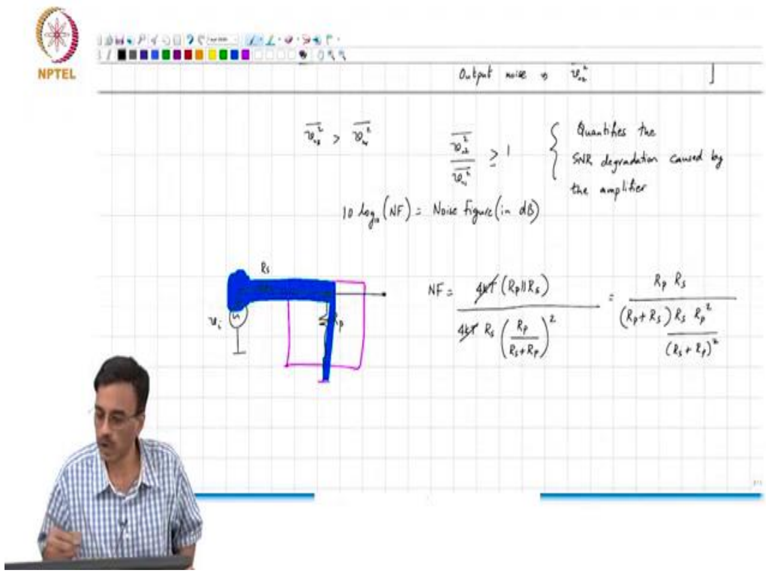
Introduction to Time - Varying Electrical Networks

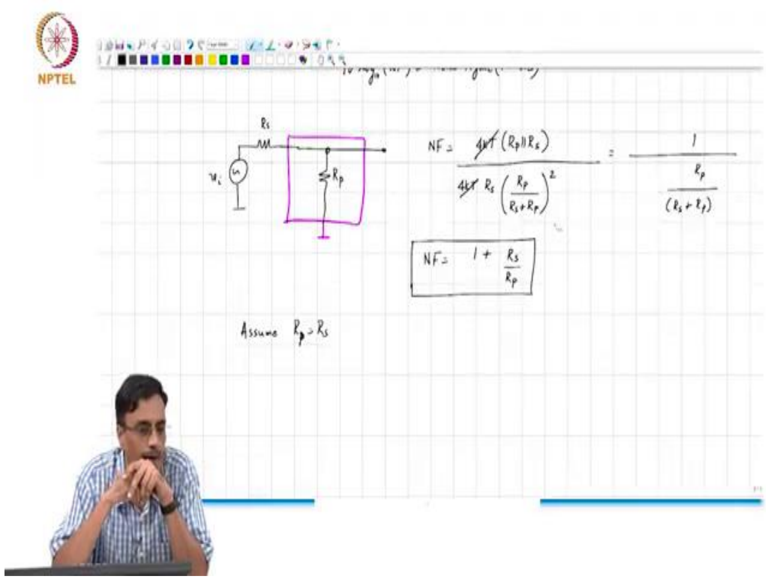
Professor Shanti Pavan
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture 25

Noise Factor Examples

(Refer Slide Time: 0:27)





So, let us do some quick examples to help you figure out what the noise factor is? So, let us say this our source, this is Rs. And this is within quotes are blackbox. And I am going to say, oh well my blackbox simply has got a resistor Rp. Now, what comment can we make about the noise factor? What do we do?

We need to find the output noise. And we need to find the output noise assuming the box itself is noiseless. So, what is the output noise spectral density, what is the output voltage noise spectral density? It is 4 kT times, pardon? Why RP by RP plus RS is the whole square? It is simply, what is output noise voltage spectral density? 4 kT Rp parallel Rs. Any confusion about this?

Now, you have to divide this by the output noise spectral density that would have resulted provided, which was noiseless? Rp was noiseless. Now, what would be the output noise spectral density? It is 4 kT times, it is 4 kT times Rs that is the noise spectral density of this resistor. And how, what is the transfer function from this noise source to the output?

Rp by Rs plus Rp. So, why should I do this? The noise spectral density is going through within quotes again. So, we have to multiply by, again square and therefore this is nothing but the 4 kT goes away, so this is Rp Rs divided by Rp plus Rs times Rs times Rp square by Rs plus Rp whole square.

And therefore, this goes away, this goes away, this goes away, this goes away, this goes away, this goes away. So, this is nothing but, the noise factor is nothing but 1plus Rs over Rp. Sanity check. Well, RP is infinity. What should we expect? Oh, well, you are not doing anything. So, presumably you are not losing SNR. So, I mean, sure enough, you get a noise factor of 1or 0 D.

And, and, I mean, well, we did not, I mean, we said this is our amplifier, so to speak, but what you are actually doing is that attenuating. So, if you simply attenuate with the resistive attenuator, as you can see, this is telling you that you are always going to be, I mean you are going to be degrading the SNR. And why does that make sense?

Student: (())(5:11)

Professor: Well, yeah, that is I mean, I guess that is right. So, what comment can you make about the absolute noise spectral density at the output has that. Let us assume, let us assume for argument's sake that Rs is equal to, Rp is made equal to Rs. So, what comment can we make about the noise spectral density of the output when compared to Rp equal to infinity? It is what comment can we make about the output noise spectral density if Rp is equal to Rs?

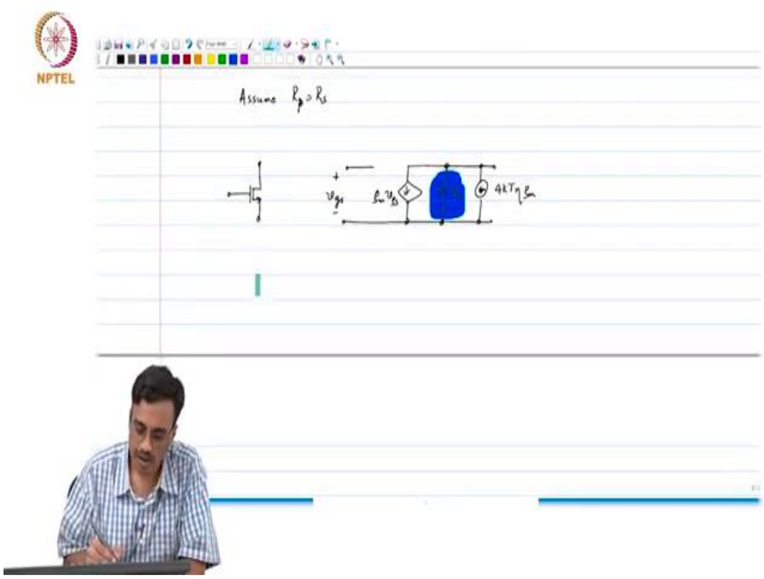
It is 4 kT times Rs by 2, so the noise spectral density at the output has actually gone down that does not mean that the signal is becoming any, I mean the signal to noise ratio is improving.

Why? Oh well, the signal is also attenuated by the same factor which basically means the signal power has gone down by, right, noise has gone down by half, but the signal power has gone down by a factor of 1four, by a factor of 4.

So, the SNR has gone down by a factor of, the noise power has gone down by a factor of 2, signal power has gone down by a factor of 4. What comment can we make about the SNR? The SNR has actually gone down by a factor of 2, even though the noise floor has actually reduced by a factor.

And that is also, that is what the equation is telling us. If Rs is equal to Rp, then the noise factor is 2 which means the output SNR is lower than the input SNR by a factor of 2. The noise factor does not say anything about the absolute noise flow. It is only quantifying how badly you have done with respect to 2 SNR. Let us do another example.

(Refer Slide Time: 7:26)

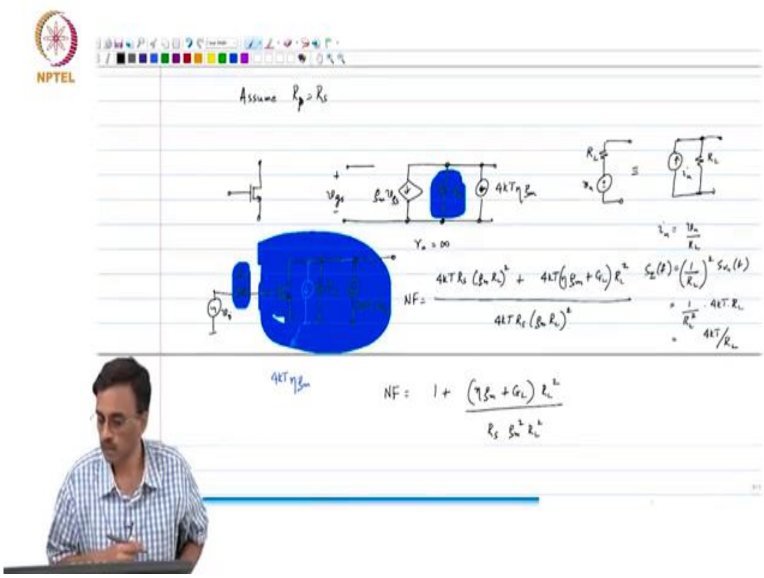


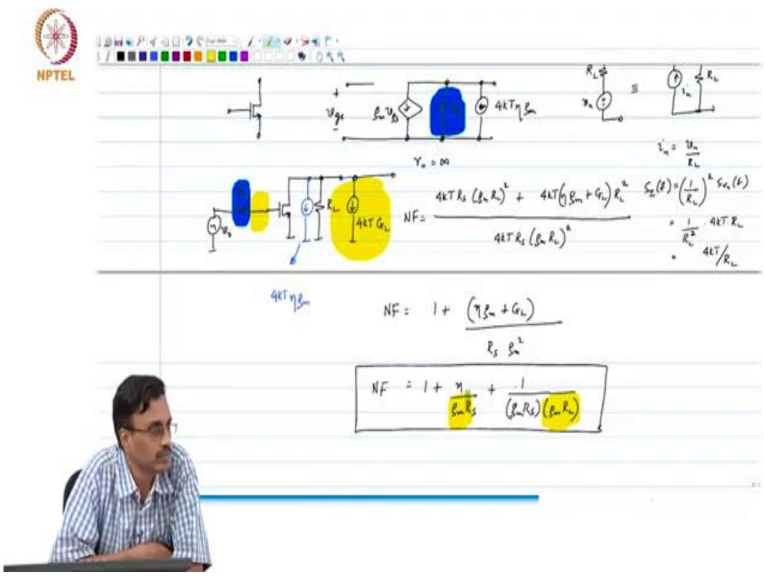
Now, let us take a transistor and at low frequency what comment can we make about, what is the small signal equivalent of the transistor? It is nothing but this is the incremental VGS, this is gm times vgs, this is ro and as I mentioned, the transistor is accompanied by a noise source whose spectral density is for the long channel MOS transistor can be shown to be 8 kT over 3 times gn but then a long channel MOS transistors only exist in the textbooks. So, what better formula is to use 4 kt eta times gm, eta is some number that fits measurements, eta or gamma or whatever they are, they have some, that is the usual non ideality factor.

Ideally, eta should be 2 thirds. In reality, it is more like 3 halves. So, whatever I mean, the designer has no control over what you get, it is what you get. And so, another thing I like to point out is that even though you have r o which models the finite nonzero lambda of the device is not a physical resistance. And therefore, there is no noise, so that current source is not the noise associated with, it is not noise associated with that r o.

And, so this is a simple model for noise. In fact, it turns out that in reality, there is also another noise source in the gate, but that only becomes, shows up at really high frequency. But for most calculations, at least at low frequencies, this is good enough.

(Refer Slide Time: 9:38)





Now, let us do a simple common source amplifier whose incremental picture is shown here. And this is VI, this is Rs, and that is RL. So, what comment can we make? How do we, what is the noise factor? What do we do? First thing is to find the total output noise. So, what will be the total output noise, noise spectral density?

Well, what is the gain from Rs to the output, the voltage source in series with Rs to the output? Well, let us assume again, let us make life simple and assume that r o is equal to infinity. So, what is the gain from the noise source to the output? Minus gm RL, So, what would be the output noise spectral density due to Rs? It is 4 kT Rs times gm RL minus gm RL the whole square which is the same as plus gm RL the whole square. Plus, what else? Is that all?

How many noise sources are there? The main thing here is the transistor man. Huh? How many noise sources do we have? 3 noise sources. Now, I mean with, even I mean, of course, the 3 noise sources we can say we can all do this in our minds, but you can imagine what will happen if you have 50 transistors like this and there is tons of noise sources, and there is 1 output, and you need to now calculate, and I made our life simple by assuming r o is infinity and there is no parasitic capacitances, etc.

In reality, all that stuff is going to come in and then it is going to be a mess. So, that is where the adjoint is, the inter reciprocal network is so useful, because you write down the (())(11:50) equations of this big network, you solve it once, and you get all the transfer functions that you are looking for.

So, back to our example. So, there are 3 noise sources. Fortunately, it seems like we can use some simple trick to convert that into 2. So, this noise source is that of the transistor and it has 4 kT eta gm, the noise source of the resistor is a voltage source. But we know how to convert from not an equivalent to a (())(12:32) equivalent.

So, Vn in series with RL is equivalent to in parallel with RL. And what is i n? v n by RL. So, the noise spectral density of the current source is nothing but well, you are multiplying that current by 1by RL times v n. So, you multiply this by a constant so you have to I mean, you multiply that by 1 by RL square times S v n of F which is nothing but 1 over RL square times 4 kT RL, so this is nothing but 4 kT over RL, makes sense guys.

So, we have 2 noise sources here, one which corresponds to the transistor, one that corresponds to 4 kT GL. So, what is the total noise at the output therefore? This is the input source plus what is the total noise voltage at the output now? Yes, Danish? 4 kT eta gm plus GL that is the current flows into RL. So, what should I, what should I do here? RL square, this is the total noise at the output of the amplifier, and you have to divide this by the noise that you would have got if the amplifier itself was noiseless.

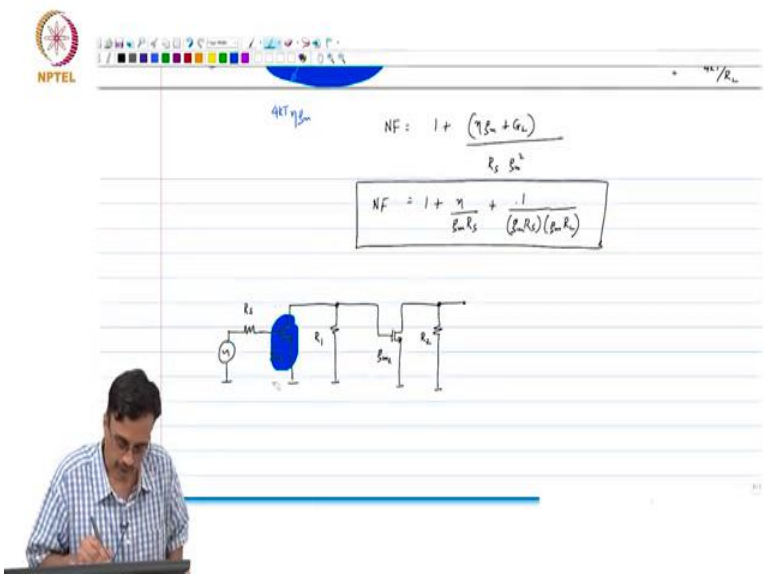
So, in other words, everything inside this blue box, if it was noiseless what is the noise you would have gone? What would you have got that is easy to do is simply 4 kT Rs times gm RL. So, this is the noise factor and therefore, the noise factor is 1 plus eta gm plus GL into RL square divided by Rs times gm square RL square.

And so, this goes away, this goes away and you have 1 plus eta over gm Rs plus 1 over gm Rs times gm R. Now, why does this make intuitive sense? Or is there a sanity check? Well, if RL was noiseless, the noise factor would be 1 plus eta over gm Rs. Now, what comment can we make about the noise spectral density added by the resistor in relation to that added by the transistor?

Actually, we missed, no I think we are okay. So, what comment can we add? Yeah, so basically, when you refer, I mean adding a noise current here is equivalent I mean, what disturbance at this node would cause a disturbance of 4 kT, I mean, in other words, what disturbance must I add here at the gate to cause a current i in the drain?

Whatever disturbance there is in the drain current that has to be divided down by gm in order to. And so, if you are looking at the output voltage, this current will cause, we will have a gain of only RL whereas the voltage here will have an effect of gm times RL. So, you can see that the noise added by the resistor is I mean RL is lot less important than the noise added by the transistor, because if you choose a large gain gm RL is a large number. Basically, it is saying that the noise added by the resistor, noise current added by the resistor, load resistor is much smaller than the noise current added by the transistor itself.

(Refer Slide Time: 18:50)



The next thing I am going to do another example, but I am not going to go through the algebra, I will leave you to do the algebra but I am going to what, I would like to discuss intuitively what we should expect. So, this is a 2-stage amplifier is gm 1, R 1, gm 2 R 2. And so, which, I mean based on our experience so far, which, what do you think? I mean which are the noise sources do you think is the most, contributes the most to the noise figure?

gm? gm 1. Why? Intuitively why does it make sense? Oh well yeah, that is, that is right I mean any, the noise source, any noise added by the transistor the noise current is now convert into voltage that is further amplified. So, a lot of the noise here will be due to gm 1and Rs. And as you keep going further down the chain, let us say we have a long cascade, the noise figure is going to be, if you have a cascade of amplifiers, each 1which is amplifying the signal, and of course, also amplifying noise from preceding stages, which of these stages will be the most, which of the stages would you expect is the most critical as far as the noise is concerned?

The first stage. And which do you think, as you keep traveling down the chain, the signal is getting bigger and bigger. So, which would you worry about distorting the signal a lot? Well, yeah, the last stages are the ones which you should be primarily worried about as far as distortion is concerned. And the first stages are the ones that you should be worried about, primarily about noise. And that is how it goes.

So, that is all I had to say regarding noise factor and noise figure. And you should be in a, I mean and again, I would like to remind you that so far, we have just discussed circuits where there is no memory so that the algebra becomes easier on the blackboard, in reality all these transfer functions will have, there will be frequency dependence of all these gains and stuff like that. So basically, wherever you see a gain, you replace it with a transfer function. And therefore, in general noise factor is a function of, it is a function of frequency because gain is a function of frequency. So, that basically is all I had to say about noise factor.