Introduction to Time - Varying Electrical Networks
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Lecture 43
Thevenin and Norton's Theorems for LPTV Networks

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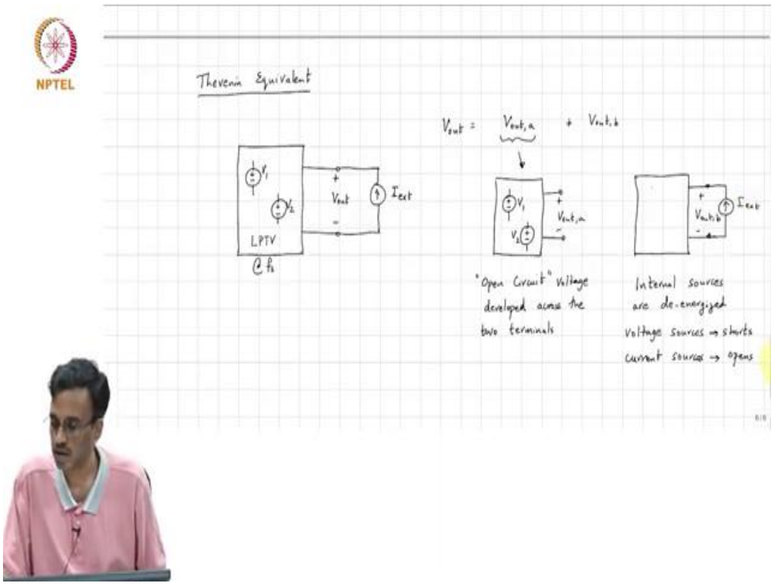

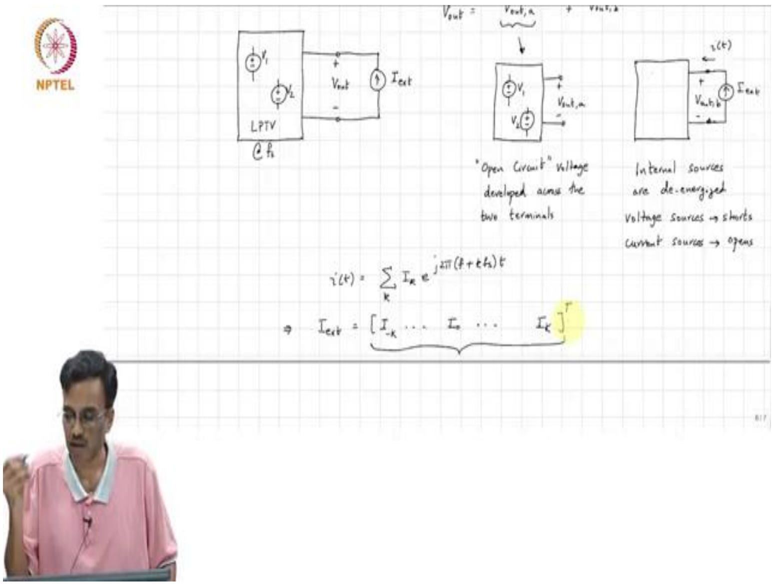

So, the next thing I would like to talk about is you know other theorems that we are familiar with, namely the Thevenin equivalent and the Norton's equivalent. The principle is very similar for a Norton equivalent. So, the idea is the following, we have an LPTV network and this varies at f s, there are a whole lot of internal sources, let us call this V 1 and then another one V 2 and so on.

And we have the two terminals and we have an external current source say I external. So, by linearity therefore, the output voltage here is simply given by, we can use superposition, the circuit is still is still linear mind you, so even though it is time vary, so the voltage V out is can be found by using superposition. So, V out therefore, is the sum of two parts.

So, let us call that V out a and V out b, V out a is the voltage developed when the external network, the external current is 0. So, this is therefore the open circuit voltage developed across the two terminals and V out b is, this is I external where all the internal sources are de-energized that basically means voltage sources are replaced by shorts and current sources replace by open.

So, clearly if I external is can be written in phasor notation as sigma k I sub k e to the j 2 pi f plus k f s times t or rather external, notation here, this I external is the phasor vector I of t is this and therefore, I external which is the phasor which is applied and the output is you know again I minus k blah blah blah I 0 dot dot dot I sub capital K. So, all transpose so this is the vector that carries the information of each of the strength of each of those tones and the voltage V out b in phasor notation is simply nothing but a matrix.

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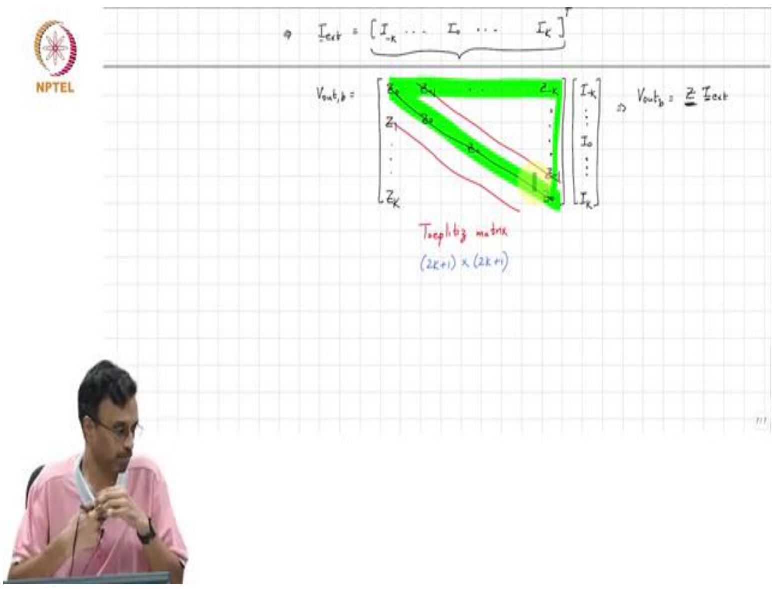

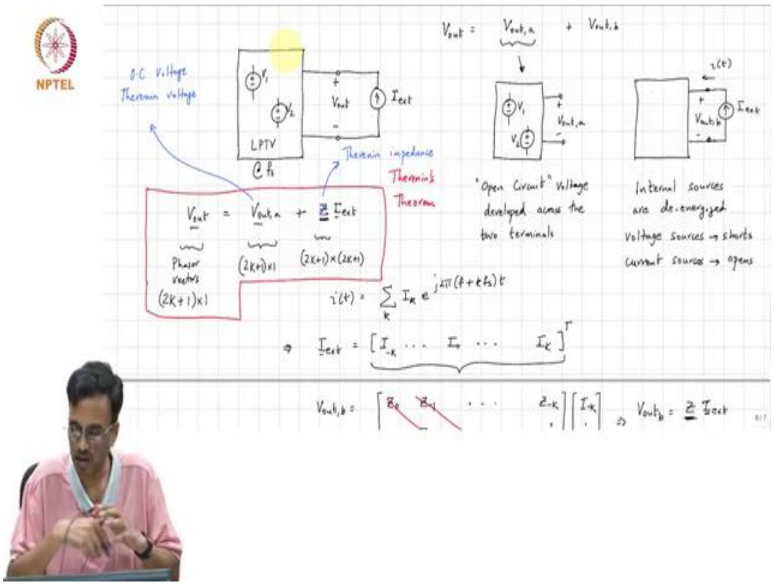

So, this is I minus k, I 0, I plus k and this would be Z of 0, this would be Z of minus 1 blah blah blah Z of minus capital K, Z 1, Z 0 dot dot dot, Z plus K, this will be Z 0, this would be Z minus 1. So, as you can see that this diagonal will be Z naught, this is Z minus 1. So, this is again what is called a Toeplitz matrix where all these elements are identical and you can simply write this relationship as simply being the V out b phaser is nothing but the Z, the impedance matrix times I external.

So, and V out a of course is simply the, I mean assuming the voltages V 1 and V 2 are at frequency f or at frequency f plus some k times f s, V out a is going to be simply can be represented by its phasor vector. So, therefore, V out in this general case when it is excited by both internal and external sources the V out phasor can be written as V out a, remember these are phasor vectors and I have size 2 K plus 1 cross 1, these are all column vectors and likewise here plus Z times I external, this is 2 K plus 1 cross 2 K plus 1.

So, as you can see this is very, looks very familiar. This is the equivalent of Thevenin's theorem and what this is basically saying is that (the) if you have a big box with multiple sources inside, and you are only interested in the terminal behaviour of this box, one need not worry about all the details about how the internal elements are connected inside. All that one needs to do is be able to find the open circuit voltage, that is, this is the open circuit voltage or the Thevenin voltage.

And this is the Thevenin impedance, which is found by de-energizing all the sources inside and applying an external current and finding this conversion, now the impedance is no longer a single number, it is a matrix because there are frequency convergence, any input tone at f will result in tone Z f plus k times f s. So while the equivalent is a lot more complicated than it was for a time invariant network.

Well it at least simplifies this time invariant system when you are dealing with LPTV networks. So in other words, you do not have to worry about all the gory details of what is happening inside the box as long as you know the open circuit voltage phasor that is developed across the two terminals and the impedance matrix you know, Z as we have seen here, we should be able to find the (terminal) we should be able to understand the terminal behaviour of this big network by looking simply at these two quantities.

It is admittedly much more difficult to find the Thevenin impedance in the time varying case, than in the time invariant case because, if you assume that the matrix Z is truncated after capital K terms, this is a 2 K plus 1 cross 2 K plus 1 matrix and even if you are going from symmetries, you are now basically looking at least finding the upper half of this big matrix. So, unlike in the time invariant case, where you know, typically finding the open circuit, the Thevenin impedances is very straightforward. It is not so straightforward in the time varying case.

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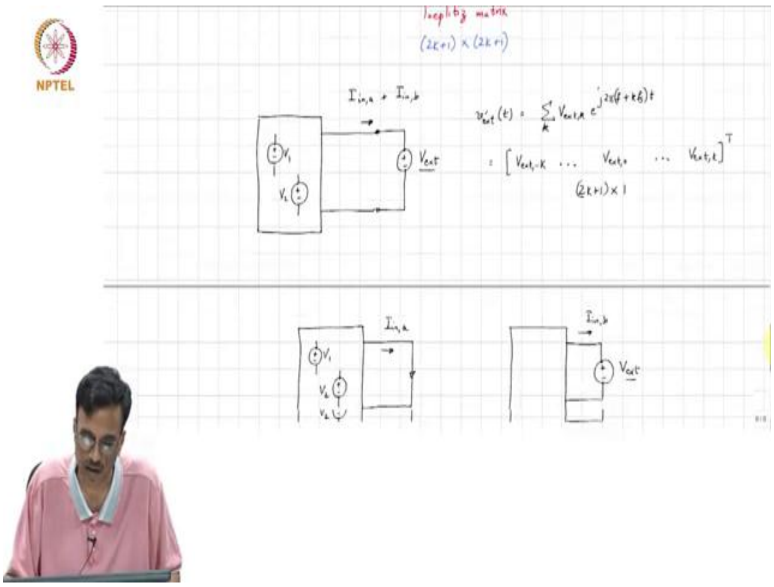

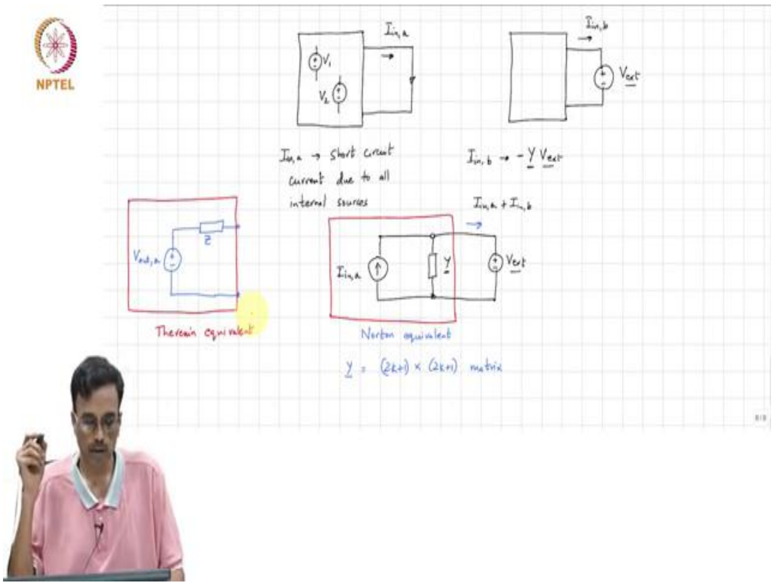

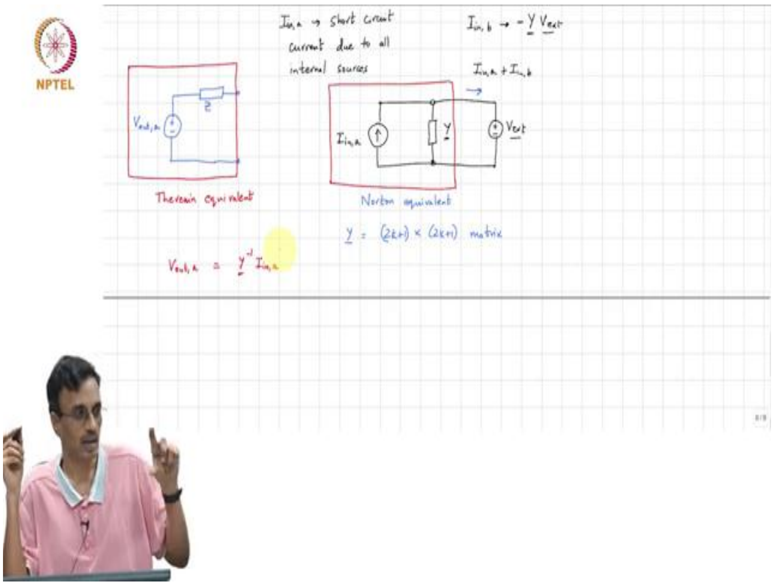

Now, likewise, we can also derive the Norton equivalent where you have a big network. I am just showing voltages here, but the network can have current sources also and we have, we were interested in the terminal behaviour here. Last time we said I mean, remember the reason why we put a current source there was, as far as the external world is concerned, you can model it by an equivalent current source or an equivalent voltage source.

What we are going to do is simply now we are going to model this as an equal and voltage source. It is now going to be a voltage phasor and V external of t is simply going to be sigma over k V external of k e to the j 2 pi f plus k f s times t and that is V external of minus k, V external of 0, V external transpose, so this is nothing but a 2 K plus 1 cross 1 column vector and the as far as the current that is going in to the network is concerned well, you can think of it as I in a plus I in b, I in a is well, you use superposition again and maybe I think we should basically do the conventional thing where I do that so, I in a plus I in b, so this is basically V 2.

And this current is I in a and we have another current source V external, all the internal sources are 0, this is I in b and so, I in is the short circuit current due to all internal sources and I in b is the current due to V external and that as you can see, it can be written just like how we did with the Thevenin equivalent. Well, you now have, this is now I in b is minus Y times V external.

So, you can therefore represent the network by this matrix, this is Y, this is I in a and this is V external and this current is I in a plus I in. So, this is the Norton equivalent, sorry, sorry, sorry. Oh, yes, thank you. Indeed. Alright so this is the Norton equivalent with Y is a 2 K plus 1 cross 2 K plus 1 matrix.

And just to put everything in one place, you can as well think of it as Thevenin equivalent V out a and what can we say about the relationship between V out a, I in a, Y and Z. Well, if you leave the box open, the voltage developed across the box is nothing, but is V out a and that is nothing but Y inverse times I in a and why is it Y inverse times Y in a?

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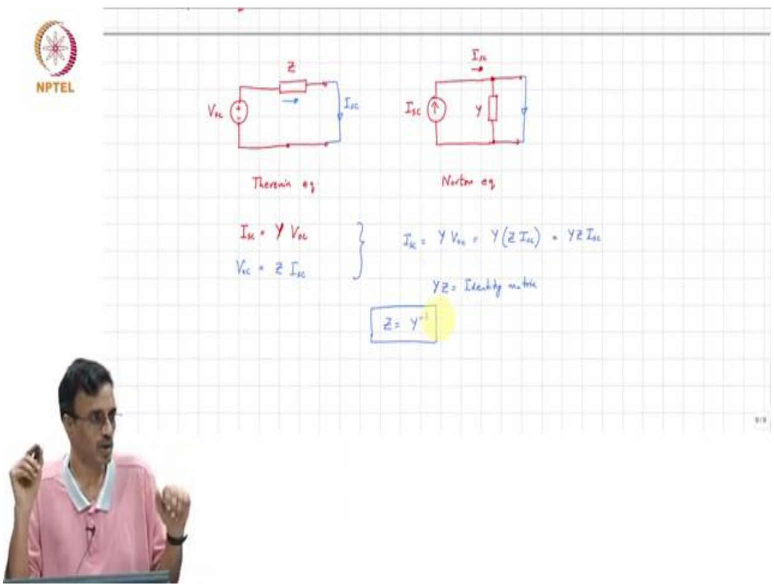

Well, the voltage across the, let us say, in other words, let us keep the circuit open. So, we basically have both are same, both represent the same network. So, however you represent the network, this is Z, this is the Thevenin equivalent and you have the Norton equivalent, which is I am simply going to call this V OC just to be I call this I SC, the short circuit current and this is Y.

And these 2 of course, must since they represent the same network, you must have the same open circuit voltage developed and the voltage open circuit voltage is V OC here. Now, this must also therefore be V OC and so, if the voltage is V, then the current that is being drawn by this admittance is nothing, but V OC times Y, the Y matrix times V OC that should be give I SC. The relationship is very similar to what one sees with a time invariant network, where there the ratio of the short circuit current is the short circuit admittance multiplied by the open circuit voltage.

Alternatively, you can you can equate the currents when you short the output, and therefore, what do you see well, this current is going to be V OC sorry, the current is going to be I SC because this current is I SC and

Student: (())(24:38)

Professor: yes, I SC times the Z matrix should give us V OC does it makes sense? So therefore, you put these two together. I SC must be Y times V OC, which happens to be Y times Z times I SC. Therefore must be Y times Z times I SC.

So, what can we conclude Y Z is the identity matrix and therefore Z is the inverse of Y and this again is very familiar, because in a time invariant network, we know that the Thevenin resistance and the Norton admittance are simply reciprocals of each other Now, Z and Y are matrices and 1 is the inverse of the

Student: (())(26:01).

Professor: Yes. Alright?