Introduction to Time-Varying Electrical Networks
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Lecture 34
Harmonic Transfer Functions

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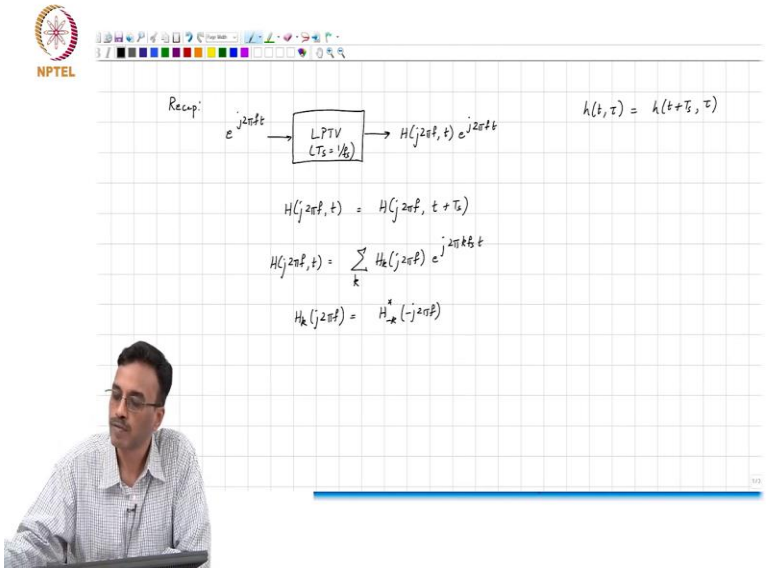

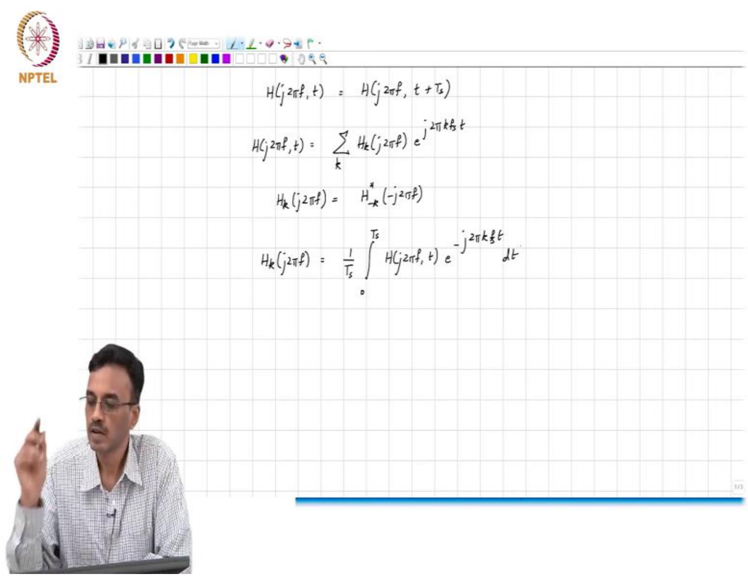

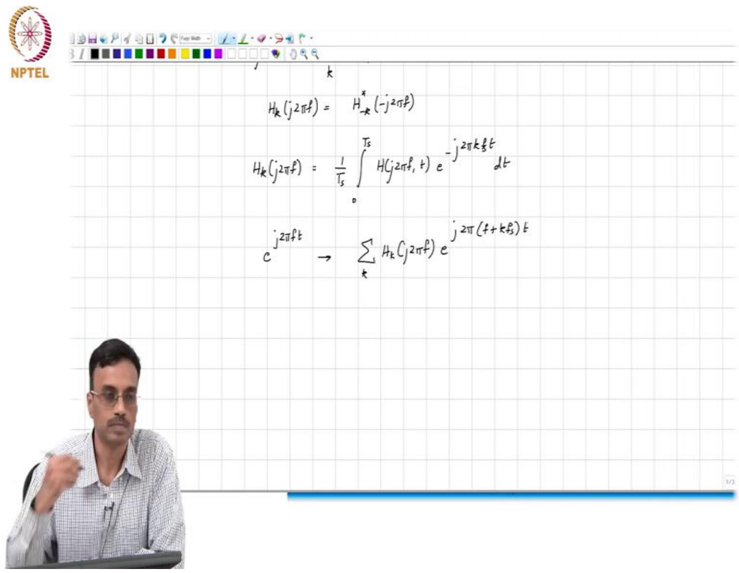

A quick recap of what we were doing yesterday. We said that if the properties of the system, of the linear system are varying periodically with time, then you can characterize its impulse response ht comma tau; also happens to satisfy the periodicity condition I have shown here. And, basically it says that, well the network, loosely speaking, looks exactly the same whether you, you look at it now, or you look at it, a time. (())(1:15)

And as a consequence, we call that, if you excite the linear system with a complex exponential, the output is this time varying gain times the same complex exponential. And since the properties of the system are varying periodically, it follows that this H j 2 pi f t, which is basically the gain experienced by the input sinusoid also varies periodically with time with period t s which is 1 over fs.

So, this is basically written as H j 2 pi f, t plus t s. Correct, which then allows it to be expanded in a Fourier series. And therefore, as we saw yesterday, it will be 2 pi f comma t is sum over k of h sub k j 2 pi f times e to j 2 pi k times f s times. And we also saw that this h sub k f must be the same as h star of minus k minus j 2 pi. And how would we find the, the h sub k?

Well, it is simply the usual equation that we know. To find the Fourier series coefficients as simply integral 0 to Ts H of j 2 pi f comma t exponential minus j 2 pi k f t dt. And the linear time invariant system is a special case of linear periodically time a system where all the Hk other than k equal to 0, all become equal to 0. Fine. So, therefore when you excite an LPTV system with a complex exponential like this, the output is signal which is H sub k j 2 pi f e to the j 2 pi f plus k f s times t.

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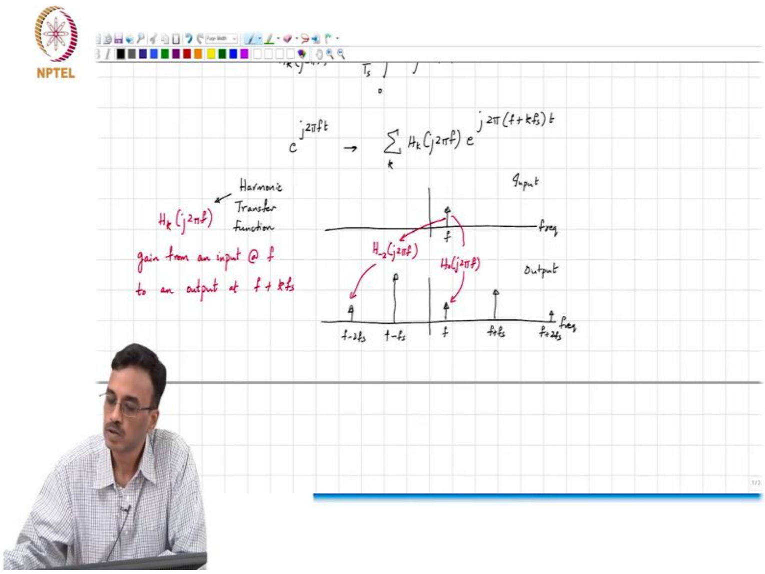

In other words, this is the input, and this is the frequency, and this is the output. So, if the input consist of a tone at f, then the output consists of tones that f, f plus f s, f plus 2 fs, f minus fs, f minus 2 fs and so on. And what comment can we make about, if the amplitude of that tone is 1; what can you make about the amplitude of, the strength of the output at f?

Is it H 0 of j 2 pi f. Likewise, what can you make about the strength of that tone? H minus 2 of j 2 pi f. So, you get the idea. So, this H sub k of j 2 pi f quantifies the gain from an input at f to an output at f plus k times f s which basically is a, which is f shifted by harmonic of the frequency with which the system is varying.

So, this is like a transfer function except that it is not, it is not the same. It, our traditional sense of the transfer function when we talk about time invariant system is that the transfer function quantifies the gain from an input and frequency f to an output also at the same frequency. Now, the output is at a frequency which is shifted from the input frequency by a harmonic of the, of the frequency with which the system is varying. And therefore, this is called harmonic transfer function. And H sub k basically stands for the kth order harmonic transfer function.

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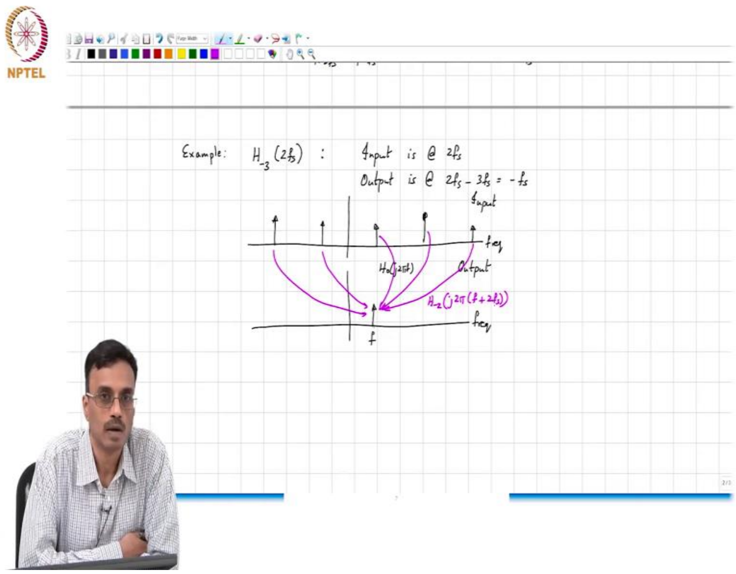

So, for example, H minus 3 of 2 fs basically means, what is the input frequency? What is the input frequency? Input is at 2 fs and output 2fs minus 3fs which is minus fs. So, just a matter of getting used to this notation. Is that clear? Next thing, as we were talking about during the close of yesterday's class, in a linear system if you look at the output if you find a tone at f if it follows that the input must also contain a tone at f. Now, if we look at the output of an LPTV system and we notice a tone at f; what does it mean for the input?

So, basically there is a little bit of confusion because, well, there could be an input at f, or there could be an input at, this output at frequency f could have come from any input, need not necessarily be at f, can be a result of tones at inputs that are given by...what? f plus kf. So, basically here this output here could be a consequence of this multiplied by H0 of j 2 pi f, or this, what would this, this gain be? H minus 2 of j 2 pi f plus 2 fs.

This frequency, remember, what you get, what you put here is the frequency of the input tone. And this is the amount by the number of fss, the number of FSS by which the input tone is moved when it goes to the output. So, if you simply look at an output, the output of an LPTV system, and see a tone f, all that it means is that there could be an input tone at any one of these infinite frequencies which are of the form f plus some integer multiple of fs. Does it make sense?

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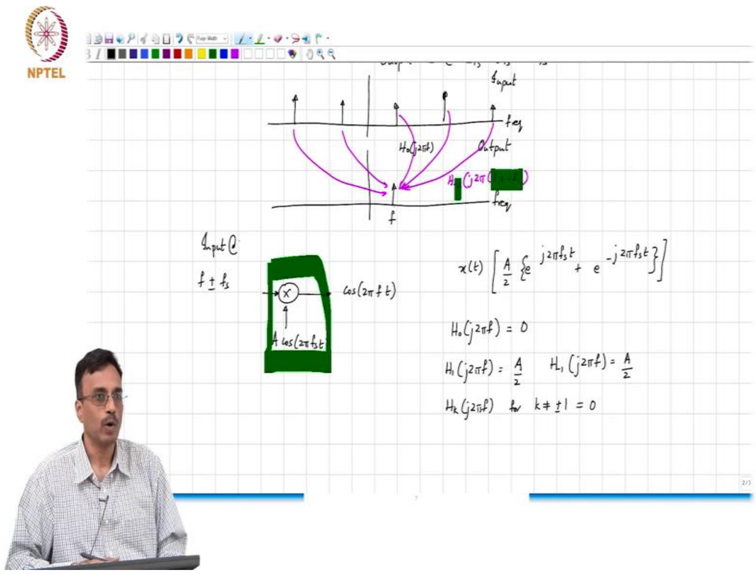

So, for example let us take the simplest possible one. So, let us say this is cos 2 pi fs times t. And the input is, let us say we see an output which is cos 2 pi f times t at, what is... the what are the possible inputs? Look at it carefully.

Student: (())(12:20)

Professor: No, look at it carefully. The system is very well known. What are the, what are the harmonic transfer functions of this, of this guy? Well, if this is the input is x of t, the output is x of t times A by 2 times e to the j 2 pi fs times t plus e to the minus j 2 pi fs times t. So, what comment can we make about H0? Yes? 0. Well what is H1 of j 2 pi f? For H1 of j 2 pi f, you put in a tone at f and look at output at f plus fs. And so, what comment can you make about H1 of j 2 pi f therefore?

Student: (())(13:32)

Professor: A by 2. And like wise H minus 1 of j 2 pi f is also A by 2, and H sub k of j 2 pi f for k not equal to 1; not equal to plus or minus 1, I guess must all be, must all be 0. So, now what comment can you make if you see an output at a frequency f, the input can only be...

Student: (())(14:14)

Professor: Yeah, that is basically either 2, by A times whatever, you do the math. So, the either plus or minus. Does it make sense? And, this is of practical importance in, for example, the system like radio because it means that when you ideally, let us say you want to demodulate certain channel; because of the multiplication you have multiple inputs that can demodulate or fold to the same output frequency. For, example here you see that the output at f basically means the input could have been at f plus fs or f minus fs. And without doing anything additionally, it is not possible to figure out whether the input is that; just by looking at the output it is not possible to figure out whether the input came from f plus fs or f minus fs.