

**Introduction to Time – Varying Electrical Networks**  
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**LTV system example: Time-varying RC filter**

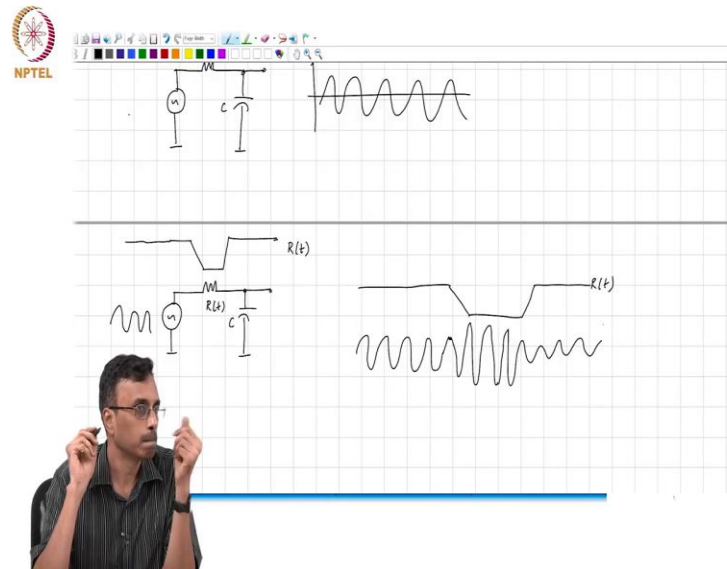
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The slide contains the following content:

- NPTEL logo** (National Programme on Technology Enhanced Learning)
- Two mathematical expressions:
$$\operatorname{Re} [ H(j2\pi f, t) ] = W_r(t) \cos(2\pi ft) + W_i(t) \sin(2\pi ft)$$
$$\operatorname{Im} [ H(j2\pi f, t) ] = W_r(t) \sin(2\pi ft) - W_i(t) \cos(2\pi ft)$$
- A circuit diagram of an RC network. It consists of an AC voltage source on the left, a resistor labeled 'R' in series, and a capacitor labeled 'C' in parallel with the output terminals.
- A graph showing a sinusoidal waveform, representing the output of the RC network.

So, let us take some simple examples and see what goes on, before it gets too messy with more algebra. So, a throughout let us keep using the simplest possible circuit which is the first order RC network. If this is a sinusoid and this is  $r$  and this is  $c$  and  $R$  is fixed and  $C$  is fixed. What comment can you make about output? Well, this is a sinusoid, so the output will also be some sinusoid you should assume that is the sinusoid. The magnitude is fixed and the phase shift with respect to some arbitrary references is also fixed.

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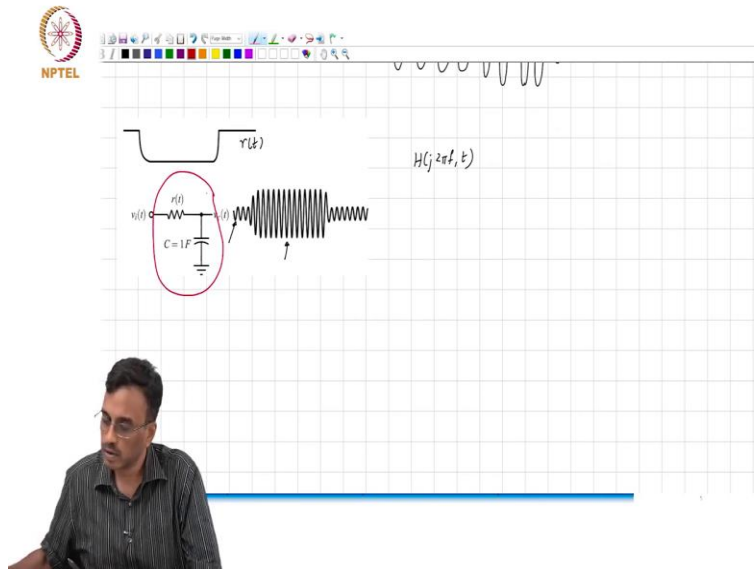
Let us now make this a linear time varying network. So, now what I am going to do is vary this resistance, say like this. So, the resistance is constant for a long time and I pulled  $t$  down, so this is  $R$  of  $t$ . So, what do you expect qualitatively to happen? The input is a sinusoid it is some frequency. What do you expect to see at the output?

Student: (())(2:03).

Professor: So, basically if the  $r$  of  $t$  did something like this, then the output would have been some sinusoid all the way up to here. Then here some weird stuff happens then what should expect to happen? Let me just make things, what do you expect to happen here? Once you settled in here, well the sinusoid must I mean you would expect that all the traces have died down and what do you see is a sinusoid whose amplitude is, the resistance is reduced and the capacitance is same.

So, the effective bandwidth of the circuit is, the bandwidth of that RC low pass filter, if you want to think about it that way has increased. If the bandwidth is increased, you should expect that the envelope of the sign wave must increase and what comment can you make about the phase shift? Will it remain the same as it was, I mean will it simply be the sign wave with a larger amplitude or do you expect the phase shift also to have changed? You expect the phase shift also to change with time. So, you will see something like this and then again it will come back to its old self and sure enough if you.

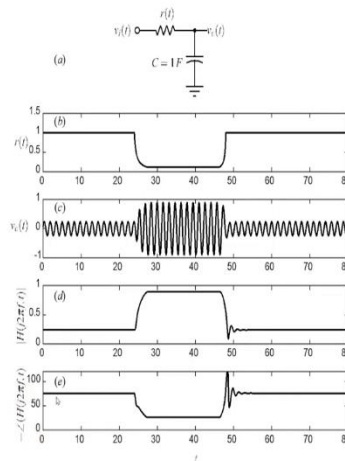
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So, here is an example. So, this is  $r$  of  $t$  and when  $r$  goes down you expect the bandwidth to increase and therefore, the output should be expected to increase. It is not apparent here that the phase shift of this sinusoid with reference to the input is different from the phase shift of that sinusoid and to therefore, find  $H$  of  $j 2 \pi f$  comma  $t$ , what should we do?

If I wanted to plot the actual gain and phase experienced by this, if I wanted to find the gain and phase of this time varying system, what should we do? What would you do? What we just discussed right now, so basically we excited with  $\cos$  and  $\sin$  and use the formulae we derived namely the real part is  $w i$  times  $\cos$  plus  $w q$  times  $\sin$  and imaginary the part is  $w q$  times  $\cos$  minus  $w i$  times  $\sin$ .

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So, this is an example where, so  $r$  of  $t$  varies as you can see here and you can see that the envelope of the sine wave does indeed increase with and eventually once the  $r$  becomes small, you can see that the envelope actually increases and it turns out that in this case the, what do you call the bandwidth initially is 1 hertz and finally the bandwidth is if you look at it I think it is one eighth of a hertz so something like that.

The resistance is gone from one ohm to point 125 or something like that. So, the bandwidth is gone from 1 radian per second to 8 radian per second and since in the beginning you can see that the sinusoid is significantly attenuated that is, at some frequency and there is some phase shift, and the so since the magnitude is significantly smaller than what do you put in, I mean the input is a 1 volt sign way. The output magnitude is significantly smaller up to about 20 seconds or so, than what you put in. So, what comment can you make about the phase shift?

Here order RC filter you putting in a input sinusoid, the magnitude of the output sinusoid happens to be much smaller than what you put in. What comment can you make about the phase shift, that we should expect between the input and output?

Student: (( ))(7:28).

P: What large? I mean, close to, you should expect the phase shift close to 90 degrees and this is indeed what the computation shows you. If you find the real part of  $H$  and the imaginary of  $H$  and then plot I mean convert them into polar form and plot the magnitude and phase, so you can

see that the magnitude is very small something like 0.25 or something and the phase shift is, since the magnitude is so small, you should expect the phase shift to be somewhere about 90 degrees. So, what I plot is the negative of the phase shift.

So, that is indeed true and the next is when the resistance goes low, we see that the what do you call the magnitude of the sign wave has significantly increased and it is increased to say some 0.9 or something like that. So, now you should expect the phase shift between the input and output to be much smaller and that is indeed what seems to happen in steady state and when the resistance goes back up again you see that the magnitude goes back to be 0.25 or whatever and the phase shift goes back to being about 90 degrees and during the transition both the magnitude and the phase do weird things, that is how.