

Introduction to Time – Varying Electrical Networks

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Lecture 23

Input Referred noise in electrical networks – part 1

(Refer Slide Time: 00:19)

The slide contains the following content:

- NPTEL logo and a toolbar at the top left.
- A block diagram of a network with input v_i and output v_o .
- A circuit diagram showing a voltage divider with resistors R_i and R_e .
$$\frac{v_o}{v_i} = \frac{R_e}{R_i + R_e}$$
$$\frac{v_o}{v_i} = \frac{R_i}{R_i + R_e}$$
- A circuit diagram showing a network with resistors R_i , R_s , and R_e .
$$\frac{v_o}{v_i} = \frac{R_e}{R_i + R_s + R_e}$$
- A handwritten note: $\frac{AR_2 + B}{CR_3 + D} \rightarrow \text{Bilinear function}$

The next thing that I like to talk about is, I mean, so this is all I had to say about passive RLC networks and the next thing I would like to talk about is well, what happens when you have a big network and you want to simplify the treatment of noise. For example, I mean, we have seen similar situations before. So, we have a big amplifier or a big network inside I mean with multiple Rs, Ls and Cs and transistors, and it turns out that every as I promised before, every transistor also has noise associated with it.

A simplistic model for the noise in a transistor, it turns out that, as far as small signals are concerned, we know the small signal equivalent to the transistor can be of the MOS transistor can be, kind of shown to be this one here. Where this is g_m times v_{gs} and this is r_o . You are familiar with all this and we are not going to go into the details.

So, it turns out that the transistor also adds noise of its own and the noise current is, well it's, to find the spectral density of that noise current there to go through the physics of the MOS transistor and then it turns out that the formula is $8KT$ by 3 times η times g_m and what are the units?

Student: () (02:25)

Professor: It is ampere square per hertz and that eta is some non-ideality factor that whose details depend on the transistor channel and then all that fun stuff. But as far as circuit work is concerned, we just assume without, we are not going to go and sit and derive how you get $8KT$ over 3η times g_m just like we did not derive or we did not question the fact that the noise spectral density of voltage spectral density of the resistor is $4KT_r$. We just take that for granted and work with it.

So, if you have an op amp for instance, with the 25 transistors, each transistor is now associated with a noise source with the spectral density. Note that the spectral density depends on the operating point of the transistor. So, when you linearize the nonlinear circuit for after calculating its operating point, you can also add in the appropriate noise sources and therefore, you now end up with a linear network, with rLC and controlled sources and a whole bunch of noise sources inside.

And so, rLC and then polar transistors and with their associated noise sources. And the question is what happens with what happened, I mean evidently, the output now not only consists of the input signal that is processed by whatever transfer small signal transfer function that the box has to offer, but also is accompanied by noise that the internal devices inside the box add to each of the branch currents and branch holders and when we have a big box like this and while it is true that the internals of the box are of interest to the person who designs the box.

I mean where I am giving the box to you for instance, I mean you, you are a user of the box. You do not really care about all the gory details of the construction inside the box. I mean, just like you have a cell phone? I mean you are not going to go and like, as soon as you get your phone, new phone, you are not going to rip it the whole thing apart and look, what ICs are there inside and what parts have been used? As far as you are only concerned about how the user interface looks.

In a similar fashion given this box, as a user of this box, I am only concerned about what the within quotes features or properties of this box are. For instance, I might be worried about the input impedance of this box, I might be worried about the output impedance, I will be worried about the gain and as a designer of the box or a vendor of the box, I have all that I need to do is to tell you, the user that this is the input impedance, this is the, this is the output impedance, this is the gain or equivalently, I will give you the four to put parameters, whichever favourite, you have.

Why your z or h or g or whatever and these four parameters, basically you know, distil all the information regarding the internals of the box, as far as behaviour across these two ports are concerned. Now likewise, there are, there could be a million noise sources inside the box. But as a user, I would like to have a simpler representation of this, these (6:30) noise sources inside the box. It does not, help me to basically, it does not help if you just gave me a big schematic with, 500 noise sources and all sorts of vague transfer functions from each noise source to the output.

All I am interested in is, I have this big box. I know it is a two port parametric representation, how do I now, I mean, what do I do now, when there is how do I represent the noise added by these multiple sources inside the box in a convenient fashion that I can work with? So, in other words, so let us assume that we have as a user, I have a source v_i a source resistance and some output voltage v_o and in every resistor associated with the noise current v_n and every current

So, every transistor with a current source whose details can be formed from the operating point. Now, at a certain frequency f , I mean clearly there is a transfer function from the noise source to the output and do you think that noise the transfer function depends on this guy R_S or it is independent of R_S ? Independent? Yeah, well, it depends.

So, I mean clearly R_S is I mean, if you think of the whole thing as a network, R_S is a network element and in general, the transfer function from v_n to v_o , depends on R_S . The question is, how does it depend on R_S and to see that let me draw your attention to first let me draw your attention to for example, another resistor R_K does it, do you think the transfer function from v_n to v_o depends on R_K ? Yes, no. Yes, and can we make any comment about the transfer function from v_n to v_o at a certain frequency, how it depends on R_K and how it depends on R_S ? Can we make any comment at all?

Given that this is a linear network Let us try and figure this out. Let us try it first, let me take an example and show you the result and then we will see why that result makes sense. So, let us say this is R_1 and R_2 and let us call this v_2 and this is v_i and this is v_o . So, v_o by v_i , therefore is?

Student: (10:36)

Professor: Well, it is simply R_2 by R_1 plus R_2 and v_2 by v_o by v_2 is nothing but?

Student: (10:56)

Professor: R_1 by R_1 plus R_2 and now, let us say somebody gave you a more complicated network. R_1 , R_2 , R_3 , R_4 and you went and calculated v_o by v_i and that turned out to be let us say you calculated it and it turned out to be, you got some answer of the form R_4 square plus $R_3 R_2$ divided by R_1 square plus $R_2 R_3$ plus R_4 square. I do not know. I mean. Well, can you comment on the correctness or the lack of it, of this result? Is that do you think it's correct or wrong?

Student: () (12:03)

Professor: So, it says R_4 tends to the okay. How about now?

Student: () (12:13)

Professor: Pardon.

Student: () (12:15)

Professor: Yes.

Student: () (12:27)

Professor: Yeah, basically the key point that I wanted you to observe or to know is that you cannot get terms of the form our R_4 square and R_1 Square and R_2 I mean whatever R_4 square in the denominator you cannot get square terms in the transfer function expression right and you know, do you know why this is, it turns out as we will see tomorrow that if you had an element say some element, R_3 in the network, any transfer function that you form will be what is called a bilinear function of R_3 and will be of the form, you will have this of the form $A R_3$ plus B divided by $C R_3$ plus D .

In other words, in both the numerator and the denominator, you will get terms that can that you will get terms that contain R_3 you will get terms that do not contain R_3 . If you collect all the terms that contain R_3 it turns out that you will get only you can pull out R_3 as a common factor, it only appears in the first degree and likewise for the denominator, it is entirely possible that you get a numerator or denominator which does not contain R_3 at all, which is simply saying that both A and C are 0.

But if R_3 appears, it will appear only in in the first part. It is actually pretty straightforward to see and we will see that tomorrow and this is telling you that if you have an element, any transfer function that you can form will be a bilinear function, this is what is called a bilinear

function. It turns out that any transfer function that you form will turn out to be what is called a bilinear function of, of a particular element.