Introduction to Time - Varying Electrical Networks
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Lecture - 4
Intuition behind Tellegen's Theorem

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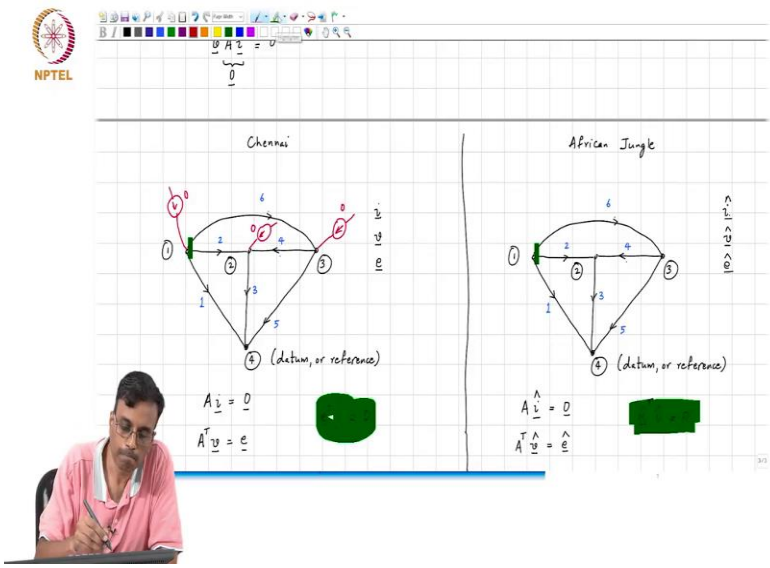

Now, let us see why this makes intuitive sense. Now, let us say, I add, here is a network, which has some node voltages and node currents and branch currents. Now, let us say I add a current source into this node of value 0.

Likewise, I am going to add another current source into that node with a current 0, and into this node with a current 0. What comment can we make about the node voltages and the branch voltages when we do this? Well, we are adding 0 currents into every node, so nothing will happen to the note potentials, and therefore, nothing will happen to the, to the branch voltages and the branch currents.

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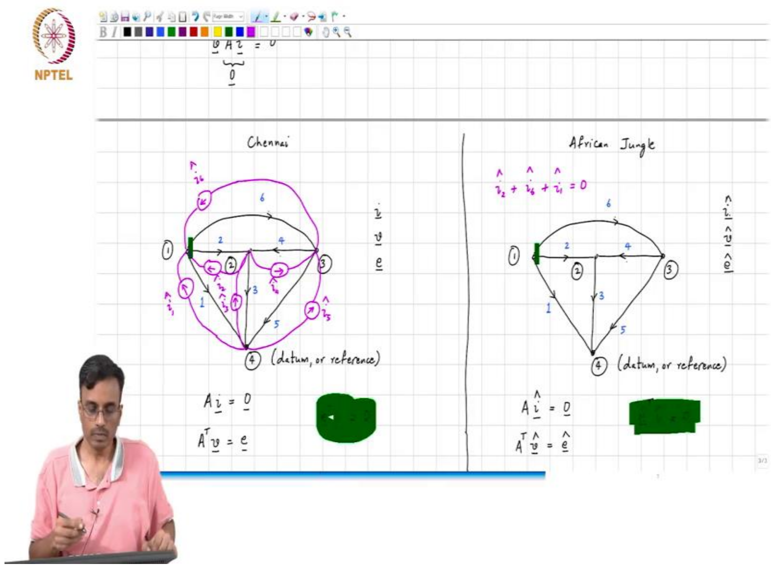

Now, what I am going to do is derive this 0 in a, in a rather special way. What I am going to do is recognize that 0 can be written as i2 hat plus i6 hat plus i1 hat is equal to 0. So what I am going to do is basically add a current here which is i6 hat, a current here which is i1 hat, and here, I am going to add a current which is i2 hat.

Likewise, in other words, I am going to replace, put in parallel with every branch in this network, a current in the opposite direction, and whose value is, where do I, I mean where am I getting these currents from? These hated currents basically are pertaining to the second network.

And please note that I am putting the current across every branch but these currents are not chosen in an arbitrary way, they are chosen in a, they are chosen in a special way. What is so special about the way I have chosen these, these currents in pink that I have added here? What is so special about the way I have chosen those currents?

Student: (())(03:30)

Professor: Pardon.

Student: (())(03:33)

Professor: Yeah, so basically all those currents, you know, basically satisfy Kirchhoff's current. So even though I have added these new currents in pink, that is not, what I have in effect done is added a current. What is the net current I have added at each node? The current I have added at each notice is 0. So nothing has happened to the branch voltages or the branch currents.

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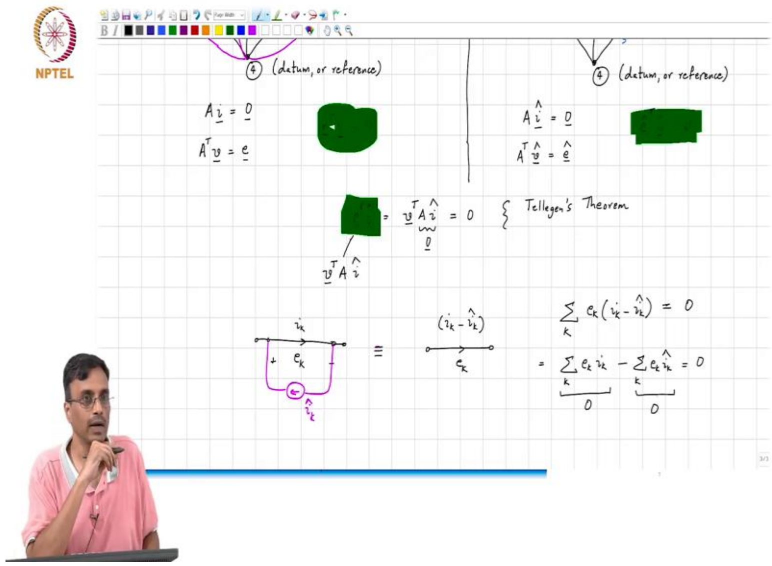

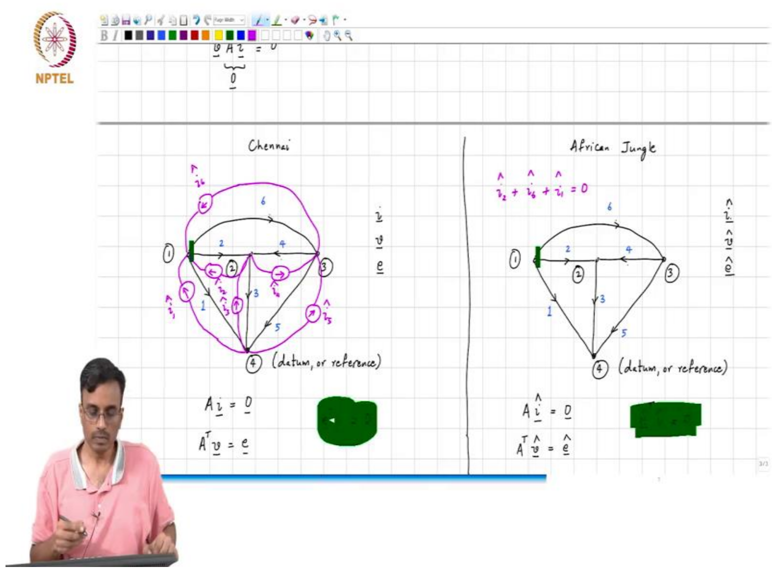

In other words, I have replaced. In the original network, if there was a branch, let us call this some ek and ik is the current; ek is the voltage. What we have done is in effect, in parallel with every branch, we have put in Ik. Now, this network is as legitimate as any other network. So energy conservation must be valid for, I am sorry, power conservation must be valid for this network too.

And, therefore, what is, what common can you think, make about this branch? This is equivalent to, if you think of this as a branch, what is the voltage across this branch? ek. What is the current to this branch? ik minus ik hat.

So what comment can we make about the power dissipation, instantaneous power in the entire network is simply k times, sum over k ek times ik minus ik hat. And so this, therefore, what must this be, what must this be? Well, energy conservation says that this must be 0. Well, this is nothing but, must be 0.

What do we know about this? This is 0 because that is energy conservation with respect to the original network. And, therefore, it must follow that this chap is also equal to 0.

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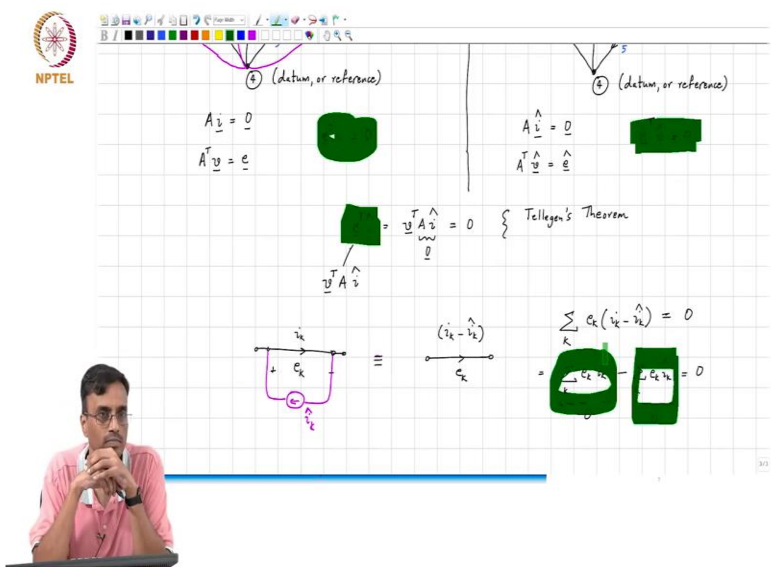

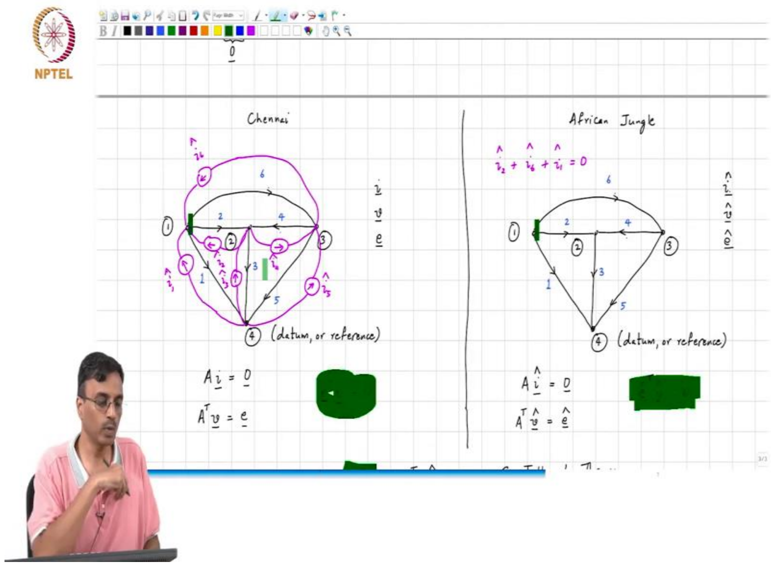

Of course, if I just showed you just this result, it would be quite puzzling as to how you multiply the voltages in one network and currents in some other network, and how the sum magically turns out to be, turns out to be 0.

I mean, if you go, think about it this way, you see that it is not that surprising after all because energy conservation must hold for this network and we already know that this must be 0. So the other term must also be 0.

So Tellegen's Theorem is basically a statement of instantaneous power conservation and it is physically appealing because you know that you know, at any instant of time, if the total power consumed in some branches is not exactly equal to the power generated in other branches, then you know, if that were true then we would not all be sitting here we would be selling power. There is a way of making more energy than you dissipate and therefore, so that is the, all right?

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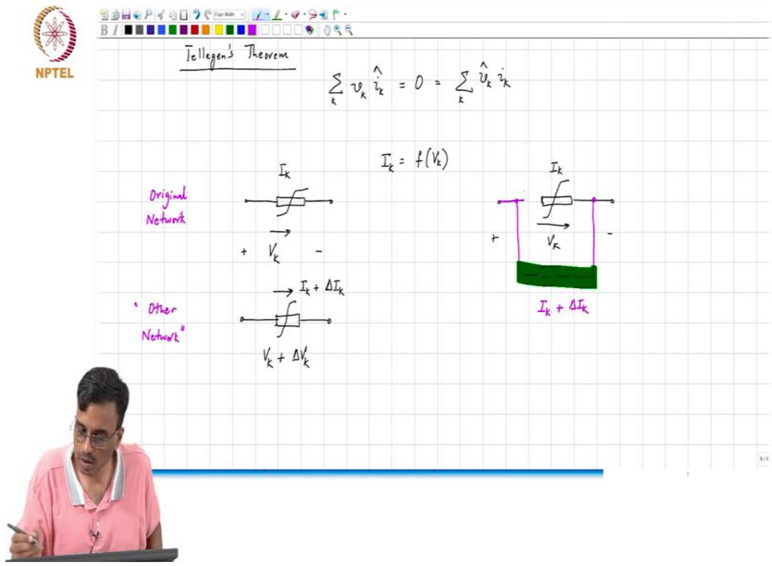

Now, it turns out the Tellegan's Theorem has you know, a whole bunch of interesting applications and let me talk about a couple of them. Tellegan's Theorem can be applied to two networks of having the same graph, which basically means that there are a whole bunch of obvious things implied here.

It could be applied, the second network could be the same as the first network. The second network could be the same as the first network but at a different time. The second network would be completely different from the first network, as long as it has the same skeleton or the same graph.

All that, this is saying is that you know, vk, some or all branches of vk times ik hat is equal to 0, which is the same as the sum over all branches of vk hat times ik. And one example, all of you have seen, taken an analog circuits' class and where all the network elements are nonlinear. And what do you call, we know that analyzing nonlinear networks is difficult, so we resort to, what do we do?

Student: (())(09:36)

Professor: We linearize the network and work with small signal equivalents. And the underlying basis for small signal equivalents are the following. I mean, you have some, I have some sources, and you change the sources by a small amount. And then you know, you only write KCL and KVL for the incremental quantities.

And the basis for that is again, you can think of it a principle that we used while deriving Tellegan’s Theorem. The original network for instance, let us say you had some nonlinear element where the branch current and the branch voltage are related in some really nonlinear fashion.

And remember that Kirchhoff's laws are not going to be or the branch voltages and the branch currents are not going to change if you, if you place in parallel with every branch an arbitrary current ik hat, where this ik hat is derived from another network.

That other network is where, this original network. The other network is where a particular source has been changed by a, by some amount. So if you go and change, say, a source voltage by some amount, then all the branch voltages and branch currents are going to be, are going to get changed.

And, therefore, this current is going to be Vk plus some delta Vk, and the current is going to be, you take a network and change one source, what happens? All the branch currents and the branch voltages change. So, therefore, this current is going to be Ik plus some delta Ik. Remember that this delta Vk's and delta Ik's need not necessarily be small at this point; they change. That changes is that capital delta.

So what I am going to do is going to replace, those who have taken an analog circuits' class have seen this before. What I am going to do, therefore, is in the original network, I am going to add a branch current in parallel, where I am going to call this Ik plus delta Ik. It makes sense people?

Now, if I do this for every branch, what comment can you make about the branch voltages, and the branch voltages and branch currents in the original network? What comment can we make? By making this construction with original network, what is the net current I have added at every node?

Student: (())(13:50)

Professor: Pardon.

Student: (())(13:52)

Professor: In parallel with every branch I have added Ik plus delta Ik, so what is the net current I have added every node in the network? It is 0. Does that make sense? Because all these Ik plus delta Iks must satisfy Kirchhoff's current law.

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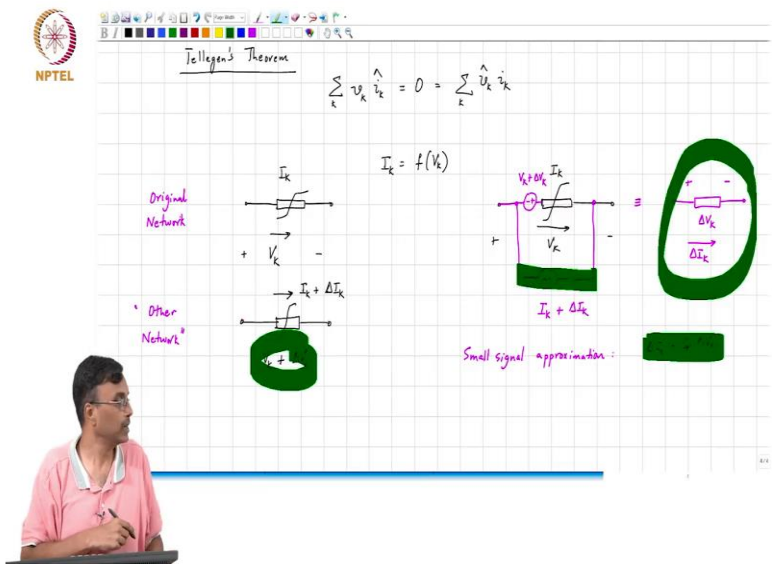

And likewise, what I am going to do is also replace, I am going to put in series with every node; every branch, I am going to put a voltage which is in the opposite direction. I am going to put Vk plus delta Vk. What comment can I make about the, what will happen if I put a voltage source of Vk plus delta Vk?

Remember, Vk plus delta Vk corresponds to the branch voltage in that branch where I have changed a source. So what if I do this for every branch, what comment can we make about the net voltage that we have added in every loop?

Well, we know that the branch voltages you know, are not some arbitrary branch voltages. They all satisfy this Vk plus delta Vk, therefore, satisfy KVL. And, therefore, the effect of adding all these is not, is that it does not change the loop currents. So, therefore, if the original network satisfies KCL and KVL, it is apparent that this must also. If I replace every branch by this composite branch, it must also satisfy KCL and KVL.

So now, what is the net voltage across this branch? The voltage across this branch is delta Vk, and the current to the branch is delta Ik. I have flipped the picture on the right, so that delta Ik flows in that direction but otherwise everything is the same. Does it make sense people? Right.

So what is the moral of the story? This is telling us that you know, if you have a nonlinear network and you change something, well, all the branch voltages and the, all the branch currents will change.

And this is telling you that the change in the branch voltage and the change in the branch current, those changes also, if you form a network where you have network elements whose branch voltages and branch currents or delta Vk and delta Ik, then it will also follow KCL and KVL. Now, the next assumption.

So this is basically, it is true for arbitrary changes in the sources, so I would like to reiterate that this delta Vk and delta Ik need not be small. The small signal approximation comes in when you assume that the changes are so small that they, you can relate the delta Ik to the delta Vk in a linear equation.

So, therefore, you can see that the constructs that we have used to derive Tellegan's Theorem in an alternative way also kind of throw light on why the small signal approximation is grounded in firm principles, it is, the only approximation is this part. This is always true.

So the changes in the voltages and currents basically still obey KCL, KVL. The moment you relate the change in voltage to the change in current throug a linear equation, that is when you have made an approximation.