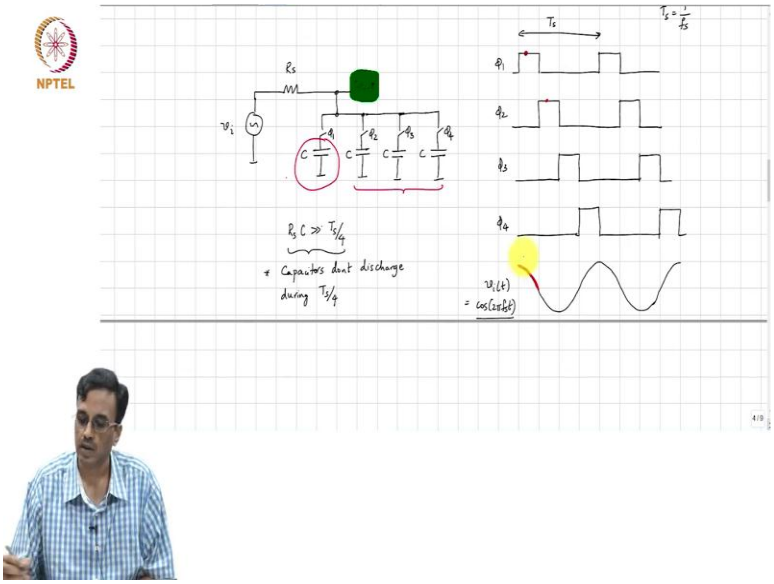
Student: 4.

Professor: Yeah. So, at remember that, as far I mean, as we discussed yesterday, if the network looks identical periodically, then it is LPTV at that small period and that is definitely true in this case, because all the capacitors are identical. So, if I took a snapshot of the network at say this point or exactly Ts by 4 later, you will know.

Because when you are at this point for instance you have been, I mean this switch has been turned on for this amount of time, and is going to be turned off after that time. But if you are here exactly the same point in phi 2, it also I mean one capacitor has been turned on for this the same amount of time before and is going to be turned off the same amount of time later.

So, even though it is. So, technically speaking, even though all the elements are switching at Ts, it is may have the harmonic transfer functions will be, many of the harmonic transfer functions only one fourth of the harmonic transfer functions will be nonzero, that is one thing. Next thing is, I would like to draw your attention to the fact that this actually behaves like a bandpass filter, which is centered around fs.

(Refer Slide Time: 7:35)



And to see that, again, as I said, we are going to derive this in a lot more detail going forward, but for today and our illustration of the N-path principle, let us assume that vi is a sine wave whose frequency is exactly equal to fs. So, let us say this is a sine wave whose frequency is, sorry. So, this is vi of t which is say cos 2 pi fs times t.

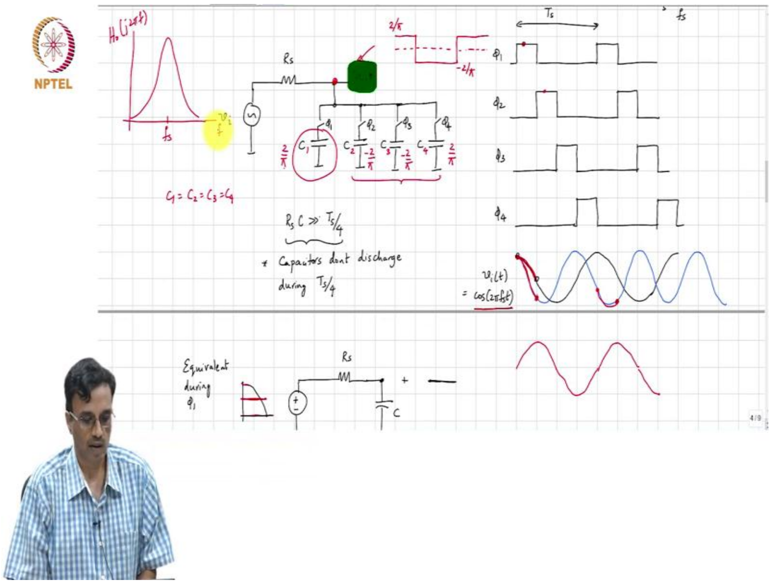
And let us assume that RS times C is much, much, much greater than Ts by 4. So, what does this mean intuitively? If RS times C is much larger than TS by 4 what comment can we make about the voltage across the capacitor, any of the capacitors labeled C? So, basically, what this means is that during when the switch is closed and the capacitor is connected to the resistance, then the voltage change across the capacitor is going to be very small, because the RC time constant needed to discharge the voltage across the capacitor is very large compared to the period in which that switch is on.

So, that is, that basically means that. The next thing is what comment can we make when the input waveform is at the same frequency as fs, in other words the input is cos 2 pi fs times C. So, let us see what happens, during I mean steady state when phi 1 is on, what do we see? This capacitor that is this guy here, what part of that input waveform is in C? When phi 1 is on what is the equivalent all these three capacitors are open, those switches are open.

So, which part of this waveform is the, is this capacitor, the first capacitor C? It is only C this part of the waveform. Because it is during that time that switch phi 1 is on. So, in other words, what comment can you make about the voltage across the capacitor, that capacitor in steady state.

Student: Steady state (()) (11:11) Like we do not have sufficient time to…

(Refer Slide Time: 11:24)



Professor: Correct. So, the voltage across the capacitor is, so, if you look at the input voltage, so during phi 1. So, you can think of this as repeatedly I mean, the, during this equivalent…

Student: Finally, it will (()) (12:05), repeatedly (()) (12:06).

Professor: Equivalent during, one second, equivalent during phi 1. So, what do you think will happen? In steady state what comment can you make about the voltage across the capacitor?

Student: That final value it will (()) (12:20).

Professor: It is, no, no, no, think carefully, during, when you close the switch two things are happening what is happening, one is the input voltage source is trying to charge the capacitor through that RS. Then the next thing that is happening is that the voltage across the capacitor is trying to get discharged through RS.

So, in steady state what must happen the voltage across the capacitor is not changing very much. So, what do you think will happen in steady state?

Student: It will be constant.

Professor: It will be constant and what will be the constant voltage? Is one guess is that okay well, maybe the capacitor voltage is the same as the peak value. What is wrong with this argument? Well, if the capacitor voltage is always at the peak value, then what is happening whenever you turn phi 1 on, capacitor is always going to be discharging. So, it will eventually it will discharge. So, the peak, it cannot stay at the peak value. So, peak value is not correct. Now, what comment can we make about, will it be 0, can the capacitor voltage in steady state be 0?

It will charge, so it has, so very good. So, the capacitor value cannot be sitting at the peak, it cannot be sitting at 0, it has to be at the average value of, the capacitor voltage has to be at the average value of the sine wave during this part of the cycle. So, this C1 I am going to call this C1, C2, C3, C4. So, C1 equals C2 equals C3 equals C4.

So, in steady state therefore with an input of cos2 pi fs times t, the average value of the voltage sitting across the capacitor C1 will be the, what do you call the average value of quarter cycle which is 2 by pi. So, this voltage will be 2 by pi on average, that is the voltage stored on C1. Now, what comment can you make about the voltage stored on C2?

Student: That is minus 2 by pi.

Professor: Very good. That will be the next half cycle. So, the voltage stored on C2 will be minus 2 by pi. What about C3? C3 will also be minus 2 by pi and C4 will be plus 2 by pi. So, as you can see therefore, if you look at this waveform, how will this waveform look therefore, when the input is cos2 pi fs times t, it will be, so it will be a square, I mean it will be a wave which does this, so this is 2 pi, this becomes minus 2 by pi and this will be a square.

So, similarly, when you put in a sine wave rather than a cosine what comment can you make? A sine wave will basically when you put sin 2 pi fs times t you basically get the input is doing this and therefore, you get a sine wave, I mean the voltage across C1 will be plus 2 by pi across C2 will also be plus 2 by pi, C 3 and C4 will be minus 2 by pi.

So, based on the response to a cosine and a sin you should be able to go and calculate what the H0 of, I mean you should be able to calculate the harmonic transfer functions for whatever H sub k of j 2 pi fs you will be able to calculate. So, but the, but as you can possibly visualize, you can see that the fundamental component corresponding to this square wave is quite large.

Now, the question I have is, what comment can you make if the input frequency and remember the reason why you are seeing a large amplitude at the output is because this capacitor, each of these capacitors sees the same portion of the waveform in this cycle, in the next cycle, in the 100 cycle. Now, if the input frequency deviates from fs by some amount, what comment can you make about the voltage that you see across C1? So, in other words, the input frequency is not fs, but say fs minus some delta f, then what happens?

Student: That wave form will be every time it will (()) (18:48)

Professor: Exactly, so every time the switch phi 1 is on the voltage across C1 sees a different portion of the waveform. So, for example, if the input frequency was higher, during the first, in the first clock period, you are basically seeing this portion of the waveform, in the second you are seeing this portion of the waveform, during the third Ts you are seeing some other portion of the waveform.

So, on average therefore, you are basically looking at all portions of the waveform and therefore on average the voltage across the capacitor will be much smaller than what you saw, if you, I mean if you kept looking at the same portion of the waveform all the time, then the voltage across the capacitor has an opportunity to build up and stay that way.

If you keep looking at different portions of the input waveform, basically in some fraction of the, I mean some input cycles, I mean clock cycles it is seeing some large value on average, during some other clock cycle it is seeing the negative of that. So on average it will be much smaller than you would see.

So, therefore, if you look at the fundamental component to the voltage across V out, then how does it look like? It will look like, if you plot it as a function of frequency. So this is H0 as a function of f. And this is of course just an intuitive explanation. It is not, I mean we should not do this a lot more rigorously. And we are going to do that later on.

But at this time, this is just giving you the intuition that this behaves like I mean, what does this remind you of, what does this response curve remind you off? It reminds you of the response of a bandpass filter and it is bandpass at fs which is a clock frequency. So, if you want to change the center frequency of the bandpass filter, what will you do?

By changing the clock frequency, you can go and change the center frequency of the bandpass filter. So, this is a tunable bandpass filter, whose center frequency depends on a clock frequency, which can be made very accurately. And if the clock frequency is changed, for example, if you divide the clock frequency by 2 using a digital divider, and then you are now basically all of a sudden have been able to tune the bandpass filters center frequency by a factor of 2, which would have been very, very difficult to do with an LC filter.

And the next thing is that as we have already seen, this is an example of a 4 path filter. So, that basically means that h sub 1, 2, and 3 will be 0, h sub 4 of j 2 pi f will be nonzero and so on and so forth. So, an N-path filter is another example which uses N-path principles to make something which is very useful.