Introduction to Time - Varying Electrical Networks
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Lecture - 3
Power Conservation and Tellegen's Theorem

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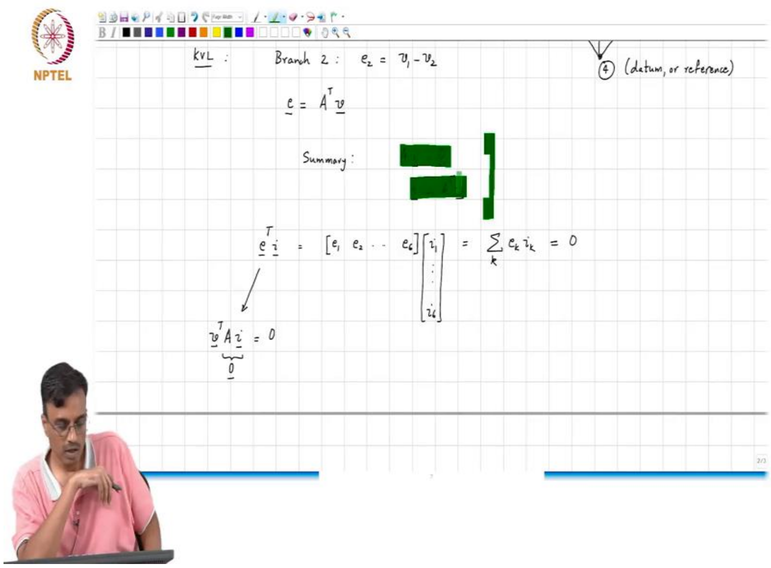

One interesting aspect that I would like to point out is the following. What interpretation can we give to this product e transpose times I, this is nothing but in our particular case, we sum over all the branches. What is this? I may write this out in full form. So this is nothing but e is a column vector. So e transpose is a row vector. And how does that look? e1, e2 blah, blah, blah, all the way to e6.

First of all, are we, is this product legal? What is the size of e transpose? 1 cross 6. i is 6 cross 1. So the product is legal. So that is simply this multiplied by i1 all the way to 6. And so, this is nothing but Sigma over k ek ik.

What interpretation can we give to this quantity? It is the sum of all instantaneous powers being dissipated in the branch. And intuitively, what do you expect that to be? If you take all the branches in a network, and compute Sigma ek ik, which is basically saying, it is telling you each ek, ik is simply the instantaneous power that is being dissipated in the kth branch, and the summation is basically adding up all the powers in all the branches at instant in time.

So what would you expect to see if you form this product, sum over all branches ek ik? You expect this to be 0 and why does that make sense? Law of conservation of power. Remember, power and energy are two different things. Power is an instantaneous quantity. Energy is an integral. Energy is of course conserved but this is also telling you that energy is conserved at, at every instant.

So nature does not, for good reason, believe in saying, you know, at this point in time, you take more energy than you generate, you return it to may at later time, right? Because nature has seen to many scams in the past, so he knows that you know, you should neither be a borrower nor a lender.

So at every time, the power is conserved. So total power generated and total power dissipated in all the branches is the same. So let us see if that is being predicted by our KCL, KVL equations. And so, what is e transpose? It is simply v transpose A and of course, this is i. And we know that this guy is 0. So this must be the scalar 0.

Student: (())(04:06)

Professor: Pardon, where does n comma n? No, it is this A. So this is simply saying, I mean, it may not be, it is not very surprising that this is true. It is probably something you expected anyway. But what is surprising is the following.

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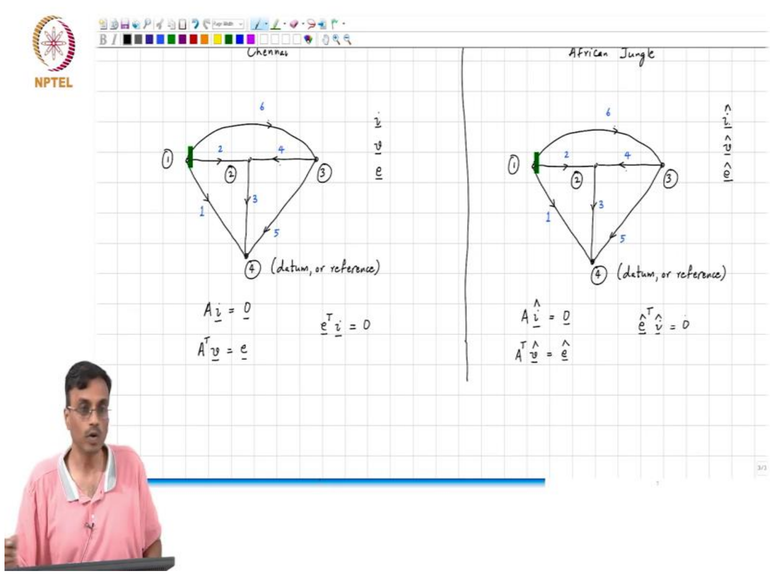

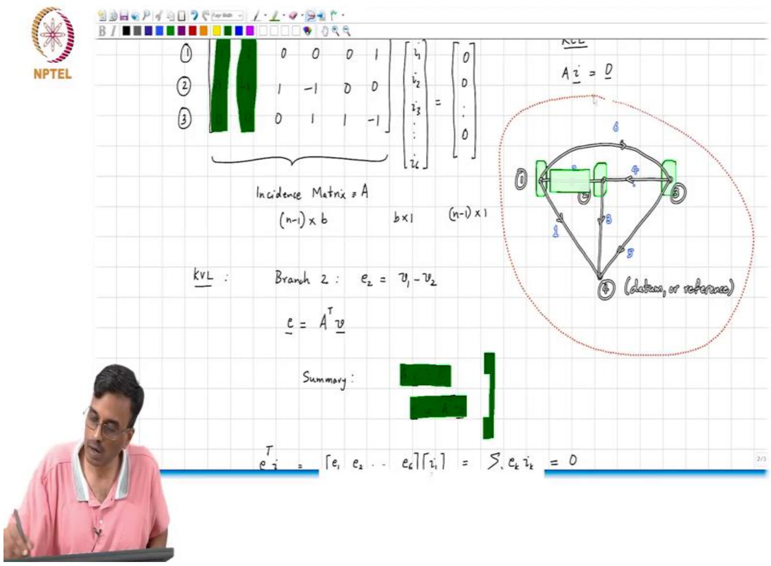

So let us say we have one network like this. So this is, let us say, a network that we have in Chennai and somewhere deep in the jungles of Africa, you have somebody who is totally unknown to you, set up another network with exactly the same graph.

The only commonality between both these networks is that their skeletons are the same. One could be linear, one could be nonlinear, one could be time invariant, one, the other could be time invariant, the time variant. It does not matter. All that it, all that we say is that these two networks are topologically identical.

Now, of course, KCL and KVL must be universally valid. So A, so what comment can we make about the incidence matrices of both these graphs or both these networks is the same. So A times, I is equal to 0, and what is KVL here? A transpose times v equals e.

And this is of course another network, so apart from the incidence matrix being the same, there is no other relationship between these two networks. And this is basically, let us call these branch currents i hat, and therefore, A times i hat equal to 0.

And what comment can we make about this KVL here? So the branch (volt), current vector and the branch voltage vector and the node voltage vector here are i, v, and e. Here, that they are i hat, v hat, and e hat. So what comment can we make about.

Student: (())(07:48)

Professor: A transpose times v hat equal to e. And we know obviously that e transpose times i equal to 0. And, of course, e hat transpose times i transpose times, i hat must also be equal to 0.

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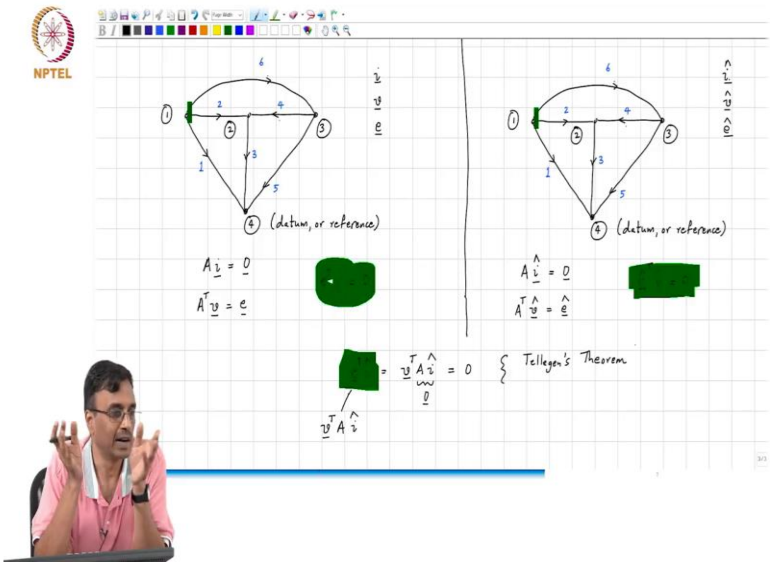

But let me show you an interesting thing, and that is, let us try and form the product e transpose times i hat. And what physical interpretation, if any, can we give to this to this quantity? What are the dimensions of e transpose times I hat?

It is scalar and it has also got dimensions of power. The only difference between what we did here and what we are doing now is that to compute, within course, the branch power. We are taking the voltage in one branch and the current in a network which is, you know, 1000s of miles away. We do not know what that network is apart from the fact that we just saw its graphs.

Now, let us see what happens when we do that. And that is e transpose is what? It is v transpose times A. And what is i, well this times i hat. So what is this, what is this equal to? And this is, well, this is the zero-vector. So what comment can you make about this quantity? It is 0, so this throws up the somewhat surprising, the fact that these two are 0 is no is no surprise at all.

That is something that you physically expect. But what is is very surprising is the fact that you take the, that you find this "some power like quantity", where you multiply the voltages in one branch and the current, you take from a network which is completely unknown to you. The only thing that you know is that it is, it has the same graph. Surprisingly, it seems as if that is also equal to 0.

We will take a look at the intuition behind why this, this is true. It is not as surprising as you might at first think. At first, you might think that you know, how does the network know what current is flowing in the branch, in some network that it does not even know, correct? And magically, all these products add to, add to 0.

So to show you why this makes, it is not as surprising as it one might seem and you know, you might also think that, well, with this, all this matrix stuff, there is no intuition. I mean, it just turns out, the math comes out to be you know, if I do the math, evidently, the answer is 0. So it must be the truth.

But is there intuition behind this result? And this result is a very important result and this is what is called Tellegen’s Theorem. And quite surprisingly, it was discovered somewhat late given that people have been working with circuits for a really long time.

This theorem came out sometime in the late 1950s only. He would have thought that somebody would have thought of this "obvious observation a long time ago" but it is actually quite recent, relatively speaking.