Introduction to Time - Varying Electrical Networks
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Lecture 45
N-path example

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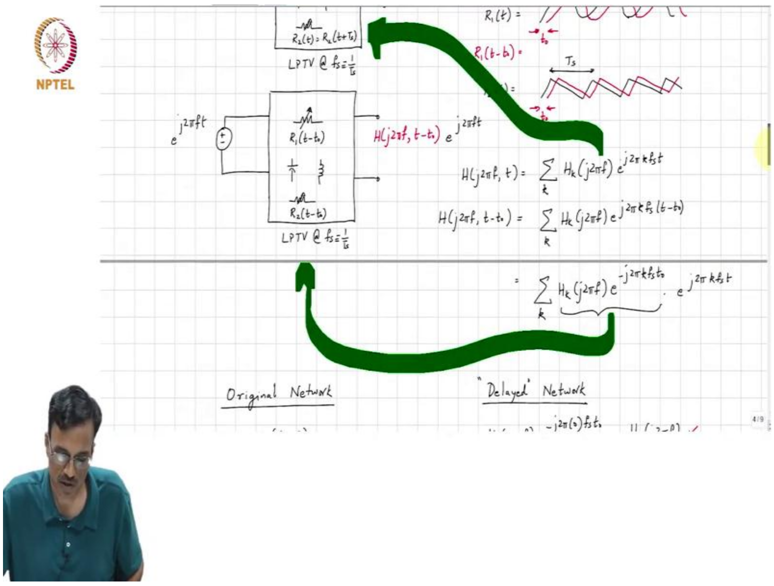

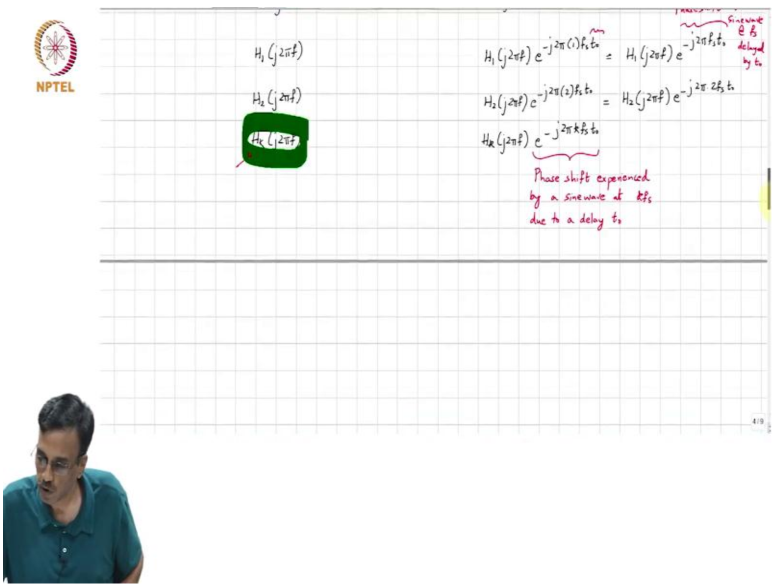

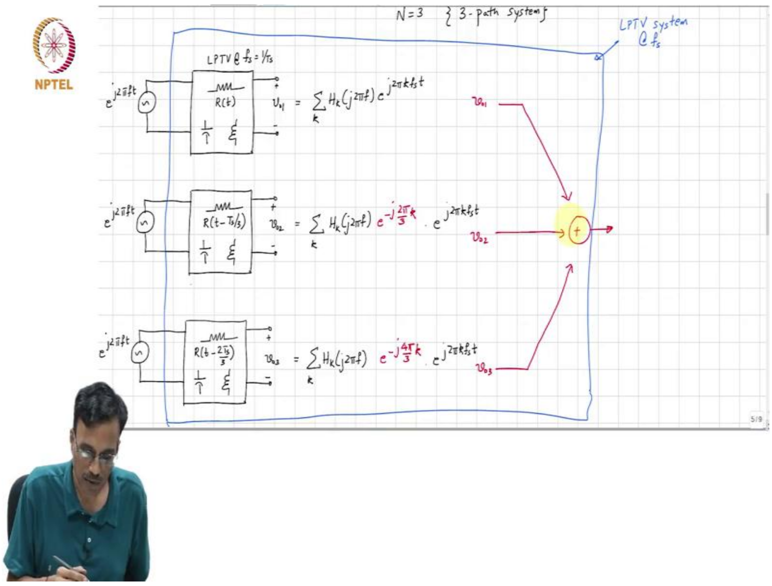

So, this is an observation to keep in mind, now let us see how people exploit that this observation. And the idea is the following, I am going to illustrate with n equal to 3. This is often what is called a 3-path system where the idea is to take the same LPTV system. Let us say this is our R of t. So, I excite this by e to the j 2 pi f t. Now, what I am going to do is, I am going to have a copy and paste of the same system, except with a small twist.

Now, I am going to delay it by R of t minus T s by 3. So, this is LPTV at fs, which is 1 over Ts. So, let us call this V o 1 this is V o 2. And now, I want to have a third chap here, which is also the same system except that this is delayed by 2 Ts by 3, so this is V o 3. Now, V o 1 as we will denote is simply H sub k of j 2 pi f e to the j 2 pi K fs times t, V o 2 is sum over k H sub k of j 2 pi f e to the minus j 2 pi times, what is the delay is Ts by 3 times fs times e to the j 2 pi K fs times t.

And the third one is simply going to be H sub k of j 2 pi f e to the minus j 2 pi 2 Ts by 3 times fs times e to the j 2 pi k fs times t. And if, let us simplify this before we go further, what is this, this Ts times fs is going to go away. So, this is can be replaced by e to the minus j 2 pi by 3 and this by the same token becomes e to the minus j 4 pi by times k I forgot the k.

Now, it is very interesting to see what happens when I, let us say we add up. So we take V o 1, we take V o 2, we take V o 3, add all these things together and so let us see what we get. I mean, clearly, if I put this whole thing inside a box, this is also an LPTV system. Of course, because each 1 of those systems is varying periodically. So it follows that when you put three of them in a box and add up their outputs, it is also an LPTV system and what is the rate at which the system is varying?

Student: Which one is varying?

Professor: The big box.

Student: Big box is varying.

Professor: The big box, I mean, we do not know anything yet. So individually, everything inside is varying at fs. So the output, also, I mean, this technically speaking, this is also LPTV system that varies with, varies at the rate fs, but there is something very special about this, as we will see going forward. So you know a reasonable question to ask is what are the harmonic transfer functions of this composite LPTV system? which is simply found by taking three LPTV systems and putting them together??

Of course, there is something special about these three things inside, which is what will, which makes it interesting. But let us see what the harmonic transfer functions are. So what is H, let us call the harmonic transfer functions of this box,? I mean, remember that, I mean, even though I have shown this as three separate inputs, and what I actually mean is that I am going to drive all these boxes with the same input.

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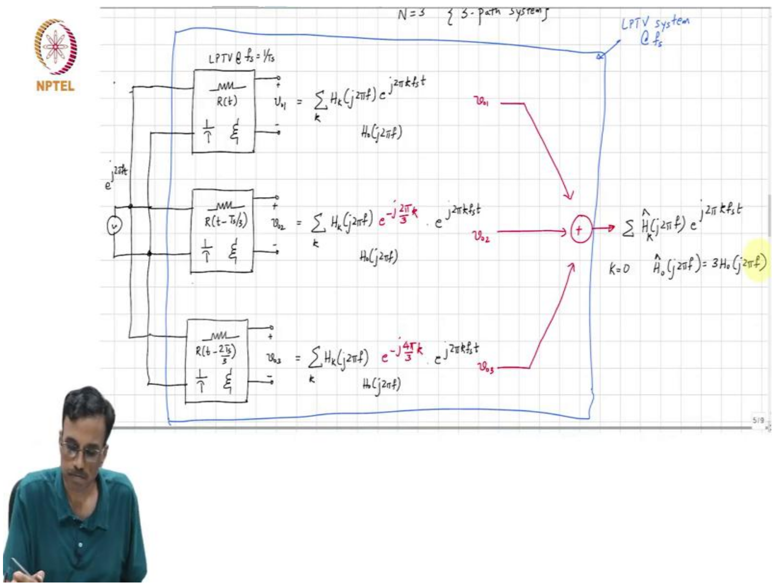

In other words, I short the inputs together. And this is my e to the j 2 pi ft. So this is an LPTV system which varies at fs, and I have an input and a reasonable question to ask is, what are the harmonic transfer functions of this LPTV system? Clearly, they must be related to the harmonic transfer functions of the individual boxes inside, and let us find out what they are. So let us, and let us call this H hat of j 2 pi f k e to the j 2 pi K fs times that is this output here.

And so we make a table. So, with K equals 0, what do we see? Oh, well, at K equals 0, this is, what comes out here is simply H 0. What comes out here is also H 0. And what comes out here is also H 0. So when you add all the three up, what do you get? You get, H hat 0 of j 2 pi f is simply 3 H 0 of j 2 pi f.

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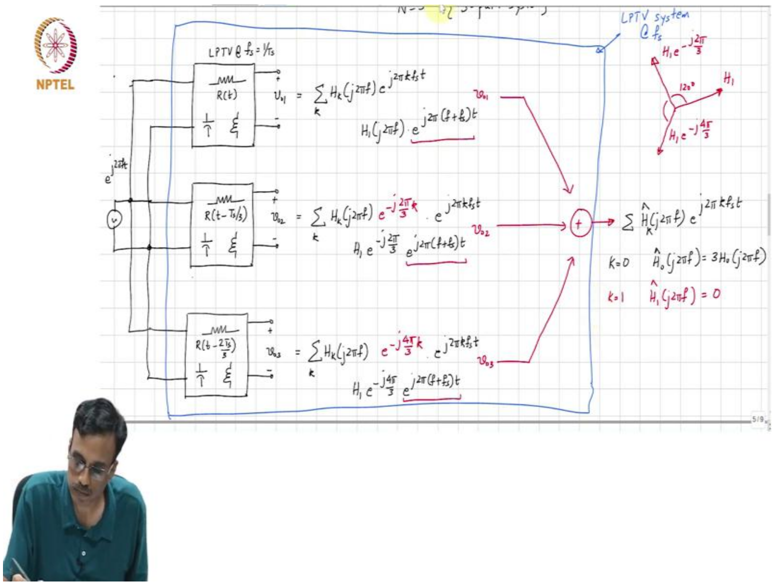

Now let us see what happens to the first harmonic transfer function. What you get here is H 1 of j 2 pi f times e to the j 2 pi fs times t plus f, what you get here is H 1 e to the minus j 2 pi by 3. So, in other words you put e to the j 2 pi ft here what you get is H 1 times this times j 2 pi f plus fs times t this is e to the j 2 pi again f plus fs times t and what you get here is H 1 e to the minus j 4 pi by 3 e to the j 2 pi f plus fs times t. So, when you, now these are all three complex sinusoids with the same frequency, so when you add them, it is simply adding the phasors.

So, if you assume that H 1 was some complex number like that, correct. What is H 1, e to the minus j 2 pi by 3 it will be 120 degrees phase shifted. So, this is H 1 e to the minus j 2 pi by 3 and the next one will have the same magnitude, H 1 e to the minus j 4 pi by 3 and when you add all the three together you basically get 0 so, K equal to 1 H 1 hat of j 2 pi f equal to 0.

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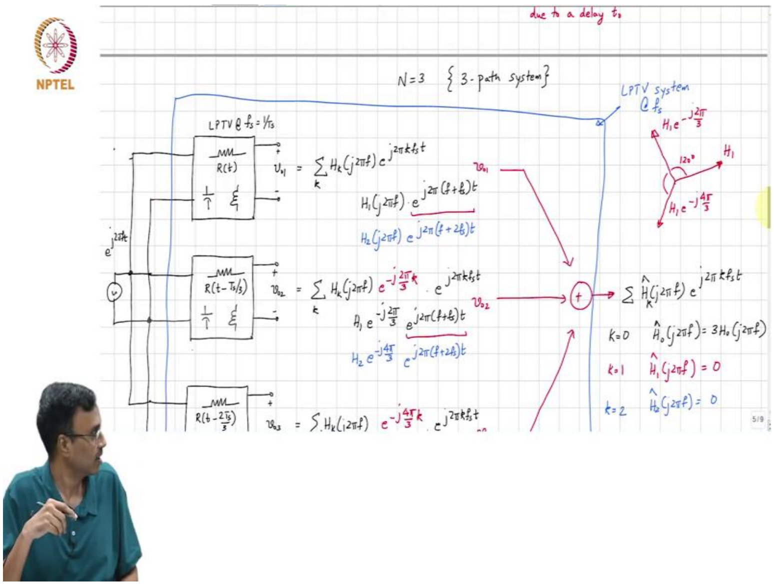

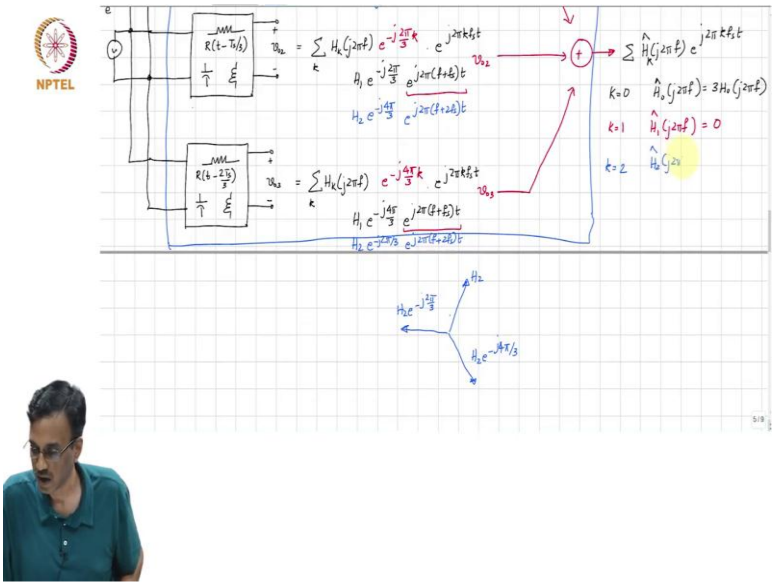

Now, for k equal to 2 what do we see? Well, you, in the original one you get H 2 of j 2 pi f, e to the j 2 pi f plus 2 fs times t, here you will get H 2 e to the minus j 4 pi by 3 e to the j 2 pi f plus 2 fs times t and likewise here you will get H 2 e to the minus 8 pi by 3, my 8 pi by 3 is the same as 2 pi by 3, e to the j 2 pi f plus 2 fs times t. So, again we see that we have some say H 2 like this, H 2 minus j 4 pi by 3 is minus 240 degrees H to the minus j 4 pi by 3 and this is H 2 e to the minus j 2 pi by 3.

And if you add all the three again you see that H 2 hat which again let me remind you is the second harmonic transfer function of this composite box which is taken three of the original systems, each delayed with respect to each other by one third the clock period and what is so sacred about one third we have three such boxes. So, the delay is related to the number of boxes by Ts by 3.

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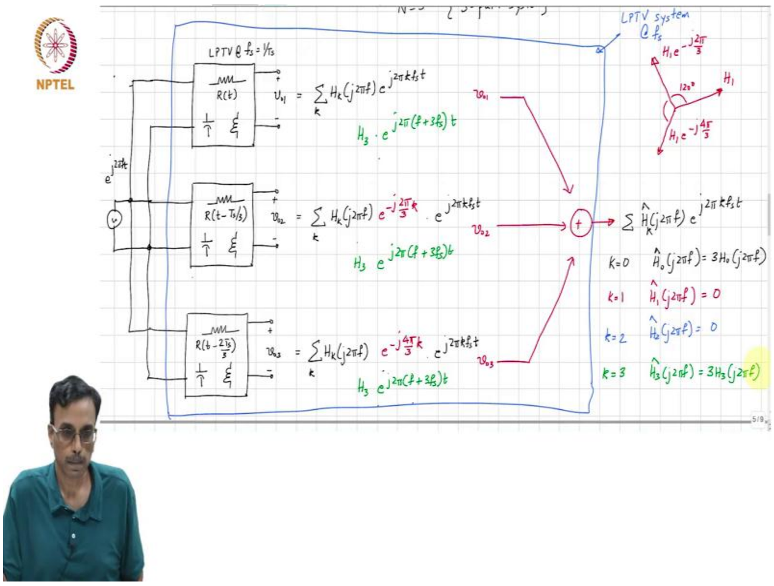

So, and now, if we look at k equal to 3, what comment can we make? Well, I am going to now, now I think you can see the pattern emerging I am going to erase all this, this becomes H 3 and e to the j 2 pi f plus 3 fs times t, this becomes H 3 times e to the minus j 2 pi, which is the same, which is 1 so it basically is (()) (15:08). And this will also be, keep the minus this will be 1 because e to minus j 4 pi is 1. So, this is e to the j 2 pi times f plus 3 fs times t. So, when you add the 3 up, you basically will get H 3 hat of j 2 pi f is nothing but 3, H 3 of j 2 pi.

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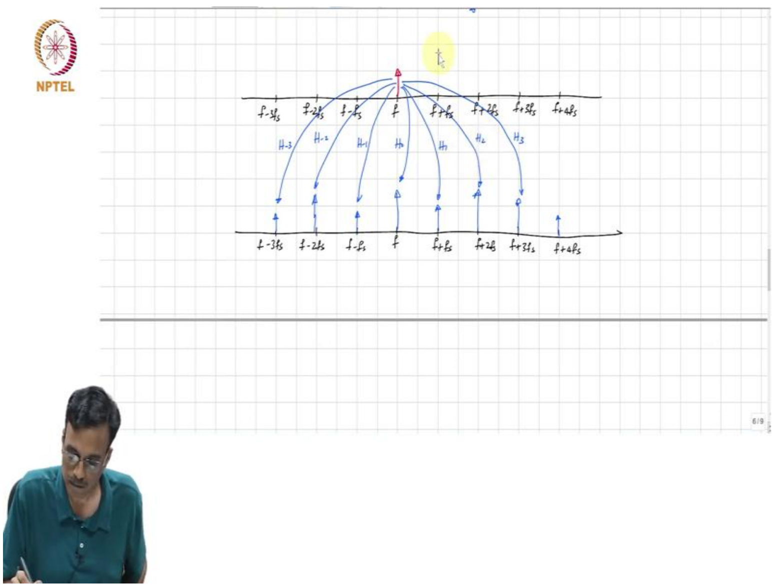

So, if you draw the input output diagram before the frequency plot therefore the original system, basically, let us say this was f, f plus fs, f plus 2 fs, f plus 3 fs and so on. In the original system, we had a frequency f here and the output frequency grid, basically, originally, you had something like this.

And so, this was H 0. This was H 1, this is H 2, this is it H 3, H minus 1, H minus 2, H minus 3 and so on. Now, by combining three such systems with the time variation delayed by Ts by 3, what have we done? The effective system has these numbers, the H hats for all those have gone or let me erase this and then draw a separate picture or copy paste.

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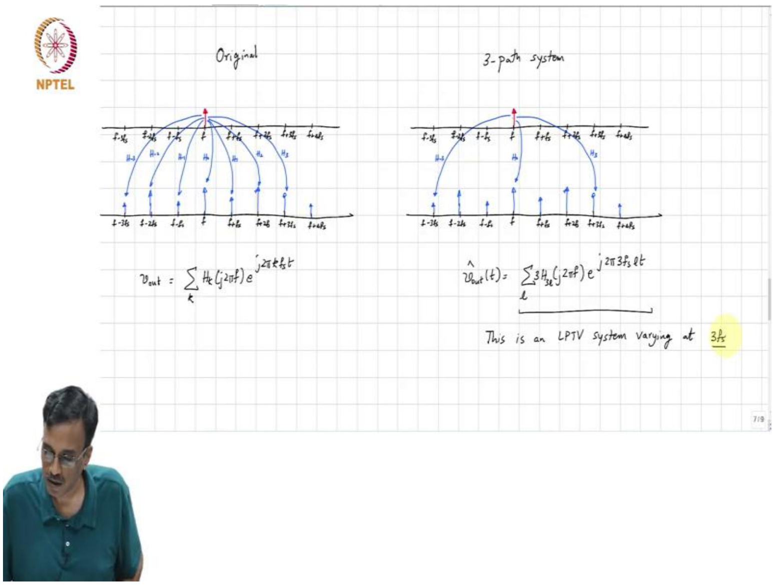

This was the original system. This is the 3-path system and the 3 path system we have eliminated this guy, H minus 1, H minus 2, H 1, H 2 and similarly, all other paths which are not multiples of 3. So, the original output could be thought of as H sub k of j 2 pi f e to the j 2 pi k fs times t. Now, we have a composite system if we call that V out hat, what can we say? It is only, yeah, we can write this as l, which is only a multiple of 3, so H sub l of j, 3 H sub l of j 2 pi f, e to the j 2 pi times 3 fs times l times t, correct, because it is only valid for k equal to 3 times l, correct. So if you, pardon.

Student: (())(20:36)

Professor: l has, l can be any integer. Oh sorry, I think l must be of a form H 3 l, this must be H 3 l, that is true, H 3 l. And of j 2 pi f times e to the j 2 pi 3 fs times l times t. So if you now look this equation, what I mean, you can think of this as being an LPTV system, which is varying with the fundamental frequency of and therefore, now let us take a look.

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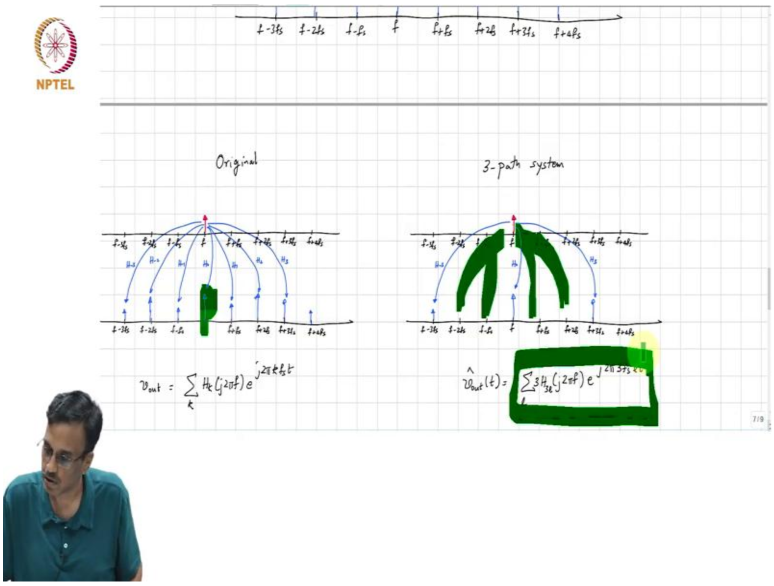

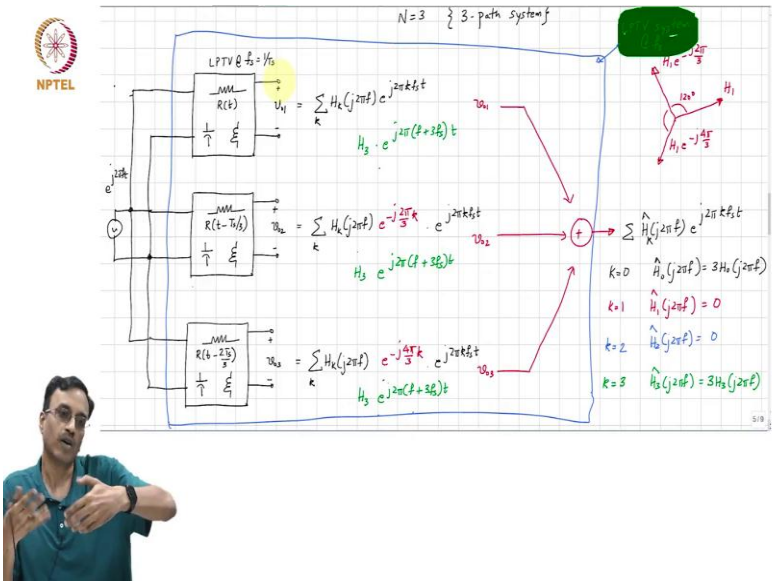

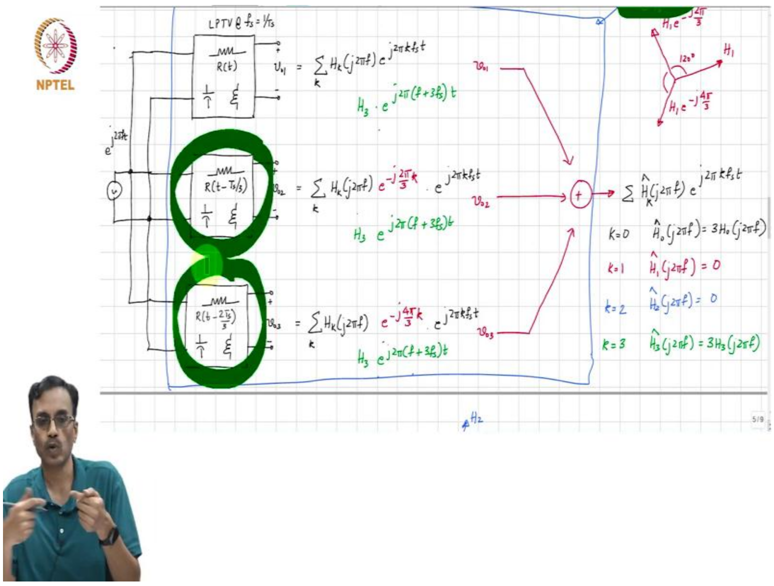

Now, if you see, in the original network, if you saw an output at a frequency f, it could be coming from any input of the form f plus k fs, so it could be f plus fs, f plus 2 fs, f plus 3 fs and so on, now what do we see? Well, it can only be coming from f plus 3 fs times l. So the number of frequencies from which the input could be coming is reduced by a factor of 3.

And another way of thinking about it, is the following. And that is that this composite system, even though technically, it is a system that is varying at fs, it appears to the external world as if it is varying at thrice the frequency. But if you stare inside the box, there is nothing here in the box, which runs at 3 fs, everything is running at fs. It is just that by delaying, by adding appropriate delays, it can and you add them all together, it basically appears as if it is running at a rate which is 3 times faster.

So I mean, an analogy is that, you know, if you have a factory, for instance, you know, the 24 hours in a day, and if you have only one shift, it is whatever the periodicity with which people come in and go out of the factory is basically once every day, it looks like 1 hertz, if you think of everyday being equal to a second.

Now, if you have three shifts, the morning shift comes in at 6 and goes home at 2, the afternoon shift comes in at 2 and goes home at 10. And the third shift basically shows up at 10, and then goes home at 6 in the morning. And if you look at the traffic pattern, you can see that it is 3 times as high as what you would get if you just had one shift. So it appears as if you know, things are running much faster than a single shift.

And of course, the output is also 3 times higher, because you are adding up the contribution of all the three shifts. And one thing that to bear in mind is that this effective cancellation of many of these paths, we have gotten rid of them, but how have we gotten rid of them? We have gotten rid of them by cancellation. So we have added three things precisely in the right magnitude and angle that when you add them together, they all become 0.

So whenever you have a system which achieves 0 by cancelation such a system is always going to be affected by mismatch. So, if for instance, these three are not exactly identical or for instance the phase difference is not exactly Ts by 3, then the addition will not be, the cancellation will not be perfect, and you will see some small residual terms there.

And then you can, you will not be justified in writing this as, thinking of this as an LPTV system that works at 3 fs it is still an LPTV system that works at fs but every you know, yeah every non multiple of 3 will have a gain, which is actually much, much smaller than what you would get when the harmonic transfer function has an order which is a multiple of 3. So, with this I will stop, we will continue tomorrow.