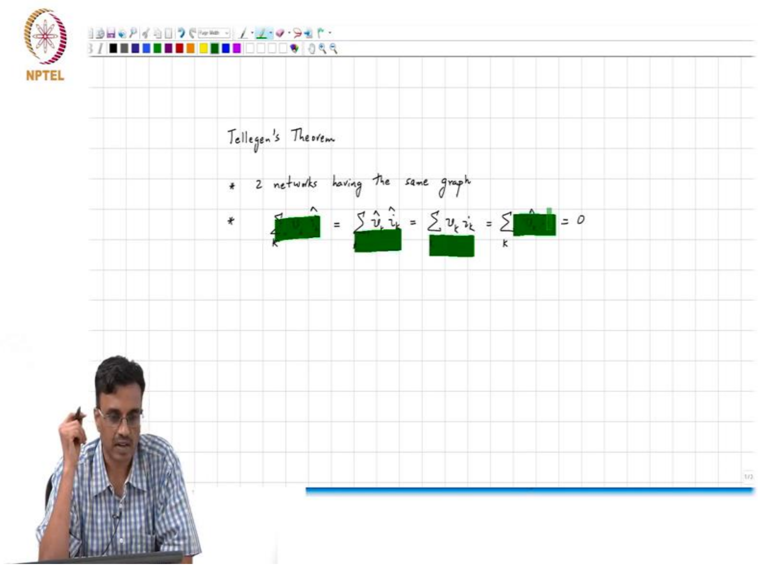
Introduction to Time - Varying Electrical Networks
Professor Shanthi Pavan
Department of Electrical Engineering
Indian Institute of Technology, Madras
Lecture - 5
Tellegen's Theorem and Reciprocity in Linear Resistive Networks

(Refer Slide Time: 00:16)



All right people. Good evening, and welcome to advanced electrical networks, this is lecture 3. In the last class, we looked at the Tellegen's theorem and it pertains to two networks having the same graph. And is it necessary that the two networks be linear?

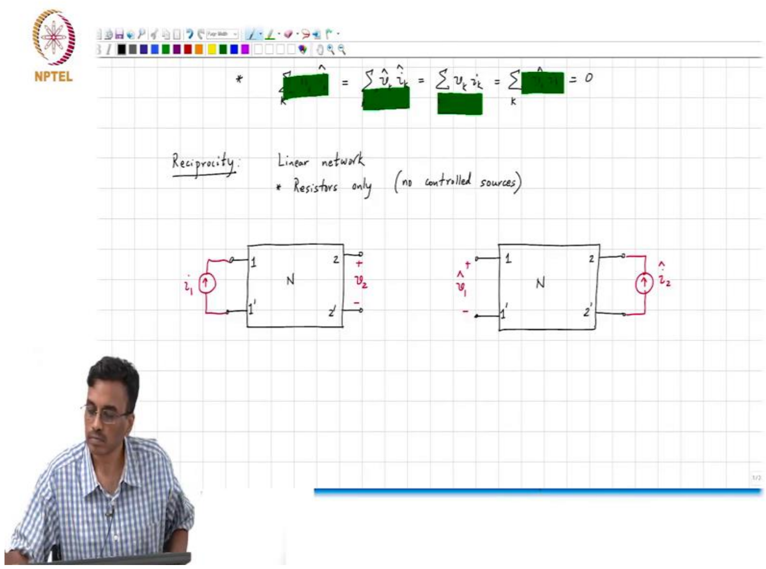
There is no, nothing that says that the networks have to be linear or time invariant. So the only requirement is that the two networks have the same graph. And what does it say? It says that Sigma over all the branches, vk times ik hat is the same as Sigma over k vk hat times ik must be equal to sigma vk times ik. Hence it is the same as vk hat times ik, is all equal to 0.

And we show how this comes about. These, these two are of course immediately understandable and apparent. You may say, well, at any instant of time, it seems reasonable that power is conserved. These on the other hand at first sight, seem a little puzzling but we saw a simple network construct that helped us get intuition into why the physical reason why this makes sense.

All that we did was to put in parallel with every branch of the network, a current which flows in the opposite direction with a value ik hat, and if you now apply power conservation to this network, it is apparent that you basically get the other two identities.

Now, it turns out that Tellegen's theorem is also very useful to prove a whole lot of interesting theorems.

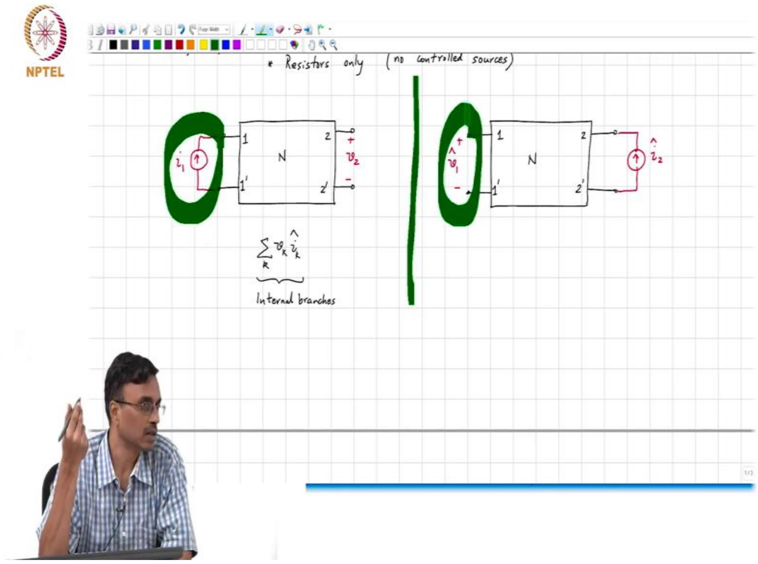
(Refer Slide Time: 03:10)



The first one that I would like to draw your attention to is reciprocity, and the idea is the following. So let us say you have a linear network. Now, and we have the same another copy of the linear network. It is let us assume, it is, it has two ports; 1, 1 prime, and 2, 2 prime.

This is 1, 1 prime, and 2, 2 prime. And this is i1, this is v2. And here, I inject i2 hat and measure the voltage at port one as v1. Further, let us assume that this is a linear network which consists of resistors only. In other words, there are no controlled sources inside the network. And we apply Tellegen's theorem to both these networks, and let us see what happens.

(Refer Slide Time: 05:11)

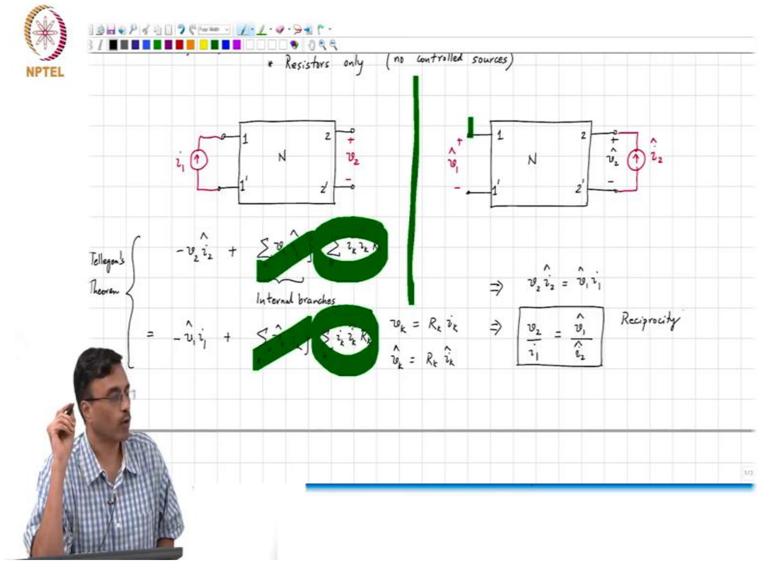


Remember, Sigma over all branches inside this network N of vk and ik hat which correspond to the branch currents in the, remember, the un-hatted quantities correspond to the network on the left, the hatted quantities correspond to the network on the, on the right.

So, and the k runs over all the branches which are sitting inside the box. So if I want to form the product vk ik hat for all the for all the branches in this entire network, you have this which corresponds to the internal branches and what comment can you make about vk ik hat as far as the port branches and currents are concerned?

What is, we do not know the voltage, we are forming the product vk ik hat. We do not know the voltage across this branch but we need to find the product of this voltage and this current. And what does that happen to be?

(Refer Slide Time: 06:43)



Well, the current in port one of the network on the right is 0. So, therefore, it must follow that. The product of Vv1 times i1 hat is 0. And the only other quantity to consider is v2 times i2 hat. Actually, I mean, this is our voltage v2 hat, it must actually be minus i2 hat simply because if we assume v2 to be in that direction, branch current should flow in the opposite direction, so it is minus v2 i2 hat.

And this must be equal, by Tellegen's theorem, Sigma over all internal branches vk hat times ik. That is the all internal branches, then we need to deal with the external the port quantities and so what is vk hat? For the first one, it must be v1 hat times minus i1. And what comment can we make about the second port? v2 hat times the current in port two, which happens to be 0. So this is what Tellegen's theorem throws up.

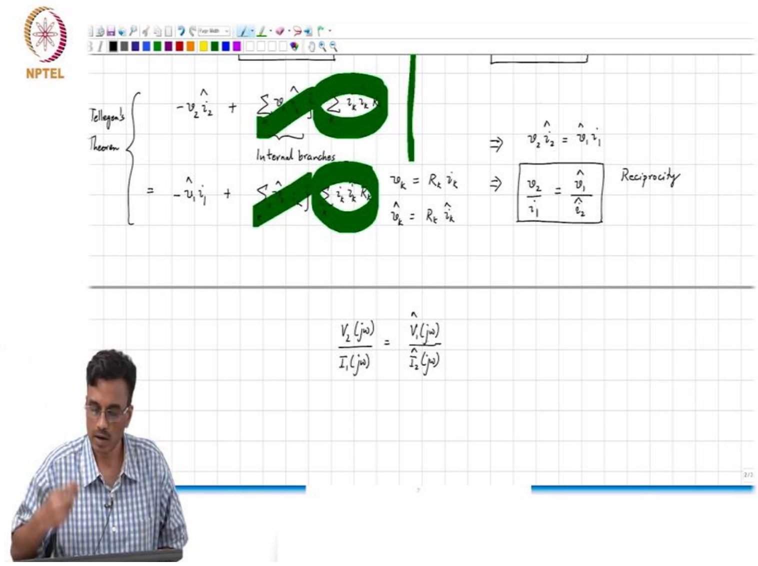
We still have not exploited the fact that the network is linear, and that every branch is a resistor. So vk, therefore, how is it related to the, how is the branch current related to the branch voltage? It is simply nothing but some Rk times ik. And likewise, vk hat is Rk times ik.

So this, therefore means that this part will become Sigma sum over all branches inside ik ik hat times Rk. And this will also become sigma over all k ik ik hat times Rk. And, therefore what common can we make about, what does this throw up if you? So these two quantities evidently are the same. So these two are therefore, the same. And, therefore, what do we get?

Student: (())(10:08)

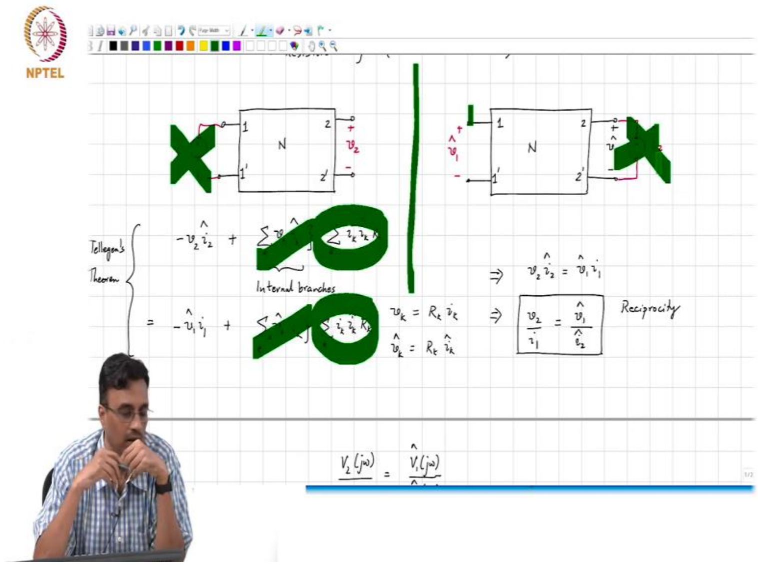
Professor: v2 times i2 hat equals v1 hat times i1, which is equivalent to saying that v2 by i1 is the same as v1 hat by i2 hat. So this is the reciprocity.

(Refer Slide Time: 11:06)



Of course, we now only did this for network with resistors. It follows, I mean it is, you go through the same proof and show that it is easy to see that if you are, if you have impedances, V2 of j omega by I1 of j omega is V1 hat of j omega divided by I2 hat of j omega. The proof is exactly the same. Makes sense?

(Refer Slide Time: 11:55)

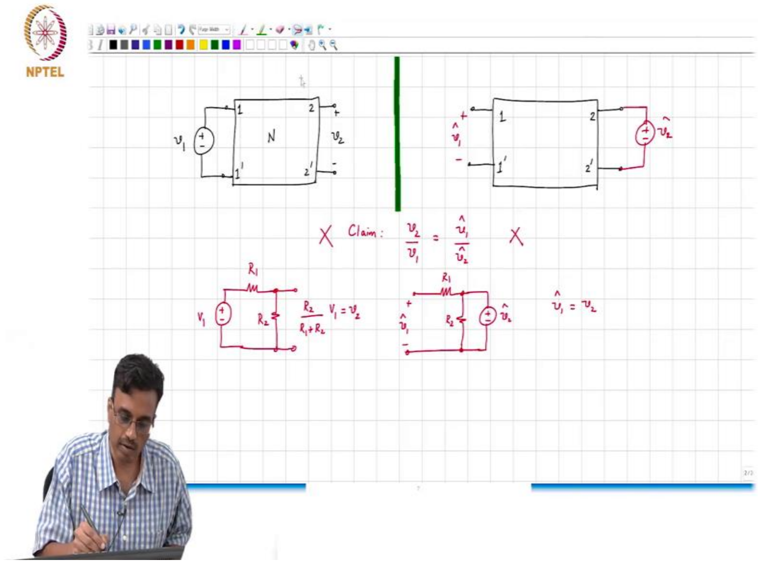


So in other words, the transfer function, I mean the what you have seen in the past is basically, the one way to say it is that, well, if you have a linear network with only resistors or impedances, then you can interchange the location of the excitation and the response, and you get the same transfer function.

One thing that I would like to point out is that if you de-energize all the sources, so in other words, this is not around and neither is this, what comment can you make about the network, about both networks?

So if I remove i1 on the left and i2 hat on the right, if look at the two pictures, what are they? It is exactly the same. So please bear that in mind when you apply reciprocity. If the graphs of the network are not the same then you will not get I mean, this condition will not be satisfied.

(Refer Slide Time: 13:05)



So we have seen current input and a voltage output. It turns out that you can work this out at home. But the same thing happens when you have or rather I would say, a similar thing happens when you have voltage input. So this is v1 and this is v2. And what should I do, does somebody know what I should do?

Student: (())(14:04)

Professor: Pardon.

Student: (())(14:09)

Professor: What should I do? So, well, the answer I get is, okay, how about exciting this and the, sorry, this is v2 hat and this is v1. The claim is that v2 by v1 is v2 hat; v2 by v1 is the same as what? v1 hat by v2 hat. This is the claim, right? And I see all of you nodding your heads. Is this is correct? Hey, you are the one who gave the answer man, now, you are saying no.

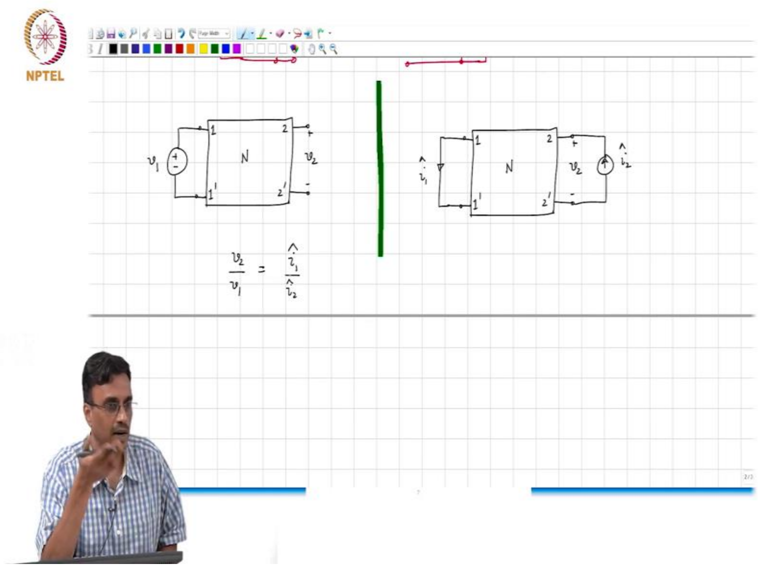
Student: (())(15:17)

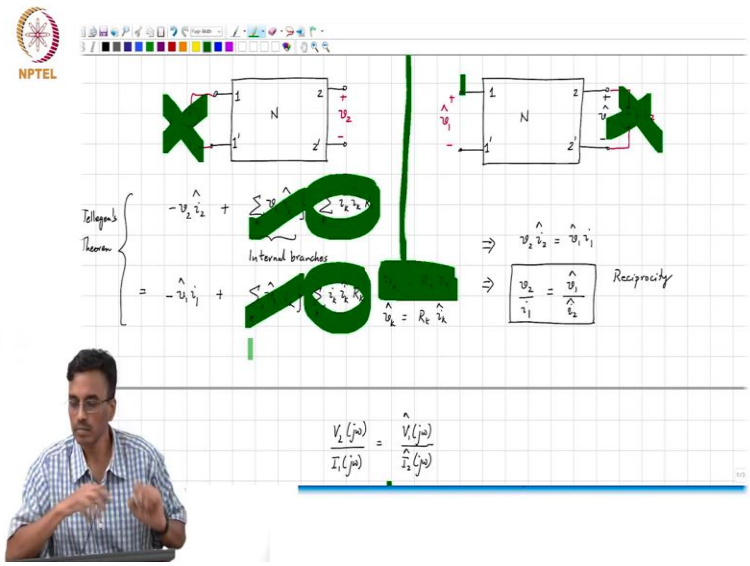
Professor: Exactly. So this is tempting to kind of say but this is actually not correct. Remember, that the graphs of the network when you de-energize the sources must be the same. So when you de-energize the sources, what do you get on the left side? Which, which port is shorted?

Port one is shorted and the, on the network on the right port two is shorted. So you do not have the same graph on, on both sides. And, therefore, this is definitely not correct. And if you want a simple analogy, a simple counter example here it is.

This is R1, I still see people kind of incredulously looking at the result and saying how can that be, so this is V1. So what is the output voltage? R2 by R1 plus R2 times V1. Now, I take the same network as you guys suggested, I will apply v2 hat here. And what is v1 hat? v1 hat is v2. So, clearly, you can see that the transfer functions are not the same. And the reason is that the graph of the network is.

(Refer Slide Time: 17:22)





So now that this is wrong, what is right is what we have to, so what do you think the correct answer is?

Student: (())(17:32)

Professor: Pardon. Very good. So the correct expression of reciprocity is basically to look at i1 hat and you apply, what should I apply on this side? Remember, v1 and v2, the input is a voltage, the output is the voltage. So what you have to apply is a current i2 hat and measure the current i1 hat.

And it so turns out, I mean, you can simply go through the same arguments that we went through to prove the other result. I will leave that for you as homework and convince yourselves that v2 by v1 equals i1 hat by i2 hat.

The voltage transfer function in the network on the left is the same as the current transfer function on the network on the right. And whenever you do this, you always will be in good stead if you run the sanity check, namely, when you de-energize the sources, the graphs must be the must be the, must be the same.

Student: (())(19:01)

Professor: No, no, no. We said already that N only consists of resistors.

Student: (())(19:08)

Professor: Pardon.

Student: (())(19:11)

Professor: And impedances, right, but impedance is not a source. It, reciprocity applies to a linear network where all the branches' currents can be expressed linearly as a function of the branch voltages.

So if you look at, so the comment was does it hold for any linear network? It holds for all network, the key point in the argument is, remember Tellegen’s theorem is valid for any network, it does not need linearity. What is allowing us to basically cancel out all the powers, the pseudo-powers kind of thing dissipated in the branch, internal branches?

We are able to do that because we are able to express the branch voltage as some Rk times ik or, in general, if there are impedances, the branch Vk of j omega is some Zk of j omega times Ik of j omega. As long as that is satisfied, you will, reciprocity holds.

Now, one in a practical case where this is automatically satisfied is when you have a linear network which is also passive, when you have only R, L and C in them, which are the commonly known, the passive elements that we use.

It is automatically satisfied and there is a general misconception that it is only applicable to passive networks, but that is not the case because, for instance, if you know I somehow managed to create an impedance which was negative. All that, these relationships are saying is that vk is Rk times ik. The sign of the R could be negative.

If the resistance is negative, it means that the network is actually active because the resistor is actually pushing out power rather than dissipating it. But that does not affect reciprocity at all, because these products will cancel out and you still end up with, you still end up with the transfer functions being identical.

Controlled sources is of a, we have still not, as far as we are concerned at this point, there can be no controlled sources inside the network. Only two terminal linear, two terminal elements whose branch currents and branch voltages are linearly related to each other. Is that clear?