Introduction to Time- Varying Electrical Network
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Lecture 19
kT C Noise in a Sample-And-Hold Circuit

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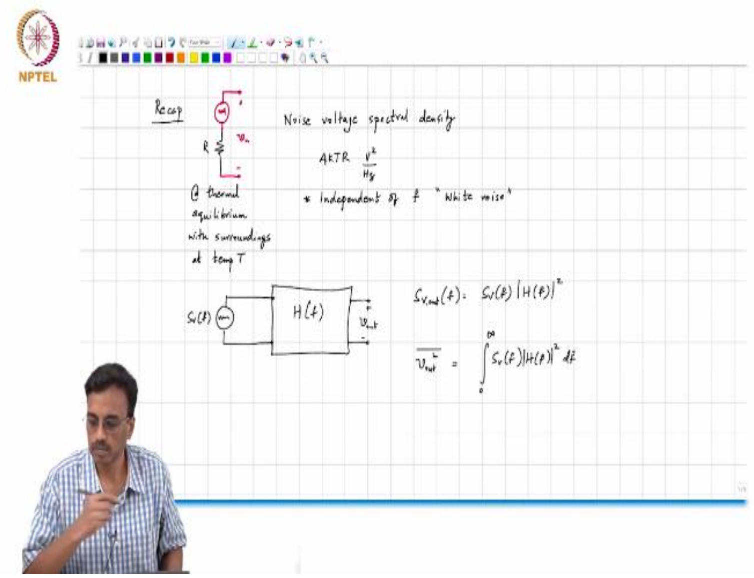

@ thermal equilibrium with surroundings at temp T

Noise voltage spectral density

$4kTR \ \frac{V^2}{Hz}$

* Independent of f "White noise"

$S_{v,out}(f) = S_v(f)\,|H(f)|^2$

$\overline{v_{out}^2} = \int_0^\infty S_v(f)|H(f)|^2\, df$

In the last class, we started discussing about noise. And here is a quick summary of what we learned in the last class. So, every resistor in thermal equilibrium is surroundings at an absolute temperature t is associated a noise voltage source in series with it, and the noise voltage spectral density corresponding to this noise source is 4 kT R volt square per hertz. And as we discussed the last time around the spectral density is independent of frequency.

In other words, this is often what is called white noise and the reason is simply the following I mean, optically, if you have all colors, the resulting color is actually appears white to us. So, and colors in optics are basically we call it light of different frequencies. So, in a similar way if you have noise corresponding to all frequencies in equal strength it is called white. Obviously, if the noise spectral density is not uniform with frequency, then it stands to reason that it was called color noise.

So, then we saw that if you have a noise source with a noise spectral density given by Sv of f and it is processed by a transfer function H of f, the output noise spectral density is simply the input noise spectral density multiplied by the square magnitude of the transfer function. And the total noise is simply the integral of the noise spectral density and the output which happens to be…is that clear people?

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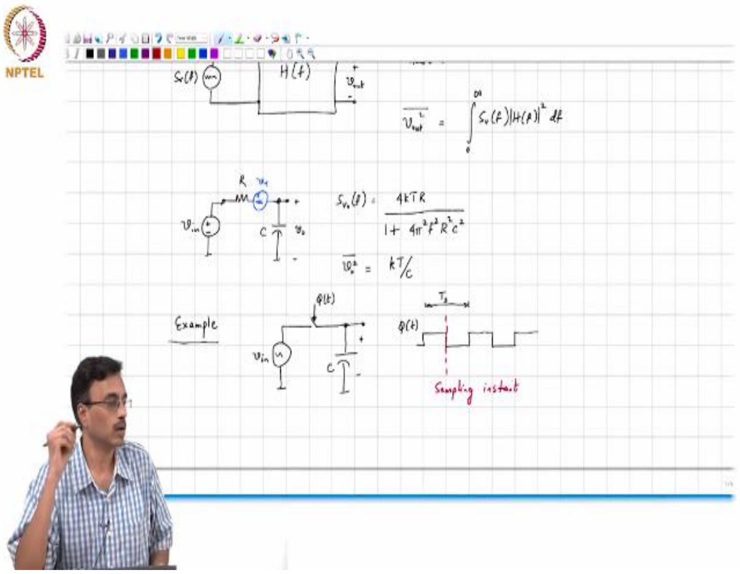

And we did our first calculation of this noise spectral density assuming we have a first order RC network, let us assume that we have an input and an RC network, the output consists of Vn filtered by the first order RC or transfer function, it also consists of noise whose Svo of f is nothing but 4 kT R by 1 plus 4 pi square f square R square C square and the mean square value of the noise as we calculated the last time was? What was it?

Student: kT over C.

Professor: kT over C where k is Boltzmann's constant and we also saw the intuition behind this apparently surprising result, even though the noise source itself depends on the resistance, the total integrated noise apparently is independent of the resistance and we reconcile that by recognizing that while it is true that the noise spectral density increases with resistance, the bandwidth of the transfer function from the noise source to the output also experiences a change in the opposite direction. And in this case, it just so happens that the two of them simply cancel. So, the total the mean square noises is kT over C.

Now, what is it apart from just being a mere curiosity it turns out that this result has quite some importance in practice and here is an example where plays a role. As we were mentioning, as we were talking about in the early part of the semester, we like to process signals digitally which basically means that you take a signal use, you sample it, and you quantize it so that you have a digital representation of the input. And one way of doing this is to basically sample the input that you want to digitize and then once you have sampled it, you kind of look at that sample and then quantize it.

So, quantization obviously takes some time. So, you need some place to store that input sample value and storage of something can only be done on elements that can, that have memory and so it is either an inductor or capacitor and for reasons of size, it is often a capacitance. And so, for example, here is the most simple-minded example one can think of, so, this is a switch, which is periodically operated, let us call this signal phi of t.

So, for instance, this is an example phi of t, this is the sampling period yes, when the switch, when phi of t is high, the switch is closed, when phi of t is low, the switch is open. And therefore, when the switch is closed, what happens to the voltage across the capacitor? Well, when phi is high, the capacitor directly comes across the voltage source and therefore, it tracks the input voltage, when the switch is open, well, whatever charge is there on the capacitor is trapped, and therefore, what comment can we make about the sampling instant, at what instant are we actually looking at the input voltage?

Student: (())(6:55)

Professor: I cannot hear you.

Student: (())(6:57)

Professor: Yes so, what is the precise instant of sampling? It is the falling edge of phi, so this is the sampling instant, of course, life is not as simple as this.

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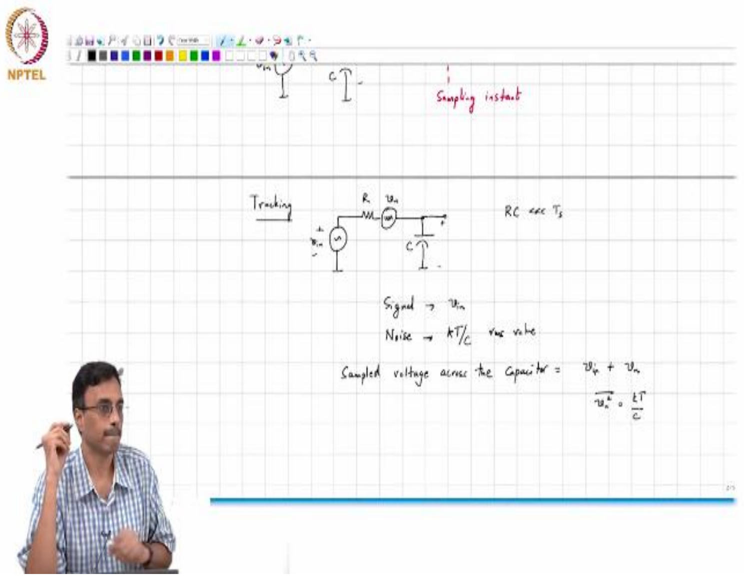

In reality, there is a whole bunch of non idealities, the first one being that no switches is ideal and it turns out that every switch will have associated with it or assistance. And so, therefore, the bottom line is that, well the output will track the input, but there will be a small delay corresponding to the RC time constant. But more importantly, the resistance also adds, the resistor is also associated with a noise source.

So, during the tracking phase how does the circuit look like? Well, there is the input and so, what comment can you make about the voltage across C, I mean, and by the way the RC time constant, what comment can we make about the RC time constant in relationship with ts? Pardon?

Student: (())(8:55)

Professor: I mean that RC time constant must be extremely small compared to half ts because you want the input to track, I mean the voltage across the capacitor to track the input as closely as possible. So RC is much, much, much smaller than ts. And so, before during the tracking phase, what comment can you make about the total voltage across C? It will of course, consist of Vn and remember at the sampling instant namely the falling edge of the of this waveform phi of t, you not only have the input plus you have a component due to the thermal noise of the switch and what would be the so, what comment can you make about the signal is basically Vn, whatever it is. What about the noise? Yes. What comment can we make about the noise? It is 0 mean, that is correct, what else?

Student: (())(10:14)

Professor: It is a random waveform, that is correct, it has got 0 mean and it has got the mean squared value and what is the mean square value?

Student: kT by C.

Professor: kT C, all right, please understand that when we say the mean square noises kT by C it does not mean that you know, you add square root of kT by C to the output, it just means that if you build a million such circuits or if you look at the noise associated to the voltage across the capacitance a thousand or a million times, you will find that the mean square value of that noise is kT by C, it does not mean that at every instant of time the every time you measure it, it is square root kT whole square, is this clear?

So, what common can you make and so this is what the voltage across the capacitor would be during the track phase and then the whole phase comes, when the whole phase comes, what happens? The switch is open suddenly, so whatever voltage is there on the capacitor, that voltage is, I mean, that charge is trapped, it has got nowhere to go. So, what you are actually sampling is Vn plus the sample voltage across the capacitor is nothing but Vn plus some noise voltage, where the mean square value of the noise voltage is simply kT over C.

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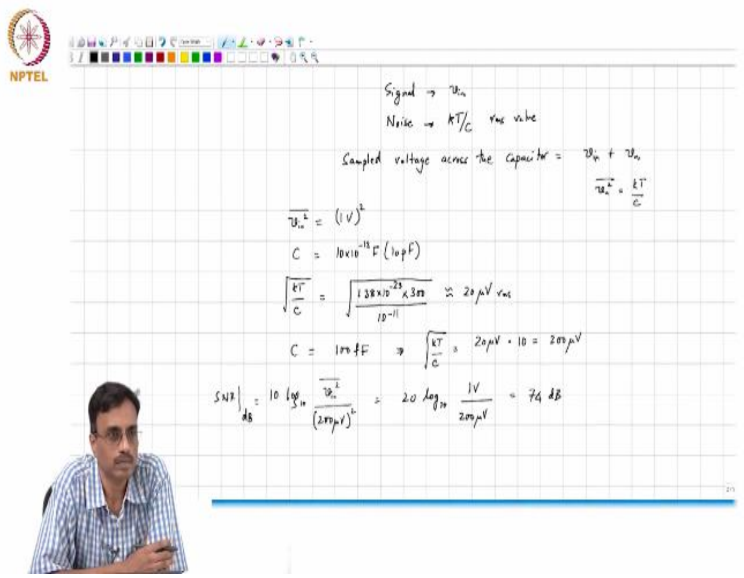

Signal $\rightarrow v_{in}$

Noise $\rightarrow kT/C$ rms value

Sampled voltage across the capacitor $= v_{in} + v_n$

$$\overline{v_n^2} = \frac{kT}{C}$$

$$\overline{v_{in}^2} = (1V)^2$$

$$C = 10\times10^{-12}F\ (10pF)$$

$$\sqrt{\frac{kT}{C}} = \sqrt{\frac{1.38\times10^{-23}\times300}{10^{-11}}} \simeq 20\mu V \text{ rms}$$

$$C = 100fF \Rightarrow \sqrt{\frac{kT}{C}} = 20\mu V\cdot10 = 200\mu V$$

$$SNR\Big|_{dB} = 10\log_{10}\frac{\overline{v_{in}^2}}{(200\mu V)^2} = 20\log_{10}\frac{1V}{200\mu V} = 74\ dB$$

For example, let us assume that the RMS value of the input signal that needs to be digitized is 1 volt and C is 10 picofarad, so what comment can we make about square root of kT over C? Somebody do the math and tell me, 20?

Student: 20.

Professor: So, if C was instead of being 10 picofarad was 100 femtofarads, what will happen to this value? There is how much times, how many times smaller? Times square root of these 10 powers minus 13 and that is 10 power minus 11. So that is basically a factor of 10 it is now it is 200 micro volts, so what comment can you make about this, tn the last instance, what comment can you make about the signal to noise ratio, which is simply the ratio of the mean square value of the signal to the mean square value of the noise? Which is and it is often expressed in dB because this is power, it is 10 log which is 20 log, 20 log to the base 10, 1 volt divided by 200 micro volts and that is how much? 74 dB.

And so in other words, this is only a factor of less than, it is about between 3000 and 4000, right, so 74 dB is between 1060 dB, 10080 dB 70, somewhere between 60 and 80 and therefore, it is somewhere between geometric mean of thousand and ten thousand, roughly around 3500 or so. So, basically, I mean the practical importance of this is that if you use a small capacitor to sample, you will be stuck with a noise voltage on the capacitor, which is large.

In this case, for instance, I know, you already made an error of the order of one part in 4000, so if the resolution of your A to D converter that you are trying to realize is much higher, then this is a very bad choice of capacitor to use. So, if you are trying to resolve a voltage to better than say one part in a million, one part in a million is 120 dB and if you do that, then this is you know, grossly inadequate, because the moment you sample the signal, already you committed a crime, which you cannot recover from.

Because you are adding some random thermal noise, I mean some random voltage in addition to the voltage you wanted to sample. That is, what do you call dependent on that capacitor, it is clear? So, what is the moral of the story? If you want to resolve or if you want to build a sampler or consequently an analog to digital converter with higher and higher resolution, based on sampling the voltage on a switch using a capacitor using a switch, then you see that the value of that sampling capacitor better be sufficiently large, so that the error you make when you sample the input voltage onto that capacitor is way smaller than what you are trying to resolve.