Introduction to Time-Varying Electrical Networks
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Lecture 35
Zadeh expansion of an LPTV system

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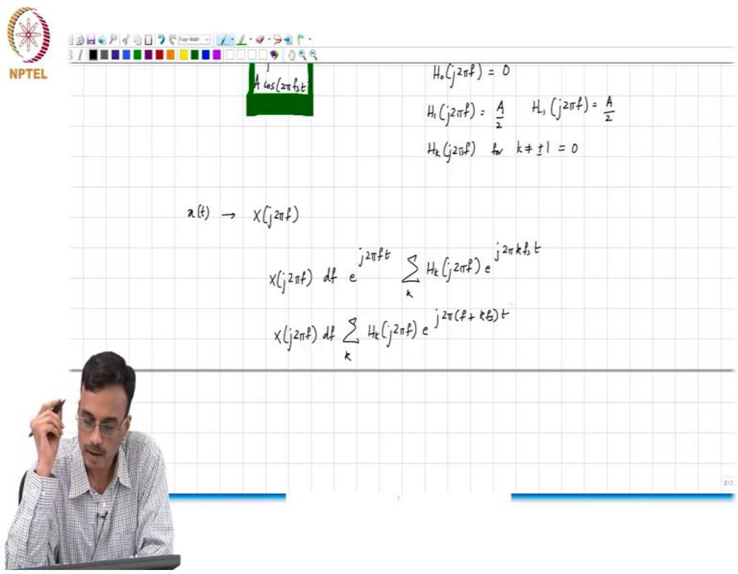

So, that was that. The next thing that we would like to do is, well, we know what happens to the output of the system if you excite it with a complex exponential e to the j 2 pi f t. So, now the question is what happens if you, if excited with a signal whose Fourier transform is x of j 2 pi f. What comment can we make? You can think of the input signal x as being composed of sinusoids with frequencies ranging all the way from minus infinity to infinity, and the strength of the sin… the complex for sinusoid between a frequency f and f plus delta f is simply nothing but x of j 2 pi f times or d f is this.

So, what comment can you make about the, output of the system due to this guy here? You know the output if you had e to the j 2 pi f t times, I mean, you know what the output would be. Now, you have a sinusoid of this form. So, you have an input sinusoid of the form, e to the, this is the strength of the sine wave and these are the frequencies f. So, what would you expect for the output? This times Hk of j 2 pi f e to j 2 pi k fs times t, so another words the output would be H sub k of j 2 pi f e to the j 2 pi into f plus kfs times t.

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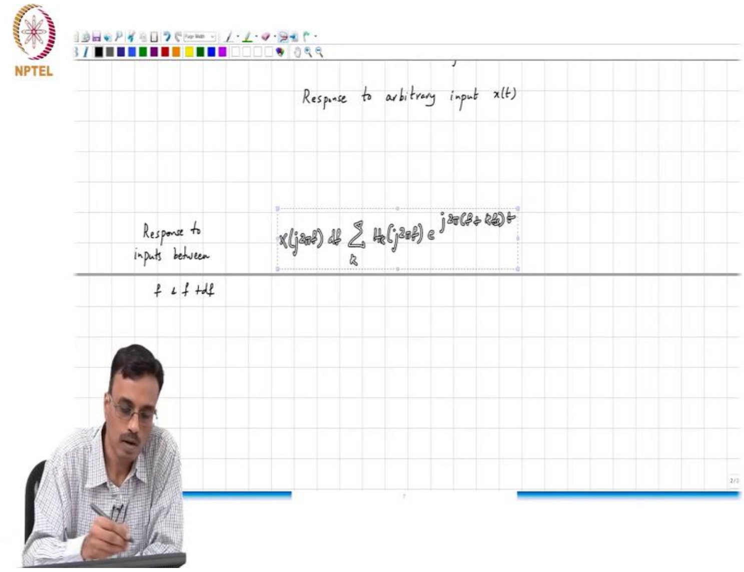

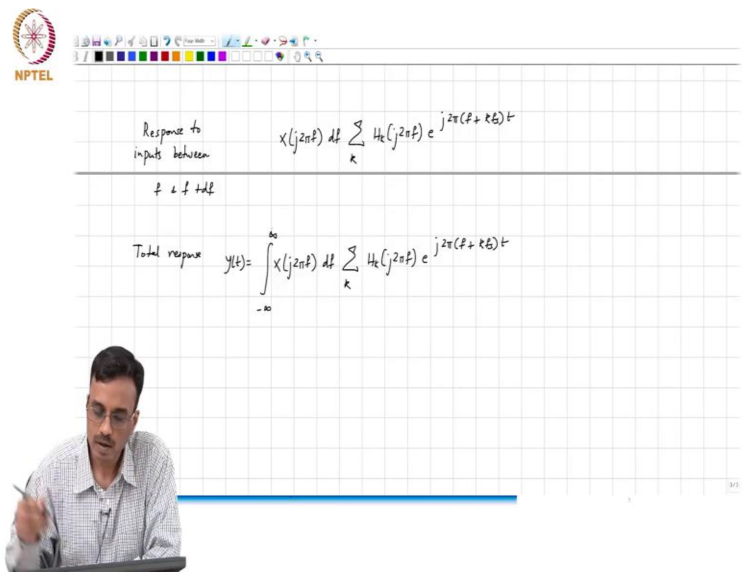

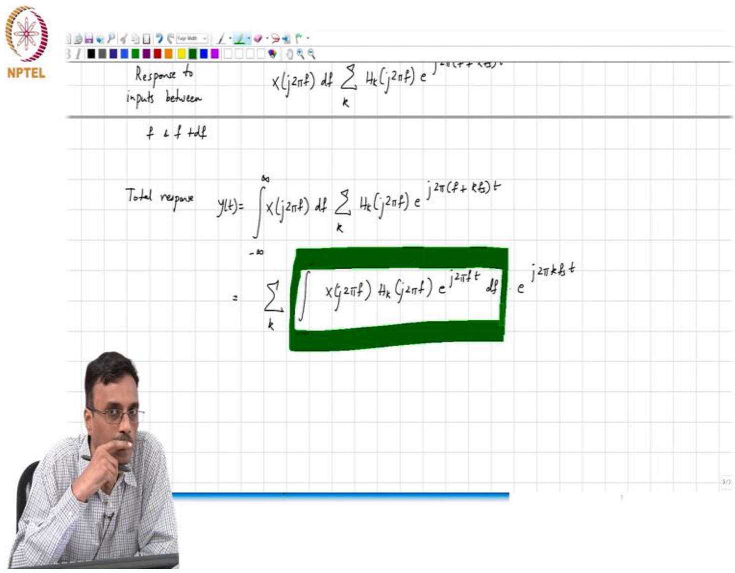

And so therefore if you want to find the total output, what would you do? So, this is the response to inputs between f and f plus df. So, what comment can you make about the total response? Integral minus infinity to infinity. So, I may not do that here, let me copy and paste this. Y of t therefore, is sum over all frequencies. Does it make sense? And now you can push the summation outside the integral. H sub k of j 2 pi f.

So, can somebody can stare at this and find something that we knew already? How do we interpret this equation? I mean, well you can say or what can I say it comes out of the math, right; but is there a way we can interpret this? What does this represent? Forget about the rest of the equation. What does that represent? Pardon.

Student: (())(05:51)

Professor: Pardon. How can you interpret that, the signal which is inside that green box?

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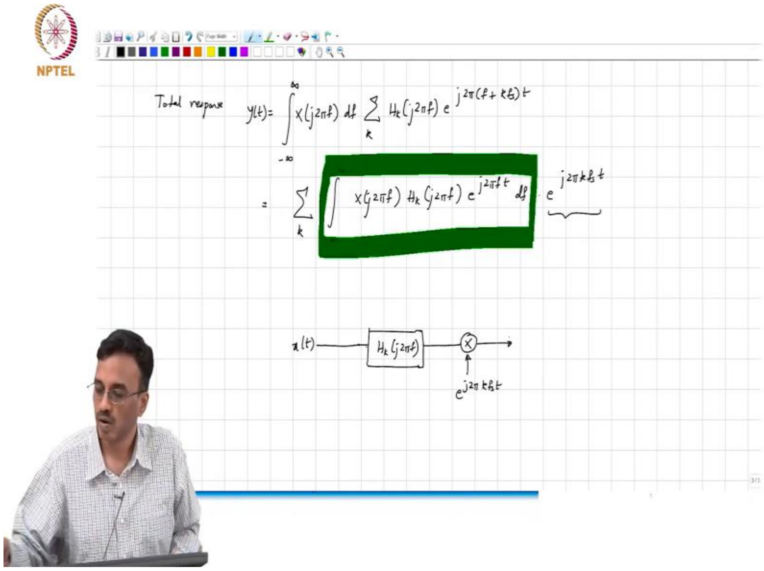

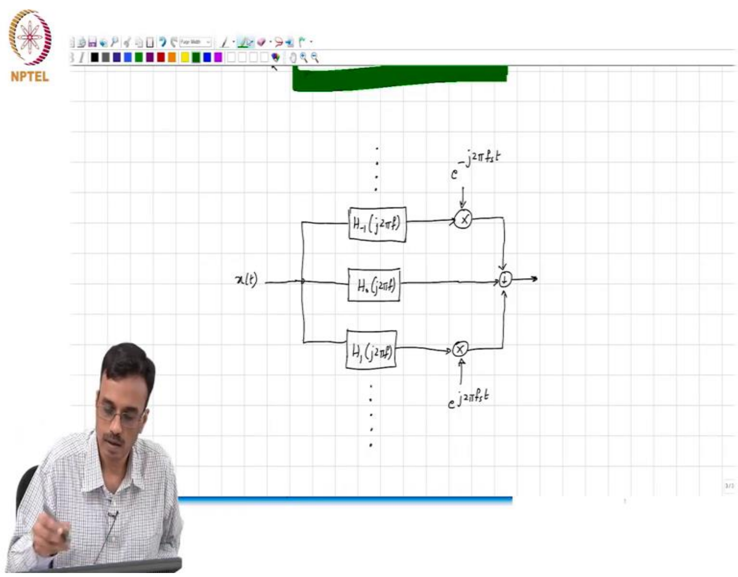

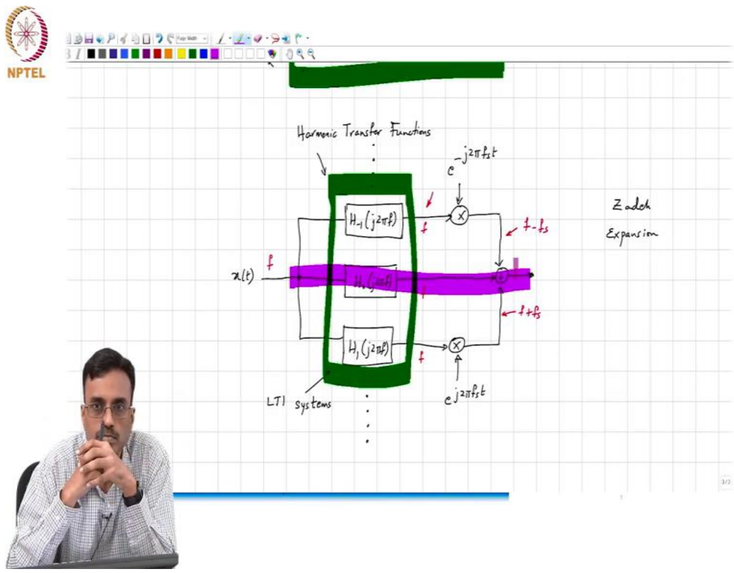

Yeah. Well, basically this is nothing but think of it is x of t exciting a filter whose transfer function is, whose transfer function is what? Hk of j 2 pi f. Does it make sense? And does this, this H of, I mean, does this filter time invariant or time varying? It is a, I mean, this is a regular, you know, LTI system stuff that you already know. So, basically you take x of t excited with the filter whose transfer function happens to be H sub k of j 2 pi f, and the output will be something with, the Fourier transform, given by this guy, and what comment, what should we do with that signal?

Remember this does not depend on small f. So, as far as this stuff is concerned it is, it can be taken out of the integral. Therefore, what do we, what do we supposed to do? Multiply this output with what? e to the j 2 pi k fs times t. And so, now that you have a summation, you basically, what this means is that, let me redraw it.

So, you have an arm which gives H naught of j 2 pi f. So, if k is equal to 0, what does this boil down to? This complex exponential outside will be 1. For k equal to say minus 1, what do you get? H minus 1 of j 2 pi f multiplied by e to the minus j 2 pi fs times t. And likewise, H sub 1 of j 2 pi f, this needs to be multiplied by e to the j 2 pi fs times t. And again, this goes all the way from minus infinity to infinity. And all these are what kinds of systems? These are all transfer functions corresponding to LTI systems.

And the outputs of these LTI systems are translated up and down and frequency by appropriate multiples of fs. So, for example, if you put in an input tone at a frequency f here, what comment can you make about the frequency content here? It will be f only. Everywhere here the content will only be f. And, if you take a sinusoidal tone f and then multiplied by e to the minus j 2 pi fs times t, what will be the content here? f minus fs. Here it will be f plus fs and in general it, what is coming here will be in the form of f plus k times fs.

So, all that this is saying is that an arbitrary LPTV system can be decomposed as a bunch of time invariant systems and the outputs of these invariant, time invariant systems is frequency translated up or down by the appropriate harmonic of the frequency with which of fs which is the frequency with which the system is varying.

So, this representation of an LPTV system is often what is called Zadeh expansion after the person who first came up with this. And these are all the harmonic transfer functions. And, we actually know how to (())(12:57) if a given LPTV system, we know how to calculate these harmonic transfer functions at least in principle at a frequency f.

If you want to calculate the harmonic transfer function, you simply excite the system with both cosine and sine, which is equivalent to saying that you excited it with e to the i 2 pi f t. Look at the output. Find the gain by multiplying the output by e to the minus j 2 pi ft. And, the moment you do that, you will get something, you will get a, you will get a complex waveform, which is also varying periodically with fs. And you can expand that periodic waveform as a, as a Fourier series. Once you have the Fourier series, you can basically, the coefficient of the Fourier series will give you the H sub k of j 2 pi f at that particular frequency.

If you wanted for a different frequency you do it. You know, for a different frequency and you know, if you want to do the frequency response, in a linear time invariant system you keep changing the frequency of the input generator over the frequency range of interest and measuring the ratio of the output to the input. Now, you have to have the additional step of actually expanding the, I mean...In a time invariant system, if you want to think about it in terms of the H sub k, all the H sub k except H sub 0 are 0.

So, you just need to, you know, look at the complex gain, and then you are done. When you have time varying system, the, gain is varying periodically the function of time. So, for each input frequency, you need to decompose the, the gain function in a Fourier series, which will then give you in one shot, the, the H sub k of j 2 pi f for all k.

Then you change the frequency and then you know, you make this measurement all over again and then you will get, you know, this H sub k for this other frequency. And then therefore you.... So, as you can see, and as you should expect, well in LPTV system it is a lot more complicated than an LTI 1. So, you should expect to do more work when you work with an LPTV system. Does it make sense so far?

Of course, the sanity check that you should always use is that an LTI system is a special case of an LPTV system. So, the results should all make sense, you know, in the special case when you have an LTI system. So, for example here, if all the H sub k were 0, other than H0, then the only arm that remains is, is this one here. And that is the standard LTI system that we do. Is clear people?