Introduction to Time - Varying Electrical Networks
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Lecture 44
The N-path principle

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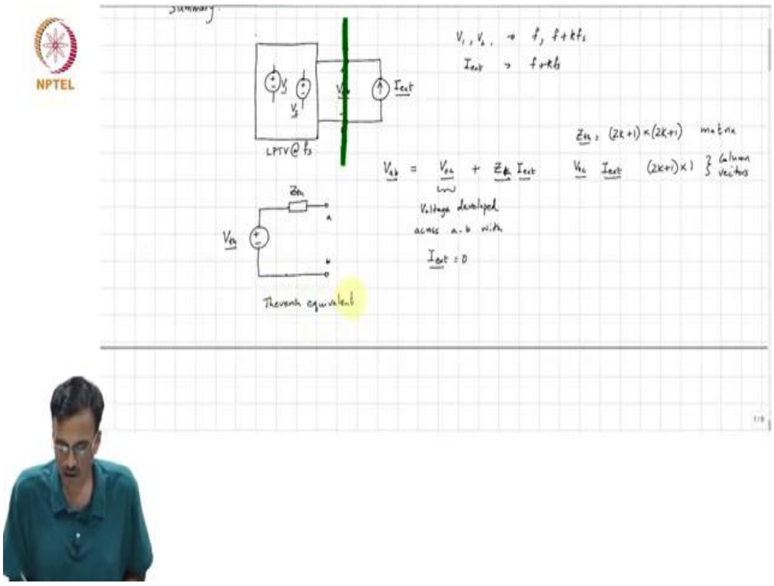

Let us quickly summarize what we learned in the last class. We learned that, in general, the impedance and admittance are not inverses of each other are not reciprocals of each other in an LPTV network. Further, we learned the Norton, the concept of Norton and the Thevenin equivalent, which I am going to summarize.

So, we have an LPTV network with multiple sources, here I am just showing voltage sources, but in general, there can be current sources inside too and what we said yesterday was that if for instance, the network is excited by an external current and the network is operating periodically at fs, the voltage across the two terminals, V external can be written as, two things to bear in mind V1, V2 etc are basically sources that have frequencies f or f plus, in general f plus k times fs, where k is an integers.

This is in direct analogy with when you have a linear time invariant system, when you do phasor analysis, you assume that all the sources are at the same frequency f and then you compute the impedance at that frequency, every impedance at that frequency and then you have a Thevenin impedance, which is at valid at that particular frequency.

Now, you have an LPTV network. So, every frequency must be of the form either f or f plus k times fs, where fs is the rate at which the network is varying and likewise, I external is also for the form f plus k s where k is some integer and the voltage across these terminals V ab. If you represent it as, if you represent all these voltages as phasers, then V ab is simply nothing but the open circuit, which is the voltage developed across a and b with I external being equal to 0.

And this is the quantity that is analogous to the terminal voltage plus Z, Z equivalent to Z Thevenin if you want, that is a matrix however, and this gets multiplied by I external Z Thevenin is a 2 k plus 1 cross 2 k plus 1 matrix and V OC and I external are 2 k plus 1 cross 1 column vectors.

Likewise, the Norton equivalent. So as far as this network here is concerned, to the left of this green line, you can replace the network by the Thevenin phasor which basically now consists of 2 k plus 1 components and has a Thevenin impedance in series and this is Zth, where this is simply a matrix which is 2 k plus 1 cross 2 k plus 1 so this is a Thevenin equivalent.

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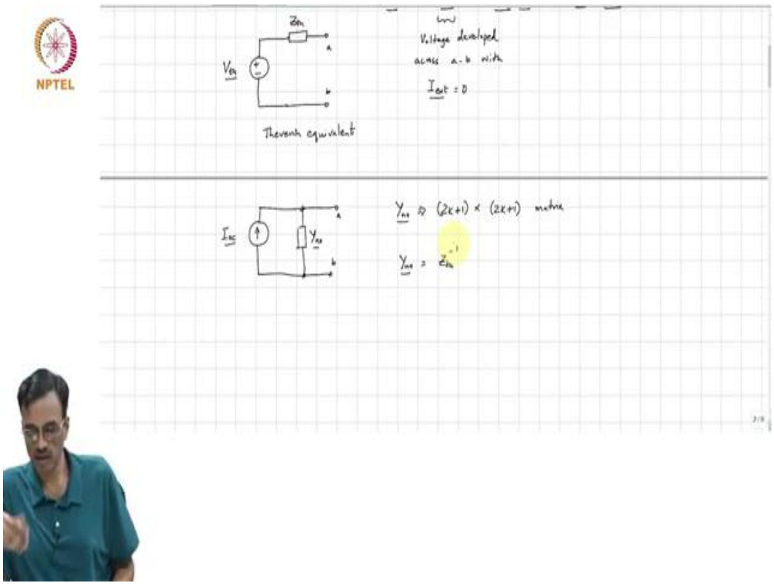

Likewise, one can have a Norton equivalent, I am not going to derive it again here, this is the short circuit current phaser and in shunt with it, you have an admittance, here n o for Norton and Y n o is also a 2 k plus 1 cross 2 k plus 1 matrix and you can write the voltage developed across a b in terms of I SC and Y n. Now, as we saw yesterday, Y n o is simply the inverse of the Thevenin equivalent. And this is exactly analogous to the Thevenin resistance or the Thevenin impedance in the linear time invariant network being the reciprocal of the Norton impedance. Very good.

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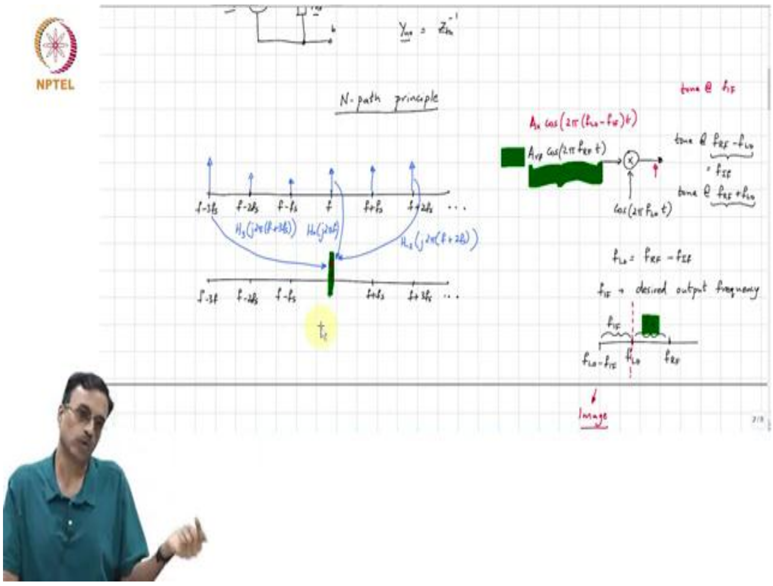

So, today let us, I would like to discuss another aspect of LPTV networks, which is used very commonly in practice. And that is what I call, that is what is called the N path principle. And is used in many areas. In RF being one example, DC DC converters is another example, where those of you who have taken a class on power management before will recognize the principle of multi phasing where you have single clock, switching frequency, but many phases of that being used to advantage in a DC DC converter.

In RF, those who have taken an RF class are no doubt familiar with the whole principle behind N-path filters. I will discuss the details later, but today, I will just go over the fundamental principle, the motivation is the following. Remember, when we had an LPTV network, let us say this is the set of input frequencies, so this is f, let us say this is f plus fs, this is f plus 2 fs and so on, and the output frequency of course also fall in the same grid.

So, let us say we look at the output, and we find that the output has got a tone at a frequency f. If the system was time invariant, then there is no confusion, we know that the input tone was also at the same frequency f, but because of the time variance, the periodically time varying nature of the network, it the, we are no longer sure at what all frequencies the input exists at, it could for instance, be coming from f with a conversion gain, which is basically H 0 of j 2 pi f.

It could for instance, be coming from another frequency. For instance, a tone at f plus 2 fs basically, will also have some contribution to the output tone at f but with a gain H minus 2 of j 2 pi f plus 2 fs. Or it could be for instance, a tone at f minus 3 fs. And a portion of that frequency could basically come here with a gain of H plus 3 of j 2 pi f plus 3 fs and so on and so forth.

I mean, you know, in other words, when you see an LPTV network, and if you find a tone at the output at a certain frequency f, all that you can say is that the input must have been at a frequency, which is of the form f plus k times fs. But you cannot say which K, the input was at. And clearly this is a problem. And why is this a problem in practice? You know, let me give you an example. For instance, let us say you have a radio. And, you know, as we have discussed before, radio basically takes a signal which is at a high frequency and demodulates it to a lower frequency.

So, for example, let us say this is a frequency at let us say this cos 2 pi, f l o times t. And let us say we are interested in demodulating the output, when the input to an output frequency, which I will call f I F so, the input frequency, let us say this is A rf cos of 2 pi f rf times t, and if I choose the l o to be, say, for example, f r f minus f I F, where I f is the desired output frequency.

So as you can see, if you multiply cos tone at rf, with a tone at l o, there are two possibilities that can happen at the output, one is basically you would get a tone at f rf minus f l o. And this is precisely equal to the I f frequency this is often called the intermediate frequency. And there is also a tone at f rf plus a f l o and this frequency you know is going to be a very high frequency and can easily be removed.

So, the fact that you put in a single input tone and you get two tones at the output is not really that much a practical problem, because the second tone is so high in frequency that it can be removed. However, there is also a rogue tone which basically, let us call this A x cos 2 pi what frequency would a tone also down convert or get convert? I mean, is there a possibility that you have another tone which can appear at f I F?

Student: Minus f rf plus (()) (13:27)

Professor: So, if you have f rf sorry, f rf, f l o is basically f rf minus f I F, if you have f rf plus f I F hold on, I think made an error. For example, so this is f l o and this is f rf and this is f I F. Now, if you had a tone which is f l o minus f I F, so that, sorry this, if we had a tone which was f I F below f l o which is f l o minus f I F.

So, for example, so, you have a rogue tone for instance, which is at f l o minus f I F times t then this rogue tone will basically also multiply with the local oscillator frequency and result in a tone at f I F, and it will also result in a tone at 2 f l o minus a f I F, which again is a very large frequency which is eliminated, which can be easily eliminated.

But the key point is that, if you look at the output here at the output of the mixer, and you see a tone at a f I F. Now, one is not sure whether the tone is coming because of an input at f rf or at a frequency which is f I F below the local oscillator frequency. So, there is a you know, term for this, if you look at this, you can see that the rogue tone is symmetric with respect to the f l o.

So, you can, there is mirror symmetry about the f l o and this is often what is called the image tone, and the phenomenon or the problem of image is primarily because of the linear periodically time varying nature of the multiplier. Now, so that is just, you know, an example. But as you can imagine, there are there are probably a whole lot of other systems where this can become problematic.

Another example I can think of straight away is sampling, sampling, as we have already seen is also a linear periodically time varying operation. And if you have, I mean, there are multiple input frequencies that can result in the same output frequency. For instance, if you sample an input tone at fs, then it will appear like dc. Or if you sample any tone at integer multiples of fs, after sampling, it will appear like dc. And therefore, you know, if you do not use an anti-aliased filter upfront, you are not sure whether the output dc is because of the true dc that is that is coming in or it is because of tones at multiples of fs.

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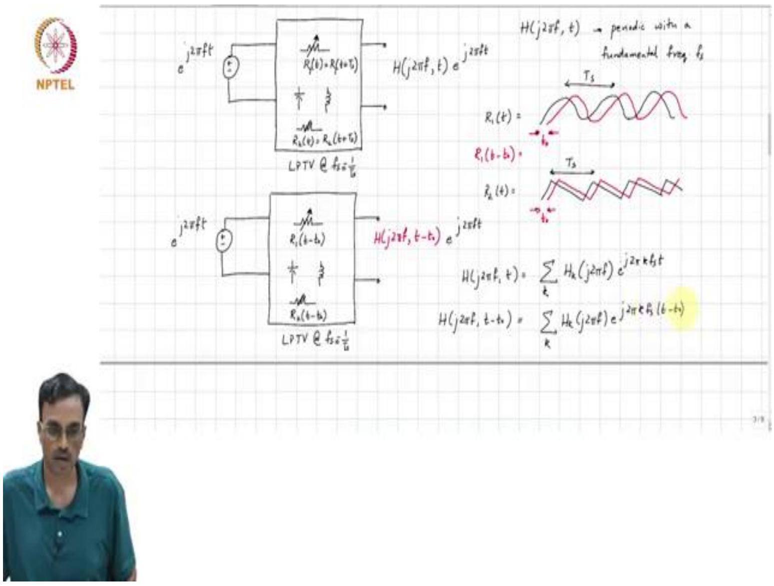

So the N-path principle is one way of addressing this problem, not completely, but to a sufficiently large extent that system design becomes easier. And I will now go over having seen the motivation, let us go the principle, and then you will see where this is useful and how this fits in and addresses this problem to a good extent, the idea is the following. And again, I am going to resort to your intuition.

So, let us say you have an LPTV network, let us say you have resistors, inductors, capacitors, what have you and the time variance, the periodic time variance is happening, because all the resistors are varying with a period Ts. Now, as we have seen earlier, if we excite the LPTV network with a complex exponential e to the j 2 pi ft, then the output of the LPTV network is has a gain which is periodic with respect to time. It is, let us call the gain H of j 2 pi f comma t, times e to the j 2 pi ft,.

And we have seen that H of j 2 pi ft is going to be periodic with the fundamental frequency fs and the Fourier series coefficients of this H of j 2 pi f comma t is periodic with a one with frequency of fs. And if you decompose it into a Fourier series, you get the harmonic transfer functions. Now, the question I am going to ask you is the following. Now, let us say I took the same network, so let us call this R1 and let us say there was another resistor which was an any number of other resistors, which basically are also periodically varying with time.

Now, what I am going to do is if I change the time variation of all these resistors, so in other words, the only time varying components in the network are the resistors. Now, if I move, if I change these resistors as per the following, in other words, let us say R1 of t was some periodic function like this, R1 of t minus t naught is basically same periodic function which is delayed by t.

Now, recall that it is similarly, R2 of t minus t naught, R2 of t can be some other periodic function of time, except that it has the same period as, so this is the time period Ts, this is the same time period Ts. So, in the first circuit R1 and R2 are like the ones shown in black, in the second circuit R2 is process is varied like this and the delay is the same T naught that exists for R1 of t.

So, now the question is what happens to the gain experienced by the sinusoid, all the varying waveforms are simply delayed. So, what comment can we make about, so if all the variation of all the resistors is also delayed with respect to time, it is intuitively obvious that the gain also is getting delayed by exactly the same one. In other words, what you would see here is H of j 2 pi f comma t minus ts, very good.

So, if we expanded this as a Fourier series you would basically get H k of j 2 pi f, e to the j 2 pi f plus, sorry e to the j 2 pi times k fs times t. So, therefore, if we simply delay this waveform what would you expect? Very simple, just simply replace t with t minus t naught H sub k of j 2 pi f e to the j 2 pi k fs times t minus t naught.

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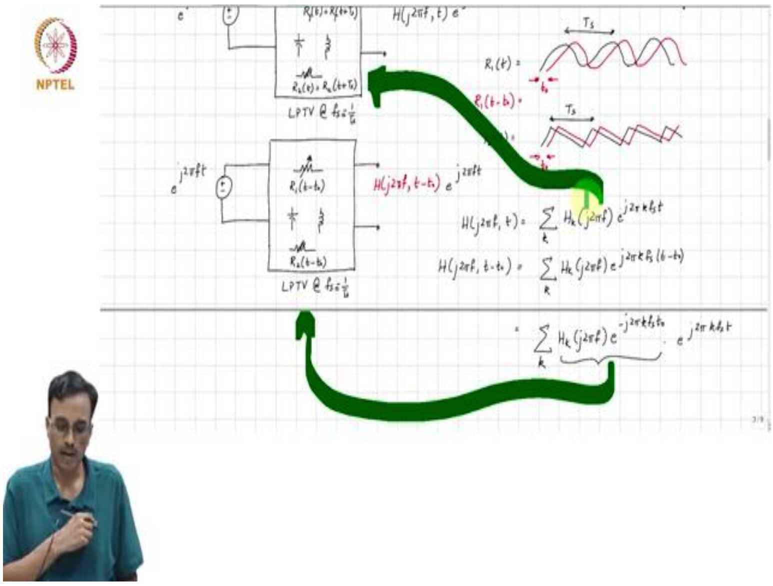

Which therefore, can be written as H sub k j 2 pi f e to the minus j 2 pi k fs times t naught times e to the j 2 pi k fs times t. So, this therefore represents the kth harmonic transfer function of this network, whereas, this represents the kth harmonic transfer function of the network on top. So, clearly it makes sense that the harmonic transfer functions must be related because all we have done is simply shifted the time variance in, of the time varying components.

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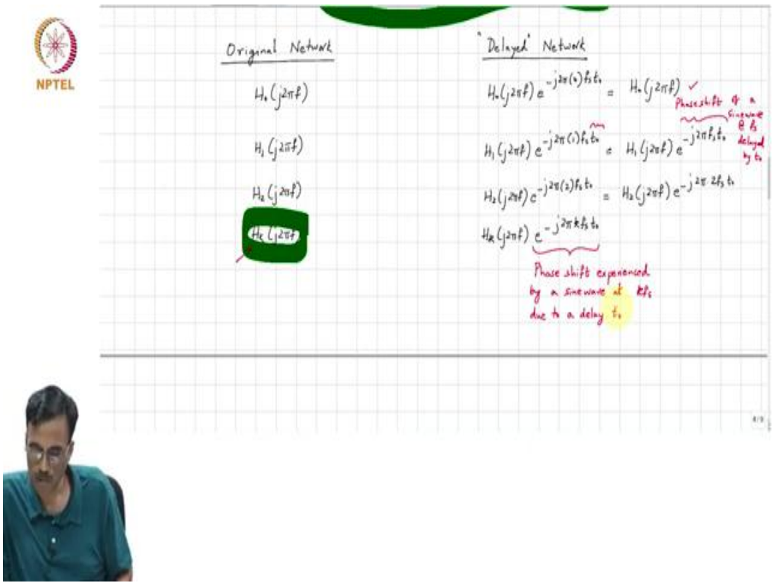

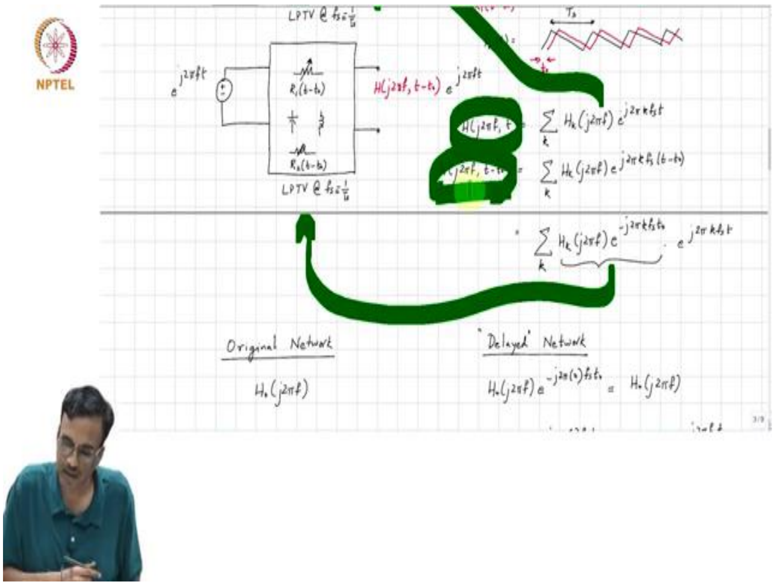

So, let us write down the harmonic transfer functions. So, this is the original network, the zeroth order harmonic transfer function was H 0 of j 2 pi f with the delayed network what do we see is nothing but H 0 of j 2 pi f times e to the minus j 2 pi k 0 times fs times t naught which is therefore, the same as H naught of j 2 pi. Why does this make intuitive sense?

Student: (())(27:58)

Professor: Yeah, very good. Remember that H 0 is simply the dc value of the Fourier series, the gain experienced by a sinusoid in the second network in the within quotes to the delayed network is has the same waveform except that it is delayed in time, if you delay a waveform in time the dc value cannot change and therefore, it makes sense that the zeroth order component or the zeroth order harmonic transfer function does not change.

Now, let us look at H1 of j 2 pi f in the original network that remember relates or quantifies the gain experienced by a sine wave at frequency f to an output at a frequency f plus fs. So, in the delayed network, it will be H1 of j 2 pi f times e to the minus j 2 pi k is 1 fs times t naught, so this is nothing but H1 of j 2 pi f times e to the minus j 2 pi fs times t naught.

Now, let us do H the second order harmonic transfer function H sub 2 j 2 pi f that is simply H sub 2 of j 2 pi f times e to the minus j 2 pi times 2 times fs times t naught and that basically is H2 of j 2 pi f times e to the minus j 2 pi into 2 fs times t naught or minus j 4 pi and therefore, H sub k of j 2 pi f will basically translate to H sub k of j 2 pi f e to the minus j 2 pi times k fs times t naught.

And so, why does this make intuitive sense? Well, remember what does this H sub k what is that, it is simply the if you expand this gain, this periodic gain function, if you expand it in a Fourier series, this H sub k quantifies the strength of the kth harmonic. Now, if you delay the waveform, the waveform this H of j 2 pi f t is a waveform it is composed of sinusoids at frequency which are multiples of fs.

Now, this lower waveform here, this H of j 2 pi f comma of t minus t naught is simply a delayed version of that original periodic waveform. Since, the original periodic waveform is a sum of sinusoids the delayed version is simply a sum of delayed sinusoids. So, each sinusoid, each harmonic of that Fourier sum is delayed by the same amount of time t naught.

So, if you delay a DC quantity by t naught, it does not change, and which is why it makes sense that the H sub 0 remains the same in both cases, H sub 1 is basically the quantifies the gain of the sinusoid at fs, if you delay the sinusoid by fs, if you delay a sinusoid at fs by a time delay t naught, how much will be the phase shift, it will be 2 pi times fs times t naught. So, this is the phase shift of a sine wave at fs delayed by t naught.

Now, if you have the kth order term here that quantifies the strength of k times fs but the delay that is also been delayed by the same amount of time t naught and therefore, this is the phase shift experienced by, very good, phase shift experienced by sine wave at K fs, due to a delay t naught, (()) (33:47) make sense.