Introduction to Time–Varying Electrical Networks
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Lecture 39
Analysis of an example LPTV Network- part 1

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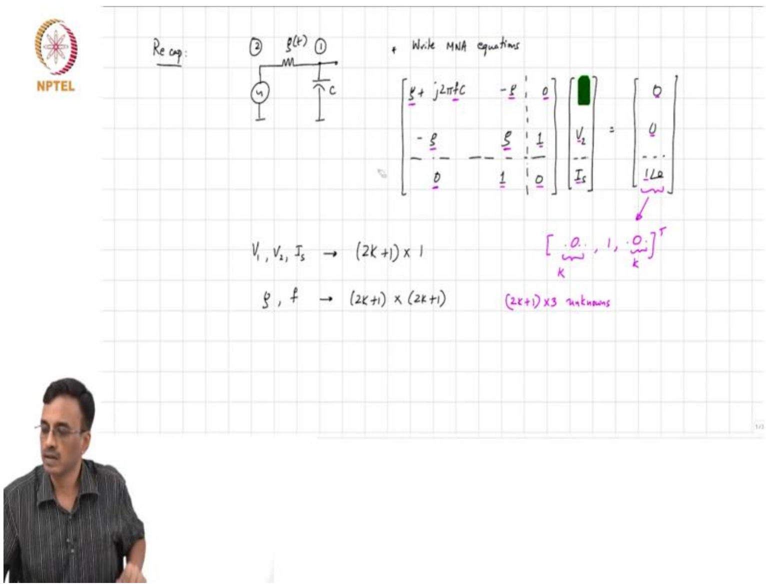

A quick recap of what we were doing in the last class, and what we said was we are already familiar with the solving for the network, when the network is linear and time invariant. And what we do is basically write the MNA equations, and in this simple case you would have you want to write the whole thing down without.

So, that is g, g plus j 2 pi f c, minus g, minus g, g; and we have this is 0 and V2. So, that is this guy here, so that is V1, V2, capital Is; and that basically equals to 0, 0. And the moment we made the conductance time varying, if this was g of t; then the process remains pretty much the same, except that what is the difference now.

Basically we replace, so the unknowns now become column vectors, and so V1, V2, Is et-cetera are 1 sorry, 2 capital K plus 1 cross 1 matrices. And g, f et-cetera become 2K plus 1 cross 2K plus 1 matrices. And the zeros and ones on the left and right side are square or column vectors of a appropriate order. And so all that we need to do in the MNA formulation is to rewrite replace each of these guys with with the appropriate matrices. And so all these are now matrices when I write this, this of course is what means is that we have we have K zeros 1, another K zeros; and while this might be difficult to solve by hand.

It is definitely a pretty straightforward go and solve by computer and in one shot. Just like how we get all the transfer functions that there are corresponding to all the branch voltages, and the currents in the voltage sources. Now, when you solve these in this particular example, as you can see there are 2K plus 1 times, 3 unknowns. So, they correspond to not only the voltages, node voltages V1 and V2. They also correspond to and the unknown current in the source voltage source. But, more importantly each with each unknown, you are actually getting all the harmonic transfer functions in one shot.

So, it is while true you are doing a lot more work, which inevitable because the conductance is varying with time. It is also (())(05:10) able to get all those harmonic transfer functions in one shot. Now, as it keep, changing the input frequency, you have to be solve solve these the set of equations for every frequency point out there; and therefore get the frequency response. Now, let I think it is a high time we waited an example.

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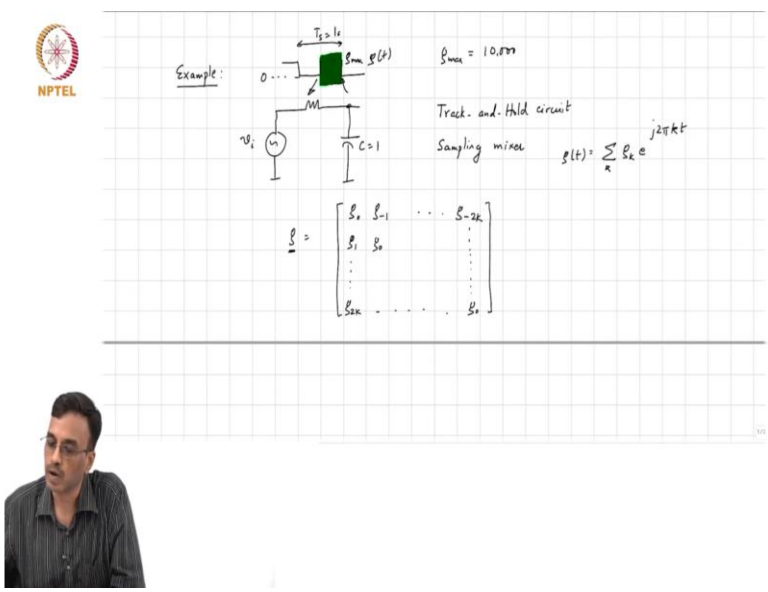

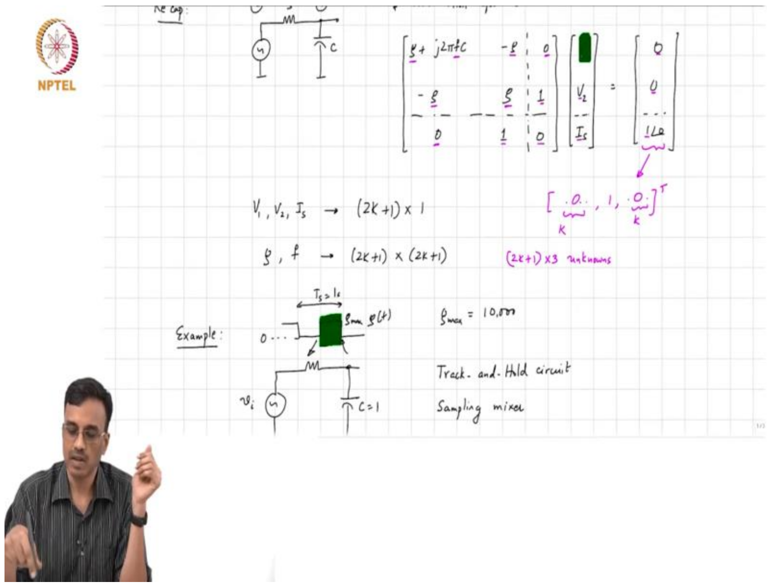

So, again let us do something that you… we should be able to understand the result. So, let us assume that this conductance is varying periodically, from the conductance is 0 for half the time period. And is consists 1 farad, Ts is 1 second and conductance is some very large number; let us call that let us call that g max; and g max happens to be some very large number. So, for all practical purposes therefore how does this this equivalent to?

Student: (())(06:57)

Professor: It is basically not strictly a sample and hold, so what is called track and hold; because during this period, when the switch when the conductance is very large. When I say very large, remember that the conductance is say 10000; and c is 1. So, the Rc time constant is c by g, which is 10 power minus 4 seconds. And as you can see that is very very small compared to half the clock period; and therefore, the output voltage will track, when the conductance is large. The output voltage will pretty much track the input sinusoid almost instantly; and when g is 0, what do you expect?

The conductance is 0, so it an open switch. So, this is an example of what is called a track and hold circuit, and sometimes they are of guys call this sampling mixer, for obvious reasons. And the effect in sampling is instant is the moment, the conductance goes from high value to a low value. So, as usual you can, you can write the MNA stamps of all this; and set up a bunch of equations. It will be this set of equations, where that g is what you call; we already saw that the conductance matrix g is nothing but what will it be now? This will be what will be the first term? It is within quotes the DC value of the conductance.

And it will be g max by 2 or let me call this g sub 0. So, g of t as you recall is sum over k g sub K, e to the j 2 pi fs; fs K fs is 1, so k times t. And so, what is the so this is g0 and this must be g minus 1 all the way up to g minus 2. And this must be g1 blah blah blah g2K and all this will be this is g0, and this is be g0. You plop it into MATLAB, you go and solve, and of course I am not going to do it here. It turns out I already did the maths.

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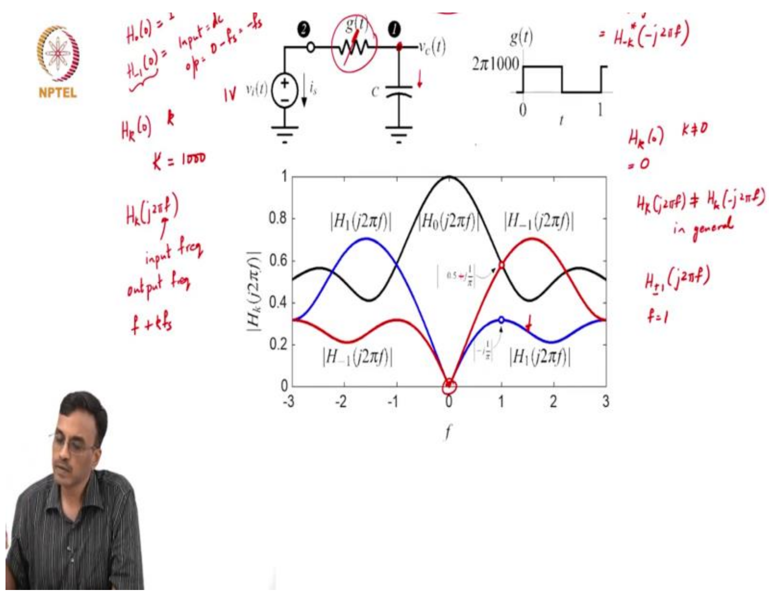

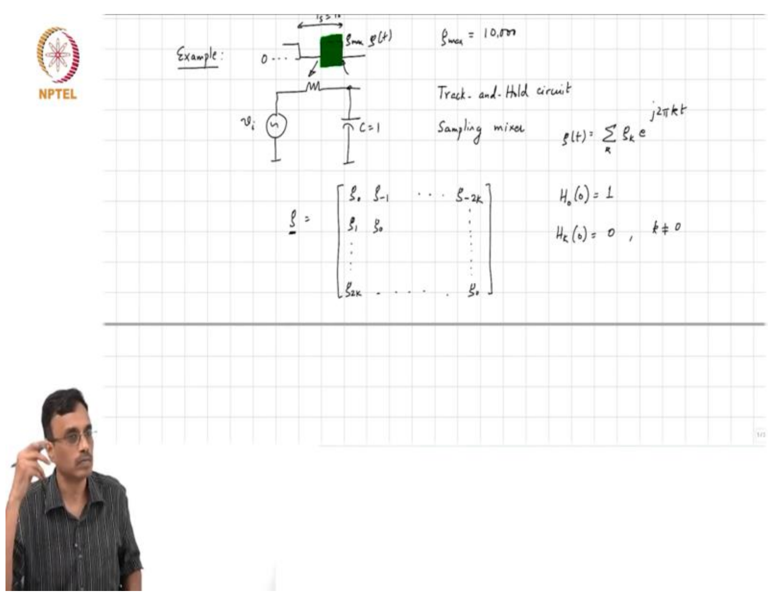

And it really does not matter, I used the g max of 2 pi into 1000; it really does not matter it is 1000 or 10000 pretty much, it is the same thing. So, first things first, so I have only I assumed thousand harmonics; so, K was assumed the capital K was assumed to be 1000. So, how many equations do we have? We have 2001 multiplied by 3; some 6000 matrix of 6000 by 6000 approximately and you inverted. And basically, then you can plot the magnitude and phase of all the harmonic transfer functions. And this is what turns out this what we get.

Now, let us try and see if runs on sanity checks, and see if we can understand any of these at least mark this special points. And then I know make sure that these things makes, absolute sense. See, you plop something into MATLAB and solve an equation and you put in garbage; and you will get garbage out and you can plot pictures. But you also want to make sure that what you plot makes make sense. The easiest point to check for is pardon dc. So, dc what should do you expect for fetch 0 of 0; what does it mean? What does H0 mean? So, you apply what you apply and what you measure?

Student: (())(12:42)

Professor: We apply a 1 volt dc and you get some way form here; and what should you measure in that wave form? The dc component of that way. So, if you putting dc here and these resistors changing periodically; what comment can you make about this wave form?

Student: (())(13:08)

Professor: Pardon, it will be constant, because if it is constant; there is no current flowing through here. If there is no current flowing through here, there is no voltage drop across the resistance. So, one way or the other you saw you solved within quotes Kirchhoff's voltage law and current law, it is hunch. Technically speaking you have to go and solve, you have to go and plop in that put f equal to 0 in that set of equations; and then go and solve it. But, in this case we see it is straightforward enough that we know what. So, if you put in 1 volt dc the output is going to be simply a dc of 1 volt.

So, what comment can you make about H0 of 0? It should be 1, does it make sense. So, well that something that fortunately works out, when you solve the equations also that confirms that (())(14:15). Now, then equation is what about H minus 1 of 0, what should you expect? What is this quantify? What should be the input and what should be the output that we are looking for? We want to put in you want to put in dc. And what are you looking for at the output? You are trying to find what the strength of the output is, at a frequency fs or minus fs?

Student: Minus fs.

Professor: See this is the input Hk of j 2 pi f; so this is the input frequency and what is the output frequency? F plus K times fs. So, H minus 1 of 0 means that the input frequency is dc; the output frequency is. So, input is dc, output is 0 minus fs, which is minus fs; in this case fs is 1 hertz, so you are looking for an output. The component at the output at minus 1 hertz, and it does not require a genius to figure this out; the output is pure dc. So, if you fourier decompose, the output what you get?

Student: (())(15:59)

Professor: Only you get dc, all the other components are 0. And therefore H minus K or H sub K of 0, for K H sub K of dc, for K not equal to 0; must all be equal to must all be equal to 0. And again sure enough you plot the magnitude of H minus 1, H plus 1, H minus 2 whatever; all of them turn out to be 0. Another thing, what it be realize about this conjugate symmetry conditions in of this harmonic transfer functions? So, H of Hk of j 2 pi f must be equal to H minus K star of minus j 2 pi f. So, and Hk j 2 pi f is not necessarily equal to Hk of in general. So, that basically means that as you can see here we computed something; we like to see if this makes sense.

So, H1 of f is not the same as the magnitude of H1 of f is not the same as the magnitude of H1 of minus f. But, for H0 if you put H0, then the negative of 0 turns out to be 0 itself. So, apart from H0, all the others will not necessarily obey; will not have a magnitude response that is e1. So, as you can see here sure enough H1 is not symmetric about the, the magnitude is not symmetric about the origin. However, H minus 1 of j 2 pi f in the positive f is the same as H plus 1 for negative frequencies. And similarly, H minus 1 for negative frequencies should be we expected to be the same as H plus 1 for positive frequencies; and that is indeed true.

So, well that seems early is (feasible) this is what we expected; so no big surprises there. Now, let us try and calculate some other salient points on, perhaps we calculate. Let see H1, H plus 1 minus 1 of j 2 pi f, for f equal to 1. How do you do this? What you suggest we do? H0 of 0 must be 1, H sub K of 0 must be 0; for k not equal to 0.