Introduction to Time – Varying Electrical Networks
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Lecture 33
Response of an LPTV system to a complex exponential input

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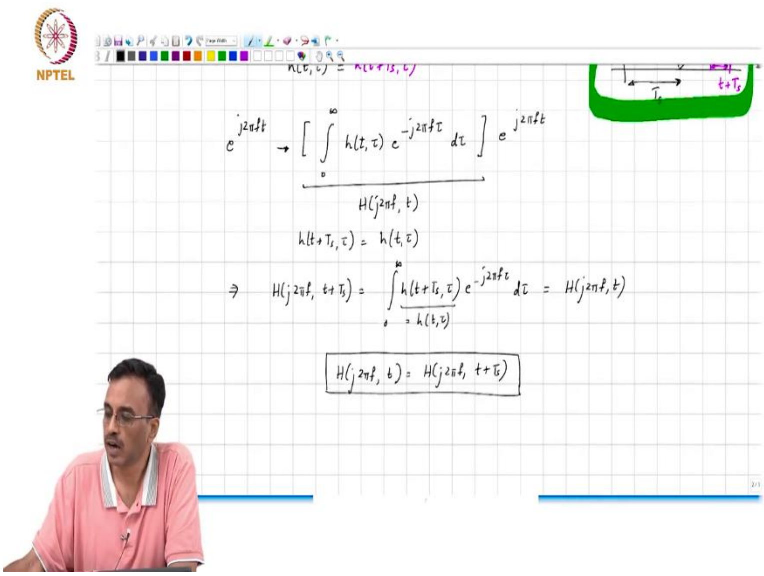

So, and remember what is the response to e to the j 2 pi ft? It was nothing but integral 0 to infinity H to t comma tau e to the minus j 2 pi ft f tau d tau. This times e to the j 2 pi ft. So, this is the frequency response, this is nothing but H of j 2 pi f comma t. Now, h of t plus ts comma tau is the same as H of t comma tau. What does that mean, as far as H of j 2 pi f comma t is concerned? What should we expect?

What do we say before we went through started all this maths, the gain experienced by sinusoid at f should be, the system is varying periodically that the period ts. So, what should we expect for the gain experienced by a sinusoid at f? Yes.

Student: (())(2:05).

Professor: You should also be periodic, so now can you use that to basically look a stare at the expression for H of j 2 pi f comma t and figure out whether it is periodic.

Student: (())(2:18).

Professor: You simply replace, H of k well H of j 2 pi f t plus ts is nothing but integral 0 to infinity H of t plus ts comma tau e to the minus j 2 pi f tau d tau and low and behold this is the same as H of t comma. So, this is therefore the same as H of j 2 pi f comma t. So, in English all that this means is that is the same as H of j 2 pi f t plus ts.

So, I mean this makes intuitive sense, forget about all the maths but you should be able to intuitively see why this makes sense because the properties of the system are varying periodically, so you expect that the gain experienced by the sinusoid will also vary periodically.

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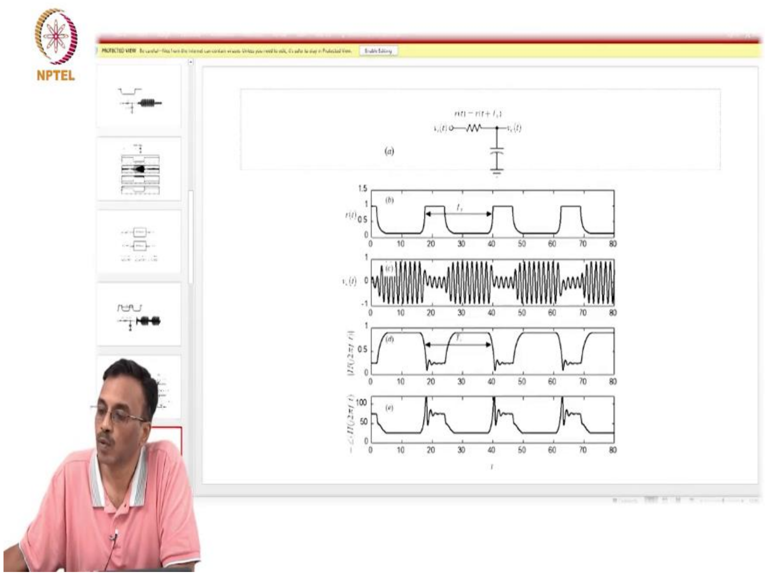

So, the output way form will therefore look like this. So, this we have already seen. So, the resistance is varying periodically and, in this fraction, and as you can see the output way form is also kind of look like this where the envelope is changing periodically.

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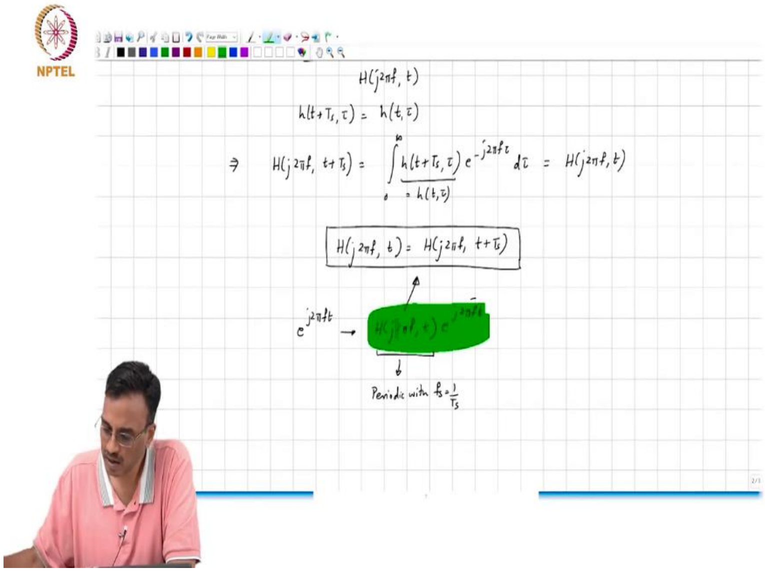

One question I would like to ask you is the gain is changing periodically, can you comment on the output being, so you put in e to the j 2 pi ft into the LPTV system. The output is H of j 2 pi f comma t and e to the j 2 pi ft, this is periodic with frequency fs which is one over t s. can you comment on with periodicity this way form? What is periodic here?

Student: (())(5:02).

Professor: What is periodic on the right hand side, is the whole right hand side periodic? Or…

Student: (())(5:11).

Professor: We have no control over what the input frequency is now. What are all periodic?

Student: (())(5:24).

Professor: This clearly is periodic with frequency 1 over f, frequency f sorry and H of j 2 pi comma t this is periodic with fs which is equal to 1 over ts. What comment can you make about this way form? Unless f is related to f s in some special way in general the way form on the right is not, please do not get be under the mistake and impression that way form, at the output is going to be periodic. The output way form is not going to be periodic, it is only the gain experienced by the sinusoid which is going to be periodic. Does it make sense? There is a difference between the two, do you understand?

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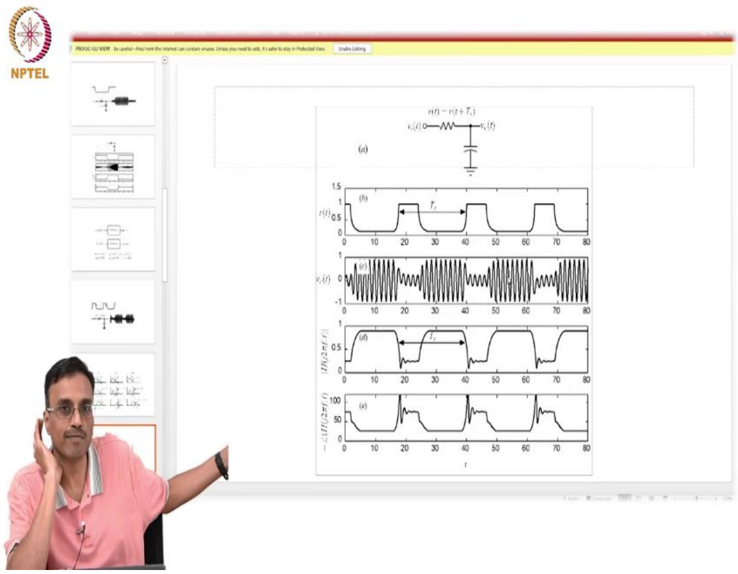

Here is an example I think it is visually obvious that this way form though it is tempting to think it is periodic, this is the period t s and clearly this is not periodic. This is the output way form and as you can see the output way form is definitely not periodic.

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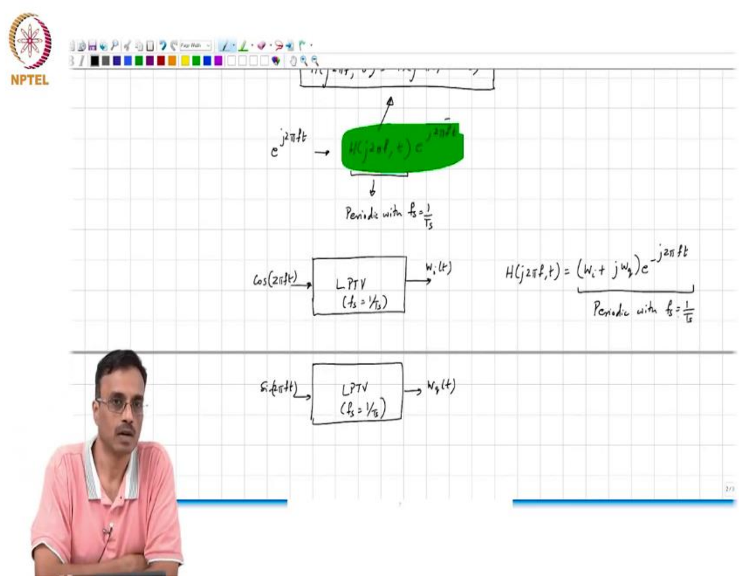

But how do you find the gain now? What would you do to find the gain? In other words, how would we find the gain experienced by the sinusoid? We have the LPTV system. How do we find what the gain experienced by the sinusoid is? Pardon.

Student: (())(7:31).

Professor: So, basically cos 2 pi ft and then the same LPTV system excited with sine 2 pi ft and then you get w i of t w q of t can you comment on the periodicity of w i of t and w q of t? They will not necessarily be periodic, I mean unless that input f is chosen to be, if f happens to be also f s for instance that is one case or n f s then you can expect the output to be periodic but otherwise in general w i and w q will not be periodic. So, what will you do to find, H to j 2 pi f comma t same old it is w i plus j w q times e to the minus j 2 pi ft, is that clear? And now is this periodic or not? This is that H of j 2 pi f comma t and this is periodic with f s, you understand what I am saying?

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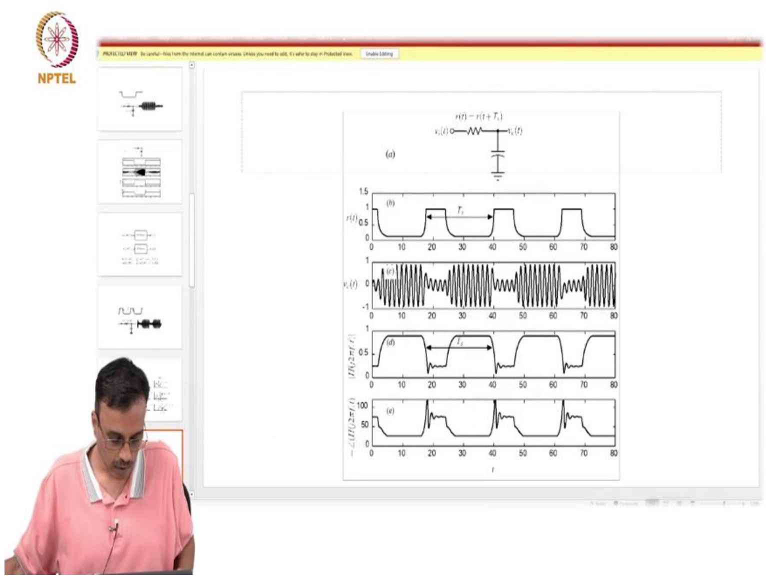

So, and sure enough, here is an example. This way form is clearly not periodic. That guy there that is the output of the capacitor, voltage across the capacitor. But if you look at the real part and the imaginary part or magnitude and phase you can see that this is definitely periodic with a period equal to t s.

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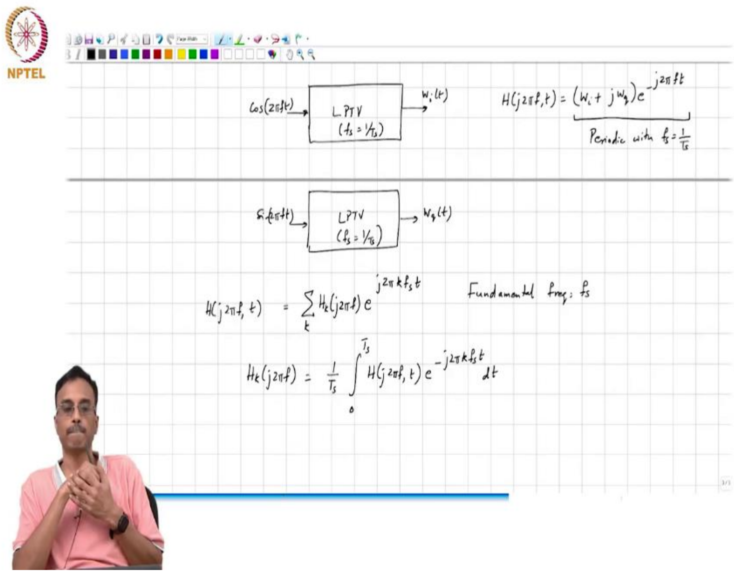

So, now let us see what this means. Whenever we are electrical engineers so whenever we see something periodic, what we would like to do? There is the only thing we know how to do which is expand this in a Fourier series. So, well how will you express this as a Fourier series? Some over k, this is periodic with respect to which variable? Periodic with respect to the time variable, so what will be the so the there, will be some A sub k which is the Fourier series coefficient of the kth harmonic and the kth harmonic is basically e to the j 2 pi the fundamental frequency is f s, so this will be e to the power k times f s times t.

And well, what is ak? How will we find ak? It is 1 over t s integral H of j 2 pi f comma t e to the minus j, 2 pi f s k times t dt from 0 to ts and this is going to be therefore a function of well if you integrated out time what is left? This is H sub k of some j 2 pi f. this is exactly like your regular four-way series I may except that you know that there is it depends on f that is all.

Intuitively what is happening is that if you put in a frequency f (())(12:15) with the sinusoid experiences gain. The gain experiences periodic with f s. Now, if you change the input frequency, what do you expect? You expect that the gain will change but that new gain will also be periodic with f s and therefore the new change f the Fourier coefficients of that periodic gain function will change.

So, that is all that this math is saying that this ak, the Fourier series coefficient happens to be function of f, that is all. So, instead of writing ak to remind ourselves we basically express this as

H sub k of j 2 pi f e to the j 2 pi k f s times t. So, the fundamental frequency of the expansion is f s and so write this as H k of j 2 pi f is this.

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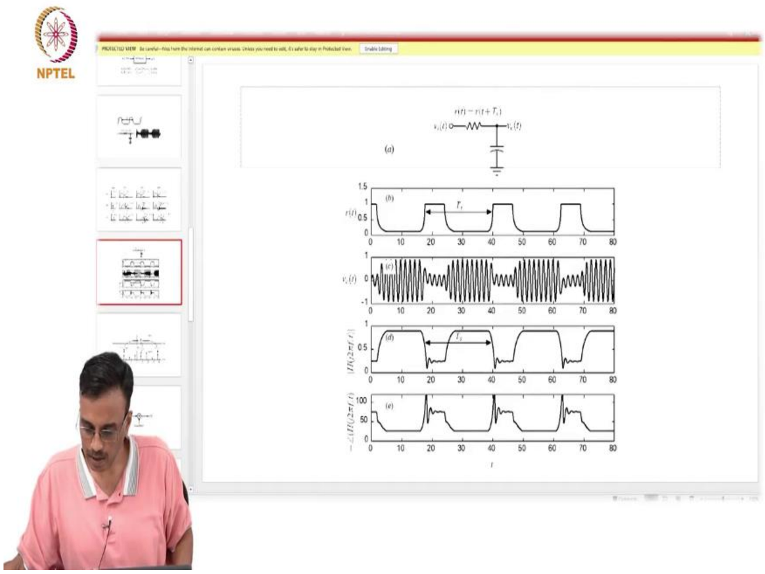

In English all that it means is that we saw this picture remember. If you combine the angle and the magnitude you will basically get a complex th form, which is periodic with time. You are expanding that complex way form in a Fourier series, that is all and therefore you will get a Fourier series expansion.

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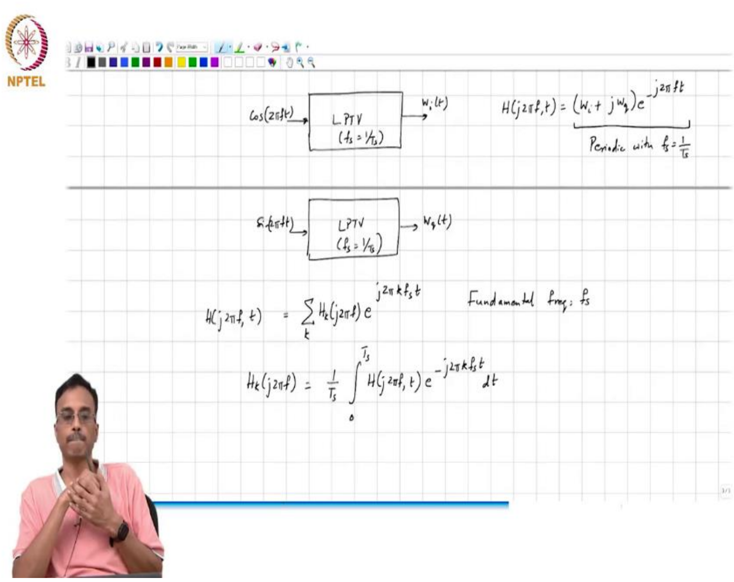

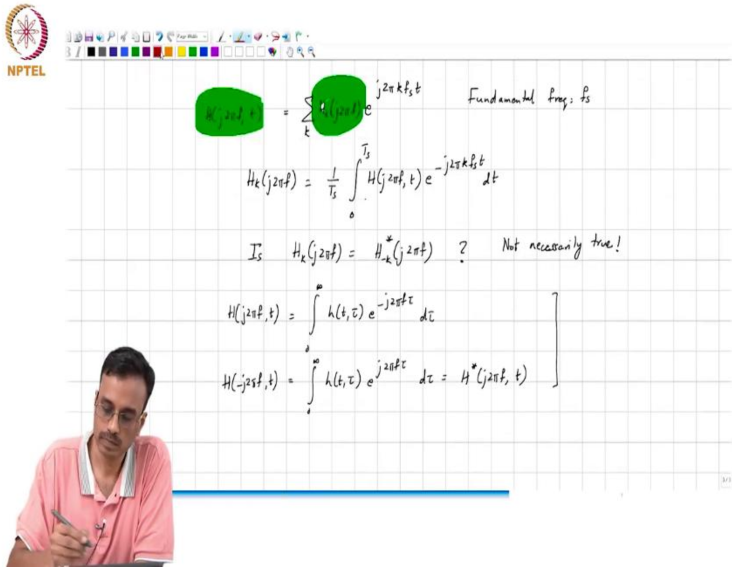

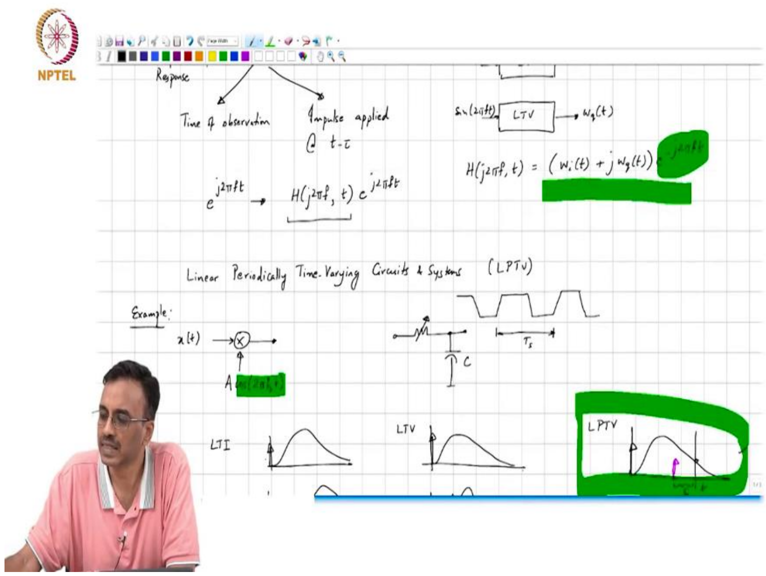

And when you had a real way form what did we I mean, what did that constrain does that impose on the Fourier series coefficient? Conjugate symmetry, so ak, would be.

Student: (())(14:19)

Professor: A minus k star but here what comment can we make? This is not real anymore. This is a complex way form, so when you expand this in a Fourier series should we expect H is get used to seeing H k of j 2 pi f being equal to H minus k of is this correct? This is simply conjugate symmetry of the Fourier series coefficients so the question I am asking you is in all the stuff you have looked till now you probably have this happen automatically, because you trying to deal with expansions of real wave forms. Now, what comment can we make? Is this true?

So, on the left-hand side you have a complex number, a complex wave form so and therefore conjugate symmetry is not necessarily. Can you give me an example where it is actually true? Can you think of an LPTV system where this is actually true? Memory less system. For example, if you have the very first thing here, what is the gain experienced by the complex sinusoid? It is simply A cos 2 pi ft which happens to be, that gain happens to be and suppose to be in general it is complex but in this particular special case it happens to be real, it is only in that case will this hope.

However, remember that H of j 2 pi f comma t must be the same as H star of what is H of j 2 pi f comma t? It is nothing but integral 0 to infinity h of t comma tau e to the minus j 2 pi f tau d tau, so H of minus j 2 pi f comma t is simply you have seen this before j 2 pi f tau d tau which is H star t, H star of j 2 pi f comma t. Now, we have seen this already, what does this mean, if H of minus j 2 pi f is H star of j 2 pi f comma t, what comment does it, what is that (())(17:55) for the H sub k?

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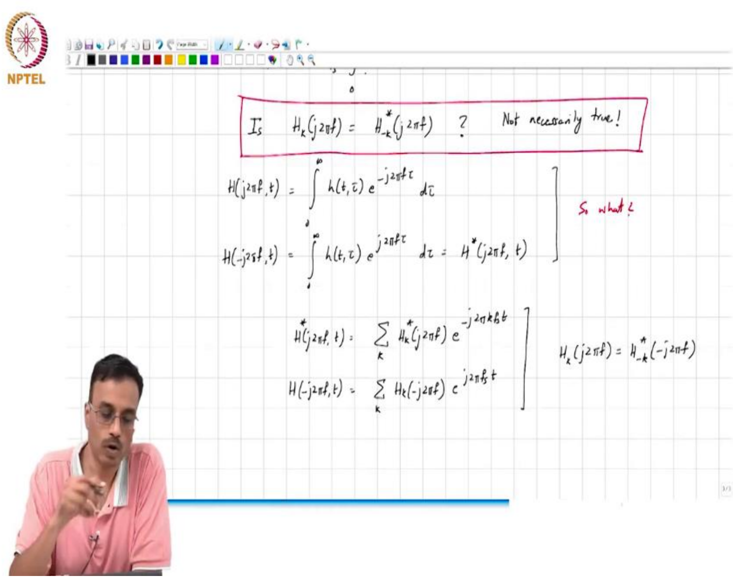

What does this impose? What constrains is that impose on H sub k? H of minus j 2 pi f comma t, therefore, is simply H k of minus j 2 pi f e to the j 2 pi fs times t and H minus j 2 pi f must be the same as H star. What is H star? So, H star is this pardon, so H star is this and now can you stare at these two and tell me how they must be related? So, H k of j 2 pi f must be equal to H minus k star of j 2 pi f or minus j 2 pi f? Minus j 2 pi f.

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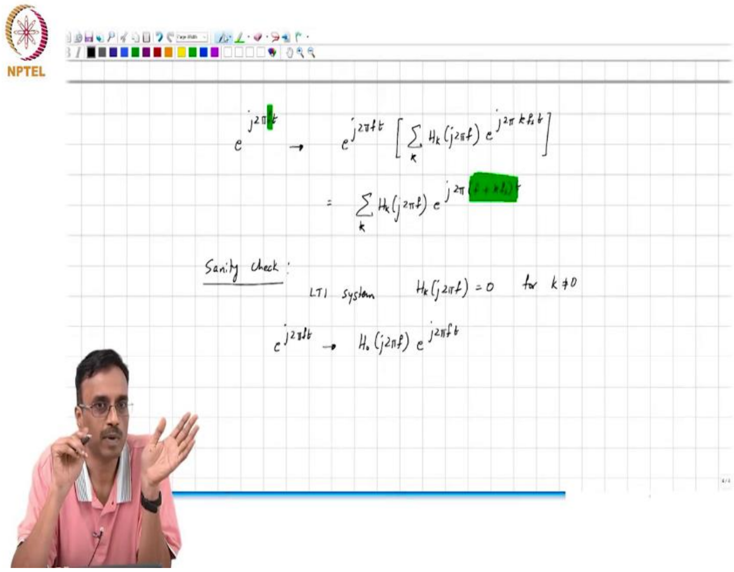

So, the next thing to note is the following. So, we excite the system with e to the j 2 pi ft outcomes e to the j 2 pi ft times a complex gain which is varying periodically with time, so that we know can be expanded into H k j 2 pi f e to the j 2 pi k fs times t. Which is equal into saying that this is nothing but k H sub k j 2 pi f e to the j 2 pi into f plus k fs times t.

Sanity check, a linear time invariant system is a special case of a, linear time varying system it is a special case of a linear periodically time varying system. Where the gain happens to be constant with time. So, if the gain is constant with time, what comment can you make about the Fourier coefficients, which of those Fourier coefficients will be?

Only the dc component will remain same, so that basically LTI system. H sub k of j 2 pi f equal 0 for k naught equal to 0 and therefore which you put in e to the j 2 pi ft you only get H 0 of j 2 pi f times e to the j 2 pi ft. Now, we knew this of course, there is nothing new about this one. So, the key point is that in an LTI system when you put in f, a frequency f at the output you only see the same frequency f. Turning this upside down, at the output of an LTI system if you saw a frequency f it must mean that the input must also have been at the same frequency f, you knew this of course.

Now, in an LPTV system the input is at f, what comment can we make about the output frequencies in general?

Student: (())(23:59).

Professor: So, the input frequency is f, in an LPTV system in general you will see output frequencies, you will see tones in the output at frequencies f, f plus fs, f plus 2 fs, f minus fs. So, you will see all frequencies of the form f plus k times fs where k is an integer. Does it make sense?