Introduction to Time-Varying Electrical Networks
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Lecture 36
MNA analysis of LPTV networks

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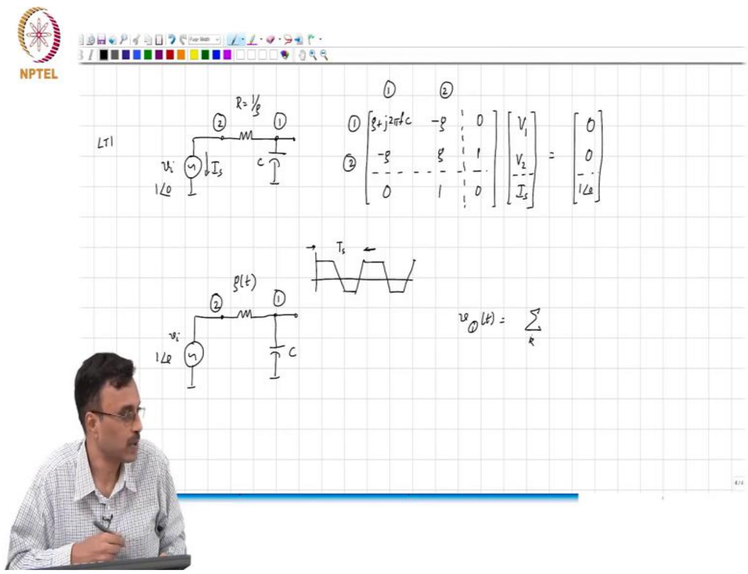

So, let us now, so that is so far as the, the basics of an LPTV system are concerned. And please remember that this is all the, these are all the steps that we did, we underwent when we talked about time invariant systems in early classes. You know the Fourier transform the input, you want to find, the Fourier transform of the output and convolution. And, what happens when you put e to the j 2 pi f t; all that stuff is what we were used to doing for a time invariant system.

Now, we have also done the same thing for a linear periodically time varying system. And, you know, I hope you see the, the analogy or rather the mechanics is similar, but it is a little more complicated in the sense that you now have to worry about output frequencies that are not merely f but f plus all this stuff.

Then, then we were studying time invariant circuits. Then we said, well, let us try and how do we, well you have, let us take a simple example of our good old first order RC filter. And, if you want to compute the frequency response of this filter, what will we do?

Oh! Well, we number the nodes and we write the, I mean, okay, I mean, you, you might argue that well this is such a simple circuit that I may simply do it by hand. But if you want to do it systematically, I mean, you will basically say, well, let us write the, write down the, the MNA equations, except that they are now, now in the frequency domain.

So, what are the, the unknowns? The two node voltages, and so we should write down the phases v1 and v2 are the unknowns. And this is Vi, which is 1 angle 0. So, I sub s is the phaser of the voltage through the, of the phaser of the current through the voltage source, and this must be, in the right hand side we find only all independent sources. And this is 1 angle 0. And can you help me if I fill out the entries of the matrix? What do we do? Yeah. I mean, I know, I knew that too good. Very good. So, this is g between 1 and 2, so this is node 1, this is node 2. g, minus g, minus g, plus g.

And, for the capacitor, j 2 pi, remember we are working with phases, that certain frequency so it is plus j 2 pi f c. And what do we do? Here this becomes 1. And here, you get 1, 0, (())(04:29). And then, you solve this equation and this, solve the set of equations. And, in one shot you have all the stuff that you wanted namely node voltage at 1, node voltage at 2 and the current through the source that is I s. All straightforward stuff.

Now, let us say, this is the LTI case. Now, the question is what happens when again, let us take our simple example of the RC circuit except that, this is again the same old phaser at frequency f, this is C, this is now, g of t. Let us say this is varying periodically. I do not know, I mean, in some periodic way where this is Ts. So, what comment can we make about the voltage and current in any branch or at any node?

Student: (())(05:59)

Professor: I mean, will therefore contain 2f and 3f and so on?

Student: (())(06:19)

Professor: Yeah, so basically any node voltage can be expressed as what?

Student: (())(06:44)

Professor: This is a linear system.

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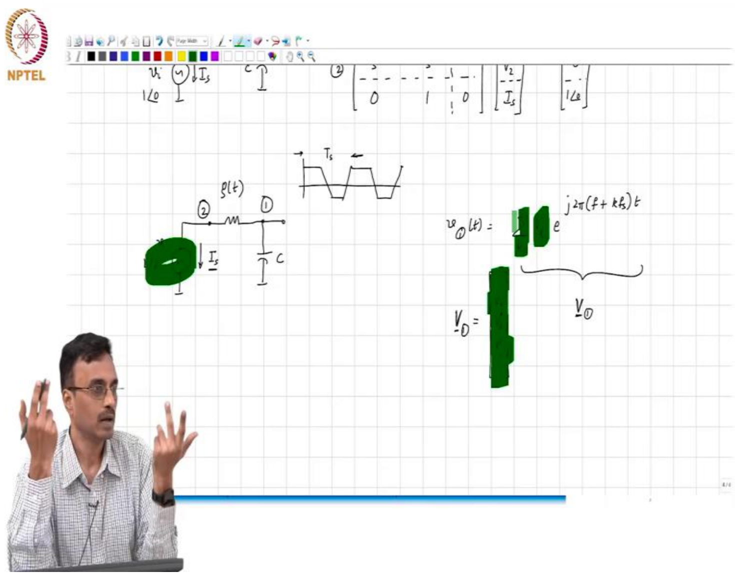

You are exciting this linear system with, with a tone at f. So, every output will therefore consist of tones at, at frequencies of the form f plus k times fs. And therefore, in a time invariant system you just simply have 1 phaser, which basically denoted the, the complex amplitude of, at the frequency f. Now, you need what?

If you want to find the specified voltages at a certain node or a current to a certain branch, and you have to express it as, as a phaser, what would you do? Earlier you need it in a time invariant system, there is only 1 frequency, f, and therefore, you, the voltage at node 1 would simply be, you would perhaps call it V1.

Now, what do you, what situations are we faced with? At any node you have multiple frequencies. So, by necessity you basically need to have expressed that as a phaser with let us say we call it...where that V sub k, I mean, now it just becomes notation nightmare, because you have a number for the node, you have a number for the harmonic.

And so, it is not uncommon to kind of look at, you open up, either a book or a paper where they are talking about this stuff and then, you see a sea of symbols. And then, as soon as you see that you close the book and then you pick up Calvin and Hobbes and start reading because it is a lot more interesting. So, there is indeed a little bit of mess involved and we have to be little careful.

So, now the, voltage at a node is now not merely characterized by 1phaser, but you need to associate complex amplitudes to every tone that is there at that node. So, in general, we basically say, well, let us keep our life simple; we are, at least our notation simple. So, we will put all these, these complex amplitudes in a column vector. And that basically, we will call this, this V is basically a column vector corresponding to the voltage at node 1. And, this column vector simply has got the set of all these, these guys here.

So now, I mean, what is the advantage of this whole thing? Well, now you do not have to write this messy expression. You just basically say, well this is nothing but, we that we underscore is just stuff that I use here to be able to distinguish a vector from a scalar. And so, we want basically is this, the set of all numbers that, that are in this equation.

So, likewise I do not know, I mean, if you have, I sub s. So, this is also now a vector of, phasor quantities where each term in the, in the vector basically quantifies the strength of, well let us say strength of each frequency in that, in that current. So now, the, now, we would write KCL and KVL. And fortunately, what comment can we make about KCL and KVL? I mean, of course it is still valid, but it must be valid for....in a time invariant system, I mean, there is only one frequency so there is no confusion. Well here, we have multiple frequencies, so KCL and KVL must be valid for, for all frequencies.

So, it is just that, so let us assume for argument's sake, that we say, well, the first 1000 harmonics of V, in other words we are only interested in, we assume that the amplitude of tones at frequencies beyond say k equals 1000 become negligibly small. Ideally that, the summation goes from minus infinity to infinity. But we say, let us say, we are happy with saying, well beyond k equal to 1000 those amplitudes are so small, we do not worry about them. So then, how many sets of KCL KVL equations will we have now?

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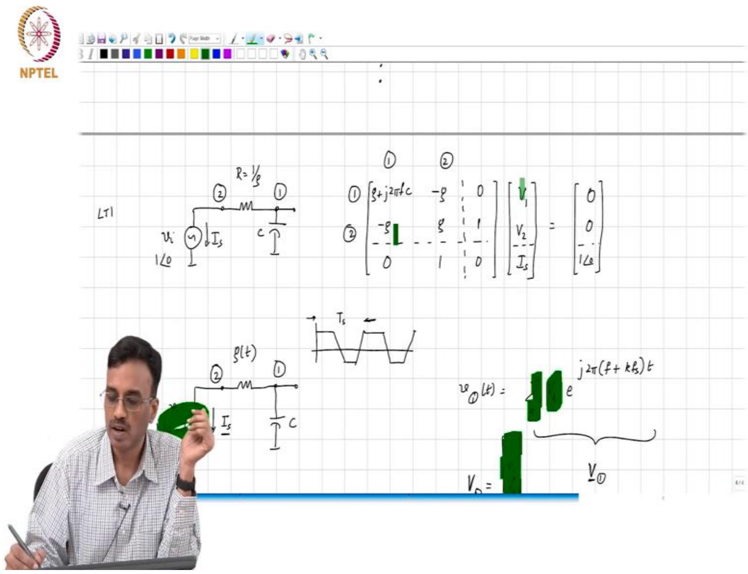

In the time invariant case, we had 3 equations, 3 unknowns. Now, how many equations will we have? If you restrict k to be 1000, then, well, it is not 3000 but a 6000, because you go from minus 1000 to 1000 for each one of them.

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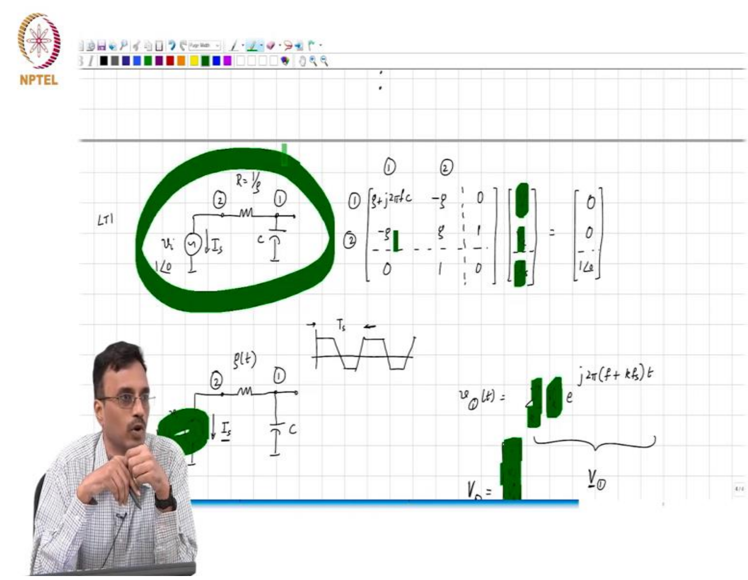

So, you have roughly 2000 unknowns here, 2000 here and 2000 here. So, basically what was, what you call, literally child's play. Now, even a simple first order RC network now becomes a set of equations, 6000 variables in.... Fortunately, once you know, you understand the principles you, you have, what you call, a whole lot of, I mean, you now, once you know how to write these, these equations systematically, you will then, can go and plop them into a computer and then get the solution, but that is basically (())(14:05).