Introduction to Time – Varying Electrical Network

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Lecture 14

The Adjoint Network

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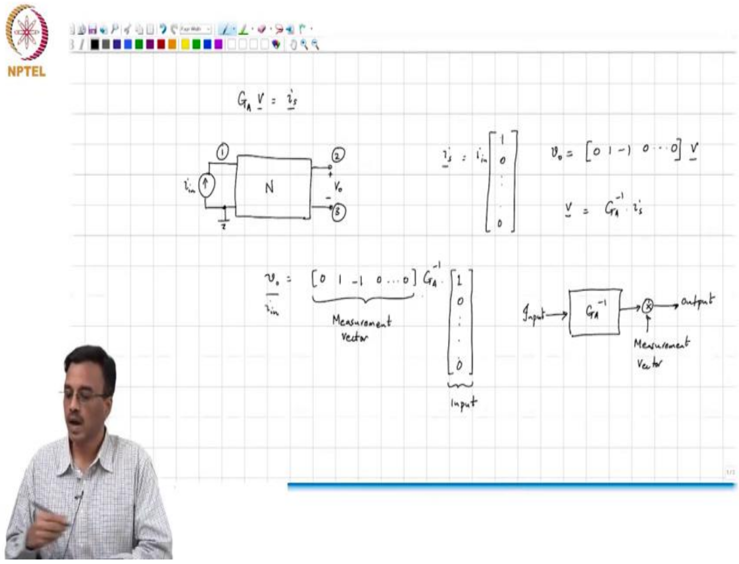

Welcome to advanced electrical networks. This is lecture 7. So, in the last class we saw another perspective of reciprocity and the basic idea was the following. So, we have we write the modified nodal equation analysis M and A equations, we basically have an equation of the system of equations with the form G sub A times V equals isub S, where G A is the augmented conductance matrix, V is the vector of unknowns, which will consist of all the node voltages as well as the currents flowing through the, through any of the voltage sources inside the network and the vector on the right hand side is simply the source vector which consists of all the independent sources namely current sources and voltage sources.

And we saw that well when we try to work with reciprocity, one way of looking at it is to basically we saw that for the case of a current excitation and the voltage output I believe and what we do is simply recognize that when we write these equations, what we are doing is injecting a current into node 1. So, this is simply becomes in times 1 followed by all 0s and V O is simply a rho vector which is 0 1 minus 1, all 0s multiplied by multiplied by V.

And V is nothing, but G A inverse times is. So, V O therefore can be written as 0 1 minus 1 all the way to zero times G A inverse times is which is a V O by in is nothing but 1 0 etc. So, this is the corresponds to the input vector. This is the circuit matrix of course, and this is the, what I will loosely call the measurement vector, the measurement matrix if you like.

So, this you can think of this as a you know as a mathematical operation. So, you have the input. It gets multiplied by a matrix G A inverse and produces a, an output which is the input times a G A inverse and then what is happening?

Student: (())(4:18)

Professor: Here you are multiplying this by the measurement vector. So, this is and this the out.

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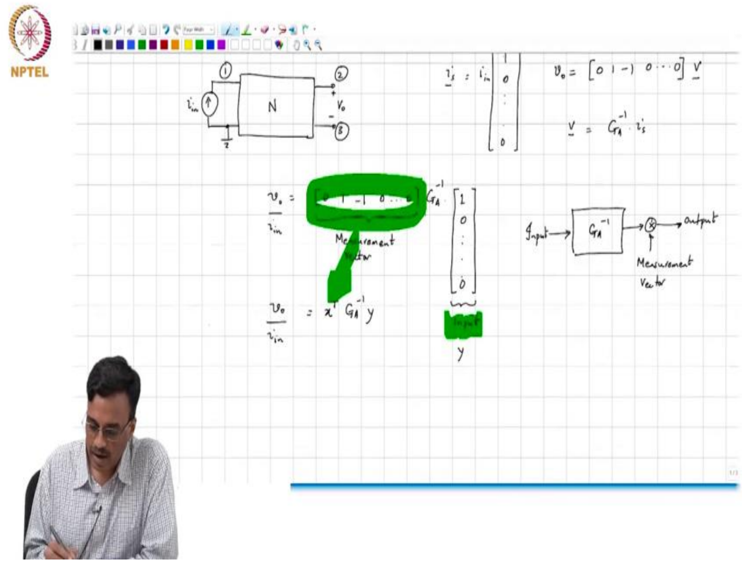

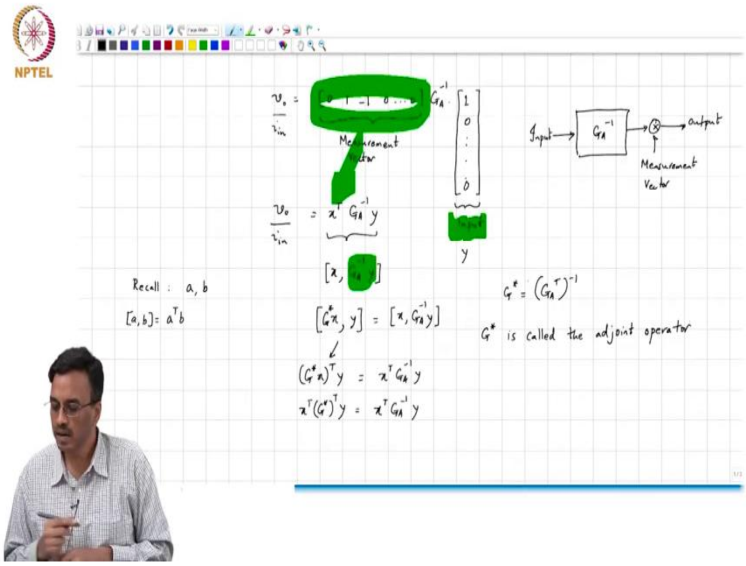

And if you think of this as x transpose, so this I am going to call x transpose simply because it is a ~~row~~ rho vector. So, x is a column vector and x transpose is that, is the measurement matrix, this times G A inverse times the input, which I am going to call the y. So, the scalar V O by in is the form G transpose, I mean x transpose times G inverse times y. And you know, and remember that if you have ~~two~~ 2 vectors or ~~two~~ 2 column vectors with the same length let us say, we want to use A and B, what is the dot product of the two vectors?

The dot product is nothing but A dot B is ~~nothing~~nothing, but A transpose B. Correct? And you look at this here and this is the Brier, This is the, I mean, this is the notation for dot product. So, how can you write this is this is nothing but…

Student: (())(6:17)

Professor: x comma

Student: G A inverse

Professor: G A inverse times y. That makes sense people?

And so, you can a it, so, you can think of it as the following, you have vector y~~, right~~, on which the matrix G A inverse is operating. And the result is another vector. And you a~~'~~re finding the dot product of that vector G A inverse times I with x, x corresponds to the measurement y corresponds to (())~~ ~~(07:10).

Now, in mathematics, it turns out that, you know, I mean, a legitimate question you can ask is the following. I am taking a vector y operating on it and taking the dot product of the result with the vector x to get a scalar. A legitimate question I can ask is, if I wanted to get the same scalar but I wanted to operate on x instead, rather than y. What should I have done to x to get the same scalar? Does that make sense?

In other words, what should I do to x before dotting it with y to get the same answer as I would get, if I is this clear people? I mean, this is simply a mathematical question. There is nothing I mean, why are we doing it knowing that this is what the math guys love to do? ~~Right?~~ And it seems like a legitimate question,

So, you are messing with y first in the in, in this computation you are messing with? You a~~'~~re messing with y. And then taking the result and finding the dot product of x, Reasonable question to ask is, well, if I messed with x instead and then compared its dot product with y, then what should I have done to x to get the same dot product?

~~So~~So, with what matrix should I multiply it to get the same data? So how do we solve this problem? What do you do? And how can you figure this out?

Student: (())(9:12)

Professor: Well, this, whatever operation you do to x, the whole I mean, that is clearly let us call that. I do not know, I mean, I am going to call that G star. So, x transpose times what does this guy G transpose G star times x transpose times y should be equal to x transpose

Student: G A inverse times y.

Professor: G A inverse times y and this is nothing, but x transpose G star transpose y must be equal to x transpose G A inverse times y. So, what comment can we make about G star? G A transpose inverse, inverse transpose, it does not matter. So, in mathematics, this G star is called the adjoint operator.

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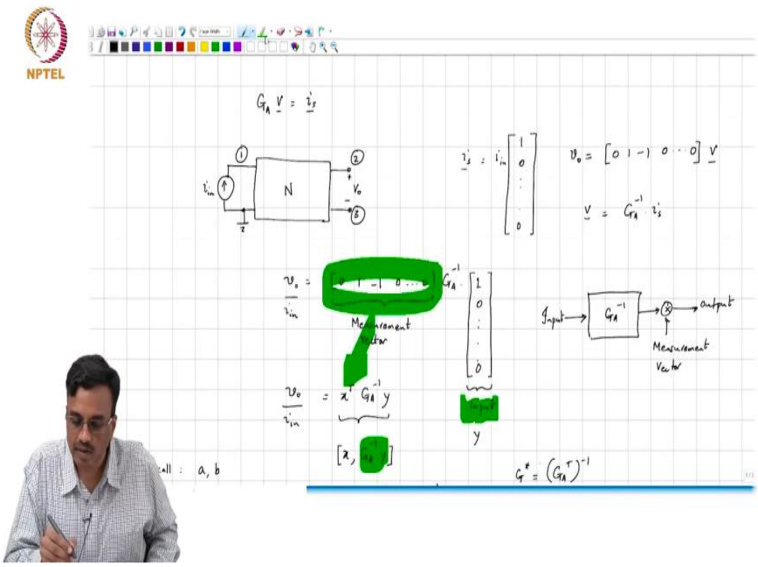

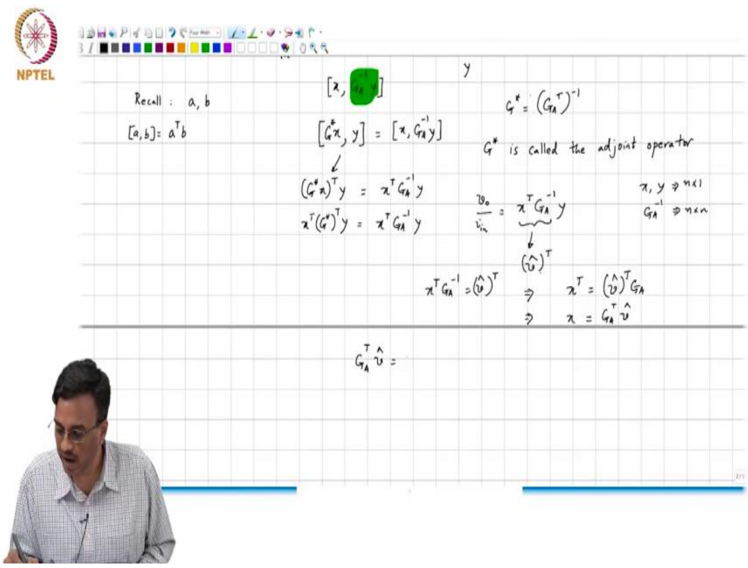

And you can see that what this is telling us is that what is this telling us if we want the same transfer function, another way of interpreting this is to say, well ~~V O~~vo by in, we know is x transpose G A inverse times y, which can be thought of as let us call this I mean, remember this? In other words, when you write an expression G A times ~~V~~ v equals is. How do you, I mean, this a mathematically this set of equations, but how would you interpret this? These are nothing but the node voltages that develop when you take a network with the with an M and A matrix G and have a source vector is.

That is the interpretation of this set of equations as far as we are concerned. So, when we have multiple input sources, as we were discussing yesterday, to do this by superposition is equivalent to finding G A inverse times is for each source separately. So, in effect, what we are doing is computing the inverse multiple times.

An alternate way of doing this, a smarter way of doing this is to basically say, we know that V out by in is x transpose, x transpose times G A inverse times y. Remember x is the measurement vector, and y is the, is the excitation rate or the input rate. So, you can think I mean, what comment can we make about the dimensions of that guy there?

Let us say there are n unknowns, let us say x, y are both n cross 1 vectors, because there are n number of unknowns and the G A inverse, what is the size of that?

Student: (())(13:25)

Professor: n cross n? So, what comment can we make about x transpose times G A inverse?

Student: ~~One~~ 1 cross n

Professor: ~~One~~ 1 cross it is a rule, ~~.~~ c~~C~~orrect? And I am going to call that v hat. I mean, it is just notation. There is nothing holy about that. So, basically this is saying x transpose G A inverse equal to v hat. Let me call that v hat transpose because v hat is, I would like to call all my vectors to be column vectors.

So, x transpose G A inverse is a row vector and if v hat is the column vector, then it must follow that I can write x transpose G A inverse as v hat transpose, which is equivalent to saying what does this mean? x transpose is G A times ~~V~~ v hat transpose which is equivalent to say x equals what?

Student: (())(14:47)

Professor: Pardon?

Student: (())(14:48)

Professor: Hold on Hold on. x transpose G A inverse equals, v hat transpose, v hat is basically a column vector. So…

Student: (())(15:08)

Professor: Pardon, sorry, sorry yes. Sorry, t~~. T~~hank you. So, is v hat transpose times G A was on the right hand side and therefore, x must be equal to G A transpose times v hat. ~~Or in other words G A transpose times v hat equals x and remember what is x?~~

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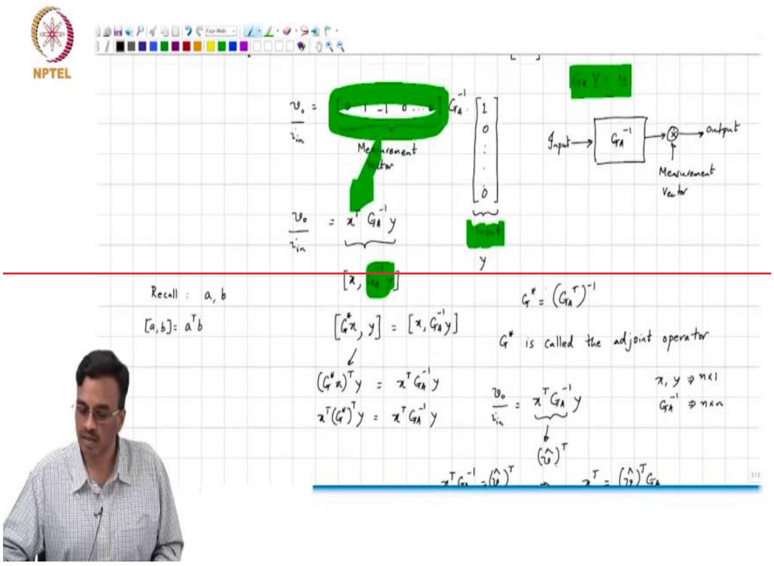

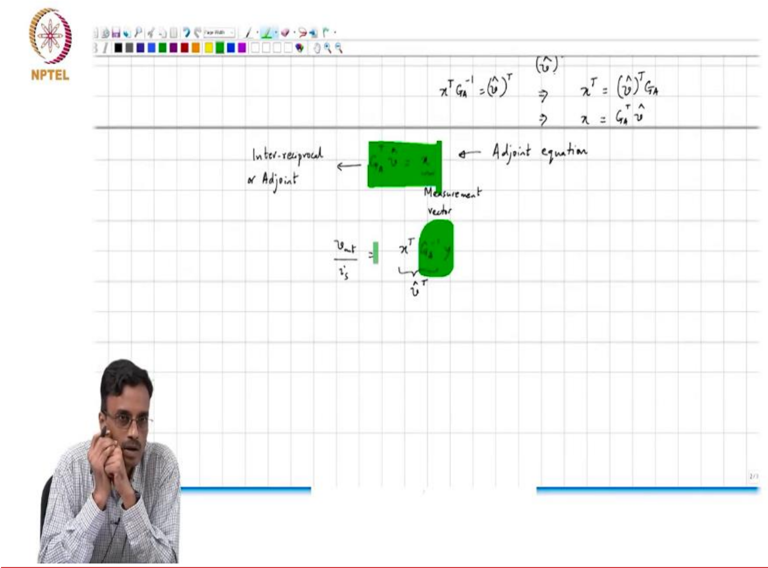

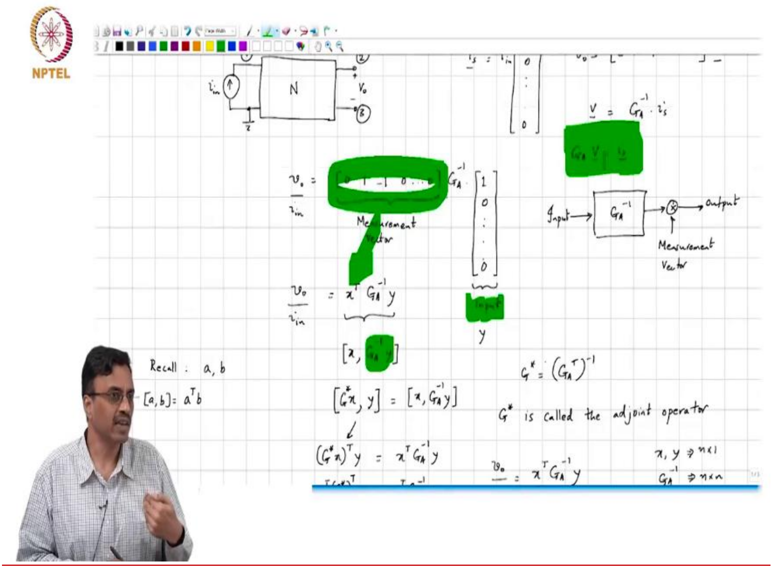

Or in other words G A transpose times v hat equals x and remember what is x?

What does this denote? What did it denote in our, this is the is the measurement vector. That was the name we gave to x. And so, but how do you interpret these equations? This when we said G A times V v equals is, how do we how did we interpret that? What is the interpretation of this of this system of equations? In English how would you describe that?

Student: (())(16:31)

Professor: So, these I mean that in English what this means is that,that V is the vector of node voltages produced when you excite a network G A with…

Student: is

Professor: is,. nNow, by the same token what comment can you, how can you interpret this? Yes what are we exerting with x?

Student: (())(17:00)

Professor: Yes. So, basically now we have, how do you the interpretation of this is that we are now exiting a network with whose M n A matrix is G A transpose with

Student: x

Professor: x, which corresponds to the

Student: measurement vector.

Professor: measurement vector. So, earlier wherever you are measuring now you have to excite and the voltages that are developed on the on this network are v hat.

And once we find V hat to find the individual transfer functions is simply from the multiple inputs is simply multiplying people, once you find V hat what do you do?

Student: (())(18:02)

Professor: Yes so, basically remember that in the in the original network v out is basically, what we call it x transpose times G A inverse times G A inverse times y and what is x transpose times G A inverse? What is it?

Student: (())(18:38)

Professor: v hat transpose. So, and once we have v hat transpose you know finding the various transfer functions from multiple inputs is simply a matter of multiplying v hat transpose with you know whatever input you want. So, in other words, v hat transpose that matrix that row vector contains information with regard to all the transfer functions, I mean the transfer functions from all the inputs to a given output and this y merely select which of those transfer functions you want. That is all.

And so therefore, so, so, therefore, if you want multiple transfer functions to the same output, this obviously entails a lot less labour, because you are solving and the how to and how do we obtain v hat? We just solve not the same network, but a related network where what is the relationship between the really that that network and the original network the M n A matrix of the modified network is simple the transpose of that of the original.

~~So~~So, we form the network whose M n A matrix is the transpose of the original network? That is what is called the, what is that called? If you have a network whose M n A matrix is the transpose of the original network, what is that called? What is it? What do you call that network?

Student: (())(20:29)

Professor: Yes, it is that is called either the inter-reciprocal network or the adjoint. Is that clear? So, and this equation, and mathematics is often called the adjoint equation. And so, the bottom line therefore, is that inter reciprocity can be simply thought of as, as evaluating the first two terms of, of this equation, rather than if we did superposition, we would be evaluating that product first and then dotting it with x transpose. It does not make sense because that output vector is not changing.

It is only the input data that is changing. So it makes sense to compute the stuff that is not changing. Because that G inverse is difficult, so it makes sense to compute that x transpose G A inverse once and then as you keep changing the input sources the y will keep changing. So the transfer finding all the transfer functions is easy. Does is it makes sense people?