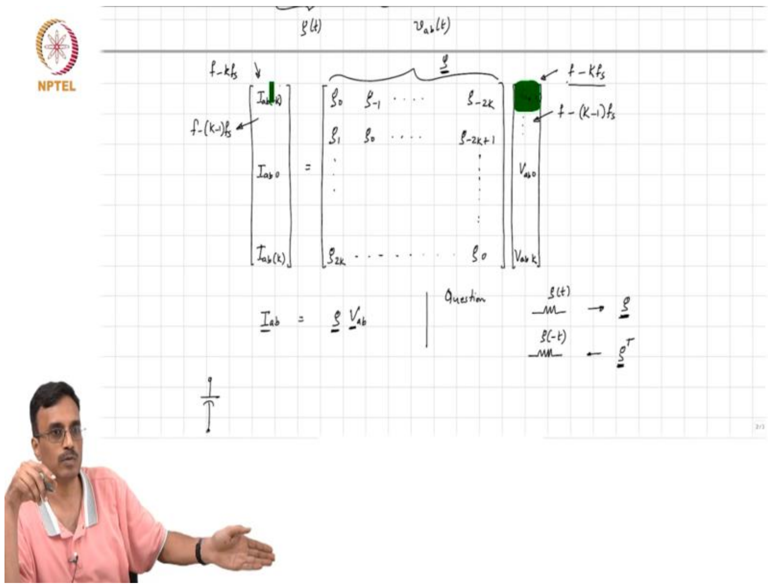
Introduction to Time–Varying Electrical Networks
Professor Shanthi Pavan
Department of Electrical Engineering
Indian Institute of Technology, Madras
Lecture 38
MNA stamp of a Capacitor and a Voltage Source in an LPTV network

(Refer Slide Time: 00:16)

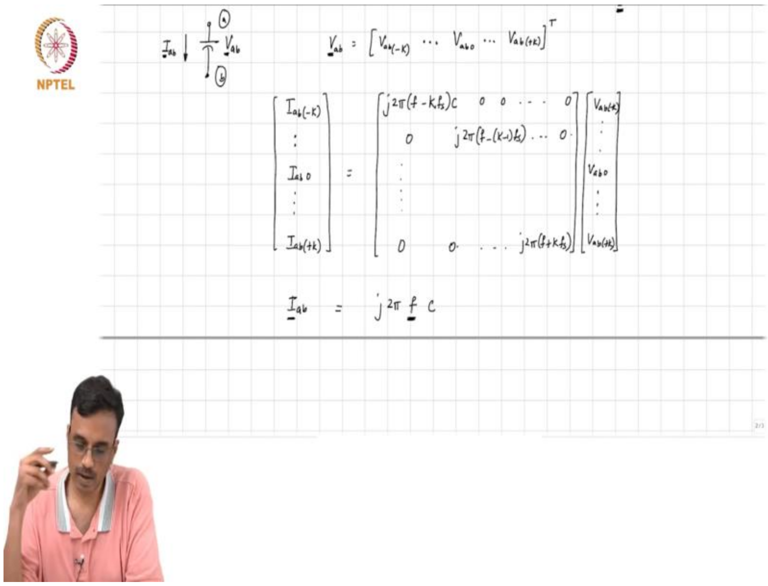


So, we have conductances and capacitors, so is a linear capacitor. Just because this capacitor is linear, does not mean the current through it has got only a single frequency component. It is embedded inside a time invariant time varying networks. So, the branch voltage and the branch current will all have these multiple frequencies. If it is linear, let see what happens how the currents are related to the voltage. And by the way, sanity check for this conductance matrix is what is sanity check?

Student: (())(01:02)

Professor: Exactly, so if if you had a time invariant resistor; just because the network is time invariant time varying does not mean all resistors will be time varying. You will also encounter resistors which are fixed; so, on obvious thing is that when if the resistors are the resistance is fixed, the conductance is fixed. Then all these off diagonal terms will become 0, and you will get diagonal matrix.

(Refer Slide Time: 01:42)



Now, we have a capacitance, again this is node a and this is node b; and the voltage phasor is Vab minus capital K, Vab0, Vab of plus K transpose. And what comment can you make about how is the current phasor is now related to the voltage phasor? So, Iab of minus K, Iab of 0, Iab of plus K. How is this related to this? Any thoughts, what will be the first row first column?

Student: (())(03:28)

Professor: No no current must be related to voltage in what way? J 2 pi f plus, plus or minus?

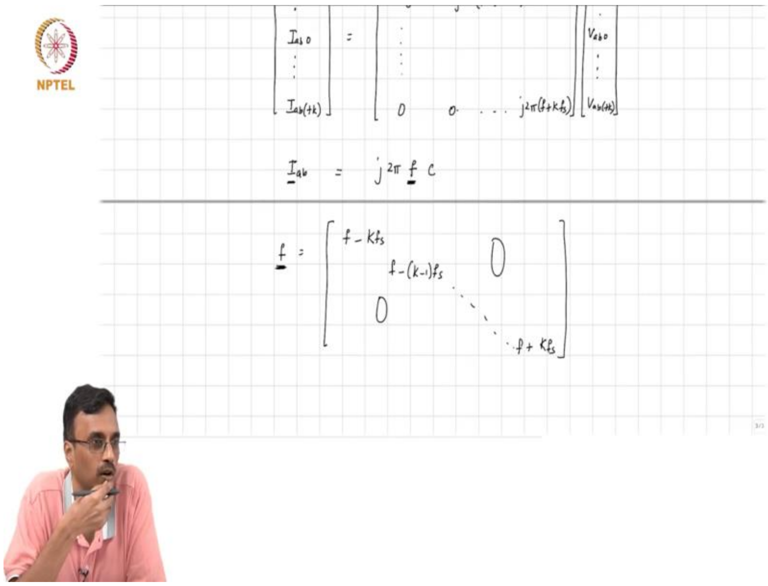
Student: (())(03:52)

Professor: The voltage is at what frequency? F minus capital K fs; it is a linear capacitor. So, it is a linear time invariant capacitor; so, this f minus K capital K fs, what will be the what would be this thing be what would be current be? It is simply j 2 pi f minus K fs times c. So, this so this would simply be this first term would therefore be j 2 pi f minus K fs times c. What comment can you make about the second the second term in the first row? It is a time invariant capacitor and therefore the output frequency and the input frequency must be the same.

So, all these other entries must be 0. Similarly, the first row in the second column will be 0, and what we will have there? We will have J 2 pi f minus K minus 1 fs 0 and so on. So, this is blah blah 0, 0 all the way up to j 2 pi f plus K fs. So, therefore the current phasor can be written as it is a diagonal matrix. And what all can you remove out of the diagonal matrix as a common factor?

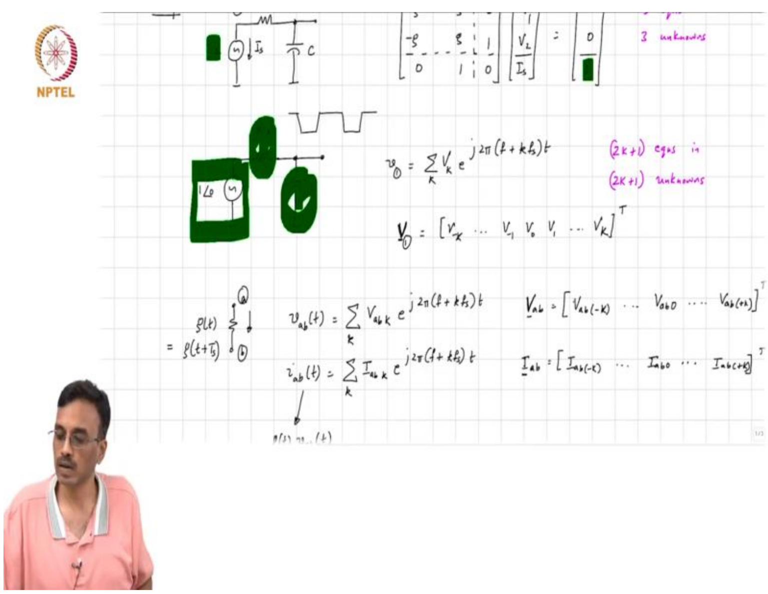
j 2 pi c; and whatever is left is what I will call the f matrix. And this is analogous to if we added, if we had a time invariant network; you would see j 2 pi the scalar f times c. Now, you see a vector f.

(Refer Slide Time: 06:43)



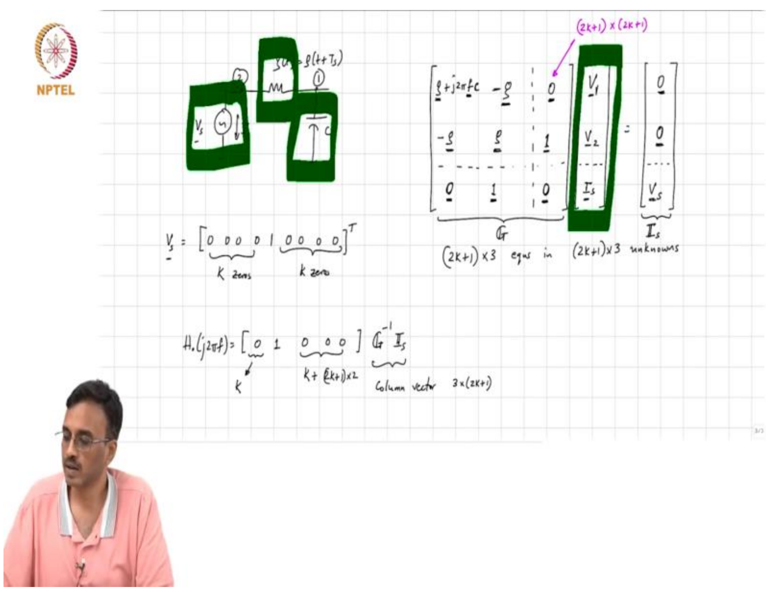
Where, this f matrix is nothing but j 2 pi, sorry is f minus K fs, rest of it is 0; F minus k minus 1 fs all the way up to f plus capital K, so all right.

(Refer Slide Time: 07:39)



So, we have two elements here, so we know the MNA stamp of this guy; we know the MNA stamp of that guy. The only thing remaining is is this one, so what you suggest we do. Voltage source, then as usual what we do? A voltage source what the unknown is? The currents through the, the current through the voltage source. Now, we do not not only have to keep track of the current at f; but the currents at all the frequencies that you are interested in, there are all also unknowns. So, in other words, in a time invariant network just that in that the Is was scalar; now Is is a vector. And likewise, the voltage source earlier was a scalar, and you would equate that two, one of the node voltages. Now, what you need to do? Let us let us actually do this in action.

(Refer Slide Time: 09:00)



So, get back to our voltage source and this is your g of t; and this is node 1 and this is node 2, and this is the unknown current phasor Is. And what do we do? We now have we will MNA formulation, except that the entries in this MNA matrix; will now all be will all be matrices rather than scalars. So, we start with the MNA stamp, so the unknowns are V1, V2 and Is; where it is now understood that V1 will have 2K plus 1 components. V2 will have 2K plus 1 components and likewise Is will have 2K plus 1 components. And on the right-hand side, what do you expect to see?

First row first column or the first row in the right hand side, is the sum total of currents going into independent currents going into node 1; which evidently we are not putting any independent sources in. So, that will now therefore be a 0 what? It be 0 vector, with how many entries? 2K plus 1. Likewise, what comment can we make about the second row? It will also be 0. And the third row, so basically this Vs phasor is nothing but 0, 0, 0 how many zeros will be there? There will be capital K zeros. Then there will be 1 and then that makes sense people. So, as you can see the mechanics of the whole process is just the same.

All that you doing is now replace he MNA stamps of a time invariant element with that of a time varying. And these all these equations are telling you, there is shortcut way of writing kcl and kvl at every frequency; at each frequency kcl and kvl must be satisfied. So, that covers the right-hand side, so let us go element by element. What comment can we make about the MNA stamp of this guy? Between nodes 1 and 2, you now have. You have the g matrix and, what must you have first row third column? You have a 0 matrix. And what are the dimensions of that 0 matrix?

Student: (())(13:11)

Professor: Just to be 2K plus 1 cross 2k plus 1 square matrix. And similarly, what comment can you make about second row third column? Basically, the currents flowing through the voltage source; basically, must all add there. So, that must be the identity matrix, and what are the dimensions of that identity matrix? 2K plus 1 cross 2k plus 1.

And the last one the last row, 0; what this should be? 1, where this is also an identity matrix of size 2K plus 1 cross 2k plus 1. What about last row last column? 0. So, that takes care off of these two elements; the last thing we to do this is at this end guy here. What you do? Plus, g j 2 pi f, sorry j 2 pi f times c.

And the j 2 pi f times c you might be wondering, why this follows embedding the matrix between two constants; it is 2 pi c times f. But we used to writing in a time invariant case; we used to writing j 2 pi f c. So, there is no change as far as the time periodically time varying case is concerned. Except that that f now becomes replace is replaced by that frequency matrix. The g is replaced by the conductance matrix, which captures the periodically time varying nature of the conductance. And all these other matrices are zeros or identity matrices of appropriate all.

So, how many unknowns are there now and how many equations are there? 2K plus 1 times 3 equations in as many unknowns. And therefore, what you do? Invert the matrix; at each f you invert this matrix, and that will give you the all the branch currents and the branch voltages, in everywhere. So, therefore you are in a position to find all possible harmonic transfer functions from the input source to various branch currents and branch voltages. Does it make sense; so, so let say you get some V1 vector. What comment can you make about, let me in other words let say, let me now is now getting use to these notations.

So, let say you want the your output of interest is voltage across the capacitor, and you want H0 of j 2 pi f. So, what will you do? Let us call this matrix as usual will call this matrix capital G; and this is Is. So, if you want H0 of j 2 pi f what what will you do? You will first calculate.

Student: (())(19:05)

Professor: To calculate the V1 matrix, you will have to invert this system of equations. So, notationally therefore it is G inverse Is, and so in that will give me what will that give me? It gives me this unknown vector, which is got 2K plus 1 times 3 numbers. But I am which number I am actually interested in? This gives me ton of numbers. But I am only interested in how many of those numbers? One of them; which one?

Student: (())(19:56)

Professor: The K plus 1 the K plus oneth row of this; this is a column vector of size 3 times 2K plus 1. So, if we want H0 of j 2 pi f for instance is simply nothing but whole bunch of zeros; 1 followed by another bunch of zeros. So, this should be K zeros here and this should be K plus 2K plus 1 times 2; this multiplied by G inverse times Is gives you that number H0 of j 2 pi f. Is clear folks? Now, if you want whatever H of plus 2 of j 2 pi f what will you do? What is only thing that will change in this equation? The position of that 1 will keep more. Yes, something was bothering you for such a long time yes.

Student: So, in doing the analysis of answers, the origin of g because I is the current (())(22:09). Here we are extending that saying the current does not 0 at every frequency.

Professor: Absolutely, well kcl the question is why is kcl and kvl, why does it have to be satisfied at at every frequency; does somebody have thought on that?

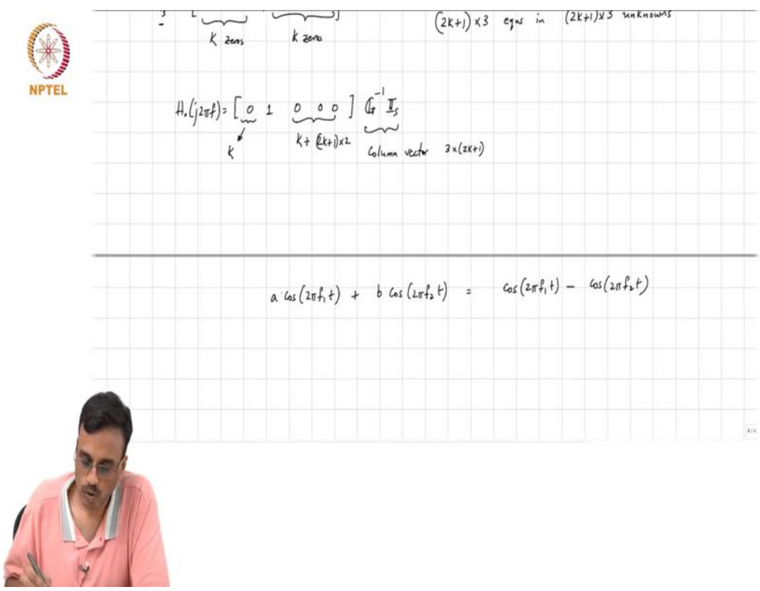
Student: (())(22:32)

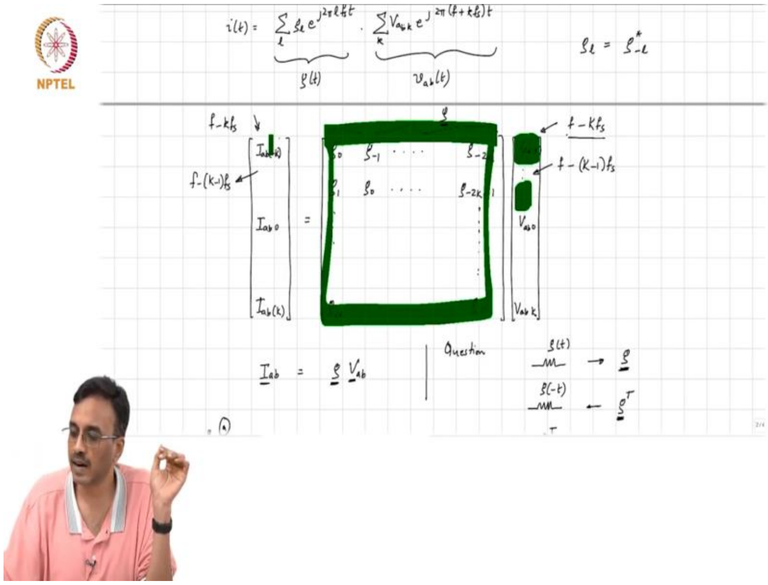
Professor: This got nothing to do with linear and non-linear.

Student: (())(22:40)

Professor: No no, remember please note that kcl and kvl are as far as we are concerned loss led down by god. They have nothing to do with linear, non-linear time invariant, time variant all that stuff is relevant. Kcl and kvl are true that is it, now his question is why should be true, at at every one of those frequencies. The fact that this network is time invariant means that the only frequencies that can that can that can exist are f. Because that is this the frequency of the exciting source. There it is very clear there is only one frequency; so we know we are all we are one happy family. Now, if we have multiple tones, the question is why should for in other words what he asking is.

(Refer Slide Time: 23:41)





If a1 cos 2 pi f1 of t plus b cos 2 pi f2 of t is equal to, let say call this cos 2 pi f1 of t minus cos 2 pi f2 of t. What are, a and what are a and b?

Student: (())(24:18)

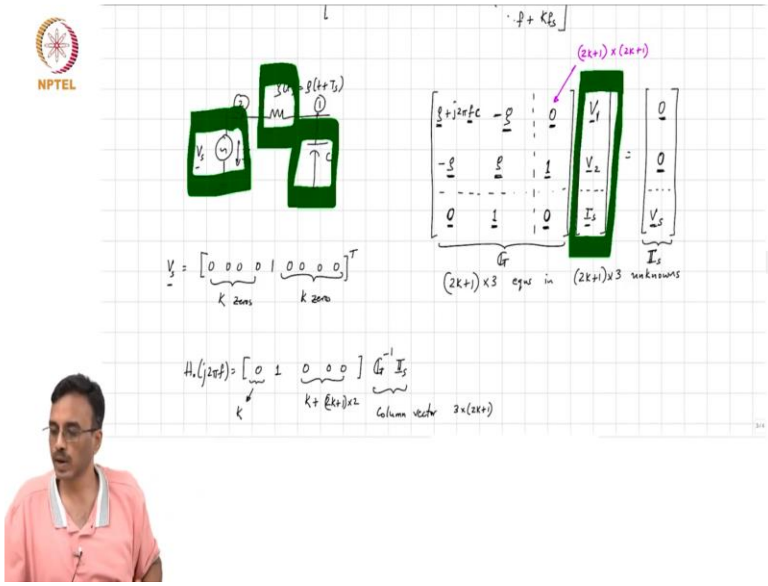
Professor: 1 and why?

Student: (())(24:22)

Professor: So, you convinced, or you are not convinced? Two different frequencies. The only way the left hand side can be equal to the right hand side, is if the stuff the individual ones correct. So, kcl and kvl must be satisfied at every frequency. Some maybe I should have I should have mention that before I went through all this; it is a good question. And hopefully all of you have convinced that it must be satisfied at every frequency. What this time variances doing? Is that because its conductance is changing with time; it is able to transfer.

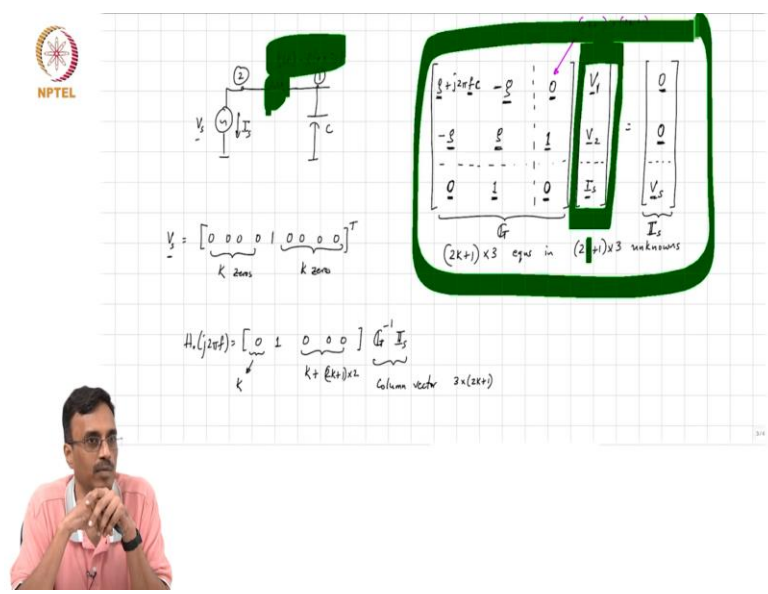
I mean fact that conductance met this small g is not diagonal; what it means is that, stuff at one frequency can jump up or down or do both. And or remain at the same frequency, depending on this matrix which is equivalent to saying, depending on the way you change that resistance with time. Frequency conversion can frequency conversion effects can can occur in that resistance. Now, it is stands to reason that if you would multiple such things together and put a network; you have a complicated mess. And that is that whole mess is quantified by is expressed by this equation, the set of equations.

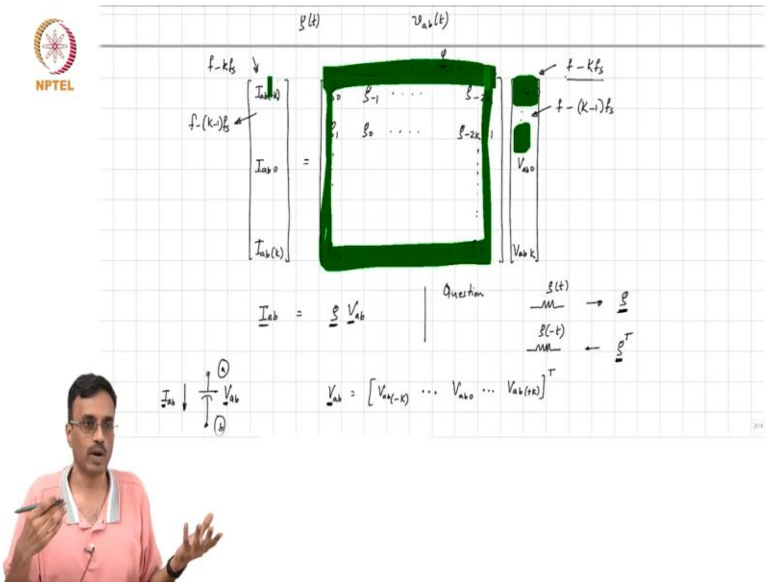
(Refer Slide Time: 26:14)



And of course if your network turn out to be time invariant, you will find that all these matrices will become diagonal matrices. And then you will find that the only unknowns, all the unknowns except the zeroth order harmonics will all be 0; it is it is admittedly messy to do the maths. But, to understand at a conceptual level, what is happening is is just straightforward as it is with the time invariant case. So, let me quickly do an example of what we should expect; I am not expecting to sit and invert.

(Refer Slide Time: 27:15)





So, by the way depending on the nature of this periodic wave form g, the number of reasonable question to ask is what should I choose K to be, this capital K to be. And that too evidently depend on; what do you think that depends on?

Student: (())(27:37)

Professor: of which element? In this case the only thing varying with time is g; so, if g is basically got a lot of Fourier series non (signi), I mean non-trivial Fourier series components. Then it stands to reason that you would this matrix would have to be of very large dimension to be able to capture the behavior of g accurately. In which case? This stuff would be a very large this K would be a very large number. For example, if you had a switch if g of t was a switch; then what comment can you make about this the g of t is basically doing something like this.

And you know that the Fourier series coefficients of a square wave, 1 by n; what the bottom line is that very slowly. Because, you have very sharp edge, and therefore it is stands to reason that you must consider a lot of harmonics of g of t; to be able to get a reasonable representation of this stuff. So, it is not uncommon for k to be thousand, 2000 depending on; unfortunately for every physical system can never, you can never turn on switch or a turn switch off instantaneously. There is always be some rise time and there will be some fall time; and that means that the higher order coefficients will are falling of to 0.

Hopefully faster than 1 over n, and which is why practice it is not uncommon to have this k of thousand and 2000 or whatever; it does not take a genius to see that. The solution of this set of equations is bound to take a lot more time and effort, than solving the corresponding time invariant network; where it is 3 by 3 matrix. And therefore, it should be prepare, to kind of spend a lot of time on simulations, when you do these kinds of competitions.