Linear Dynamical Systems
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Tutorial on State – space solution and realization
Lecture – 06
Tutorial - 1

So today, we will be seeing the Tutorial for the first week a topic which is on the Stage space solution and the realization.

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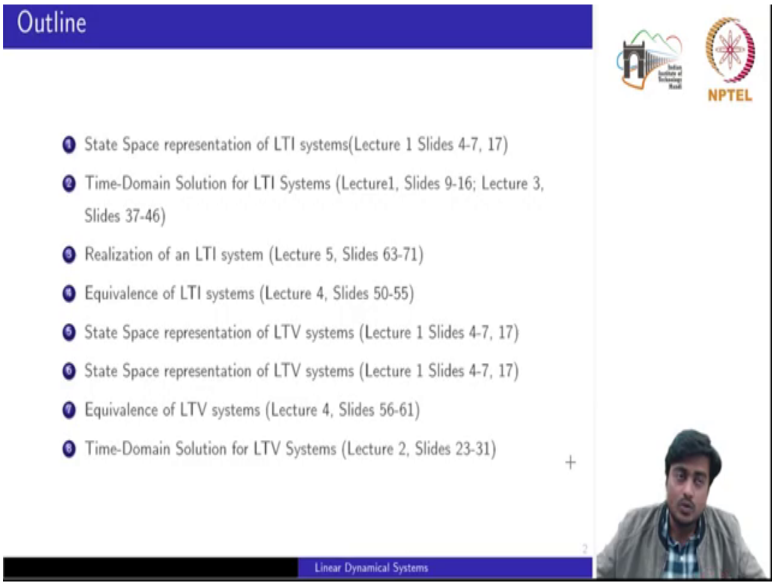

So, the idea first of all the idea behind this tutorial is that we had discussed many theoretical results in the week 1. So, with the help of this tutorial; we will see the direct application of those results on to some numerical examples. So, if it requires some solving some equations or solving some matrices, we will go through them. So, this would be the outline for this tutorial where the basic idea is to cover all those subtopics which we had covered broadly into the; into this week.

So, we will discuss the state space representation of LTI systems basically given any system; how you can obtain the state space representation; then we will see the time domain solution of those systems. So, we will discuss both for the LTI systems which in the first and second point and the fifth and sixth and this last point. Then we will see the realization how to realize the LTI system and its equivalence between the given to LTI systems.

So, for here is we have also included that what are the slides number associated with the problems with which we or which we will going to discuss.

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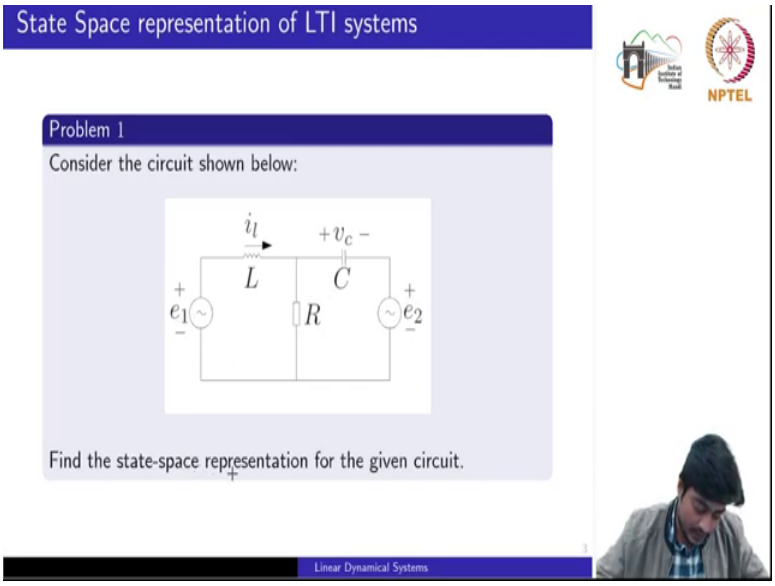

So, the first problem is consider the circuit shown below where an electrical circuit is given to you containing some passive parameters, inductor, capacitor and the resistor. And there are two sources of inputs the voltage sources e 1 and e 2; we need to find the state space representation for this given circuit.

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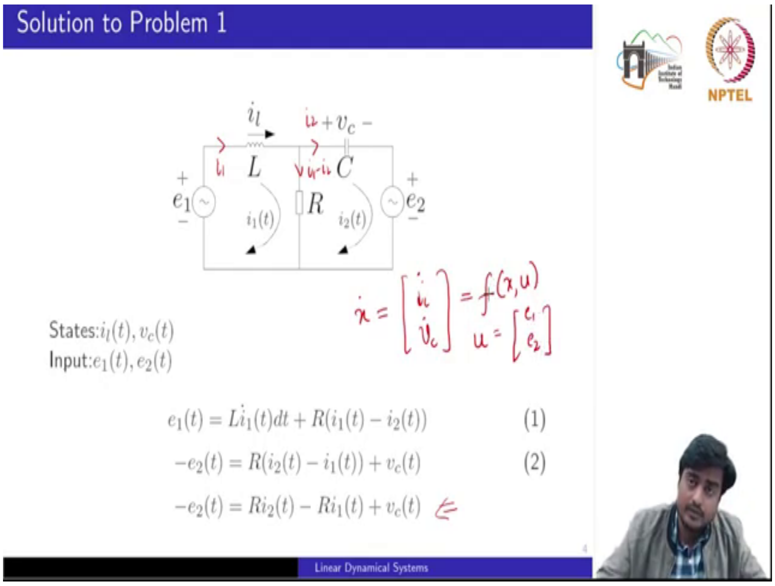

So using the Kirchhoff's law; we can analyze two independent loops. The first loop is this one in which the current i 1 is flowing and if the second loop where the current i 2 is flowing. So, at the outset we have specified two states first is the current flowing to the into the inductor which is i l and another state is the voltage across the capacitor.

The inputs are already pre specified which are given by e 1 and e 2. Now, if we take this first loop and if we see that in this branch i 1 current is flowing which is nothing, but equal to i l and in this and we suppose that in this branch i 2 current is flowing, then in this branch we would have i 1 minus i 2 by using the simple current law.

So, first we will write this first equation for this loop 1 which is nothing, but e 1 is equal to L d i y; d i 1 by d t plus R i 1 minus i 2; will be take the entire loop. Similarly, for the second loop we can write minus of e 2 because the current is flowing in the opposite direction which is equal to R times i 2 minus i 1 plus the voltage across the capacitor. So, after this simplification we obtain this next equation which you can see more closely.

Now, since we have chosen two different states one is i l and vc; we want to represent these two basic equations in the form of; there are derivative of the i l and the derivative of the v c so that we can form our state vector which is basically x dot. So, on the left hand side we want the first derivative of the state vector and then we end on the right hand side; we want to express them as a function of the state x and u; where u is here e 1 and e 2 and this function f; we know that it should be a linear map.

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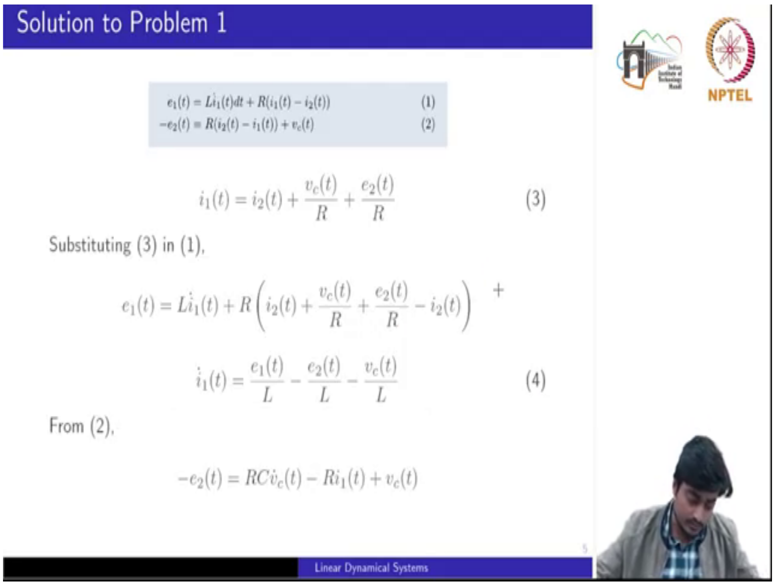

So, proceeding forward using; so using these two equation it just a simplification that we can write this i 1 is equal to i 2 plus v c by R plus e 2 by R. Then substituting this third equation into the first equation we obtained this one; all these steps we are doing so that we could have the first derivative of the state variable onto the left hand side. So after some simplification; we get this first derivative of the first state which is i 1 or is equal to i l and on the right hand side; we have the function of in the input which are e 1, e 2 and another state v c ok.

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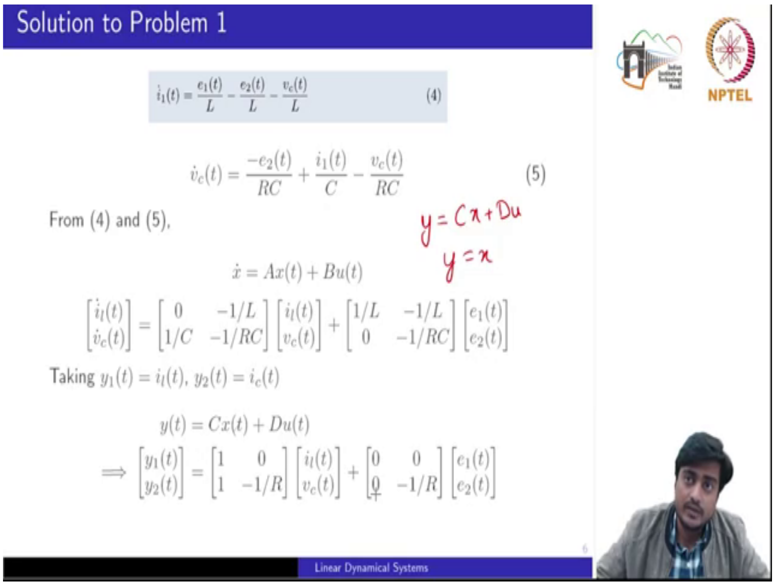

Now, going forward to some more simplification; we obtain in this equation 5; which is vc dot is equal to again the function of the input variable, the state both these states and i 1 is nothing, but equal to i l. So, after combining this equation which we have obtained in the last slide and combining this equation, we can write the state equation the linear state equation in this form where A and B matrices are obtained; these two matrices.

So, y here are selected as the so in fact y could be the state also because if you recall the output equation output equation is given by Cx plus Du ok. So, its depending on what measurements are available to us directly; we can choose the C matrix; the if we choose C matrix is an identity matrix and D matrix is 0; in that case we would have y is equal to x ok.

So, here we had chosen two different outputs; first is one of the states i l and another is current flowing through the capacitor. So, if we choose these two outputs, then the C and D equations or C and D matrices can be obtained like this. It just the simplification of those two basic questions; which we have discussed in the first part of the problem ok.

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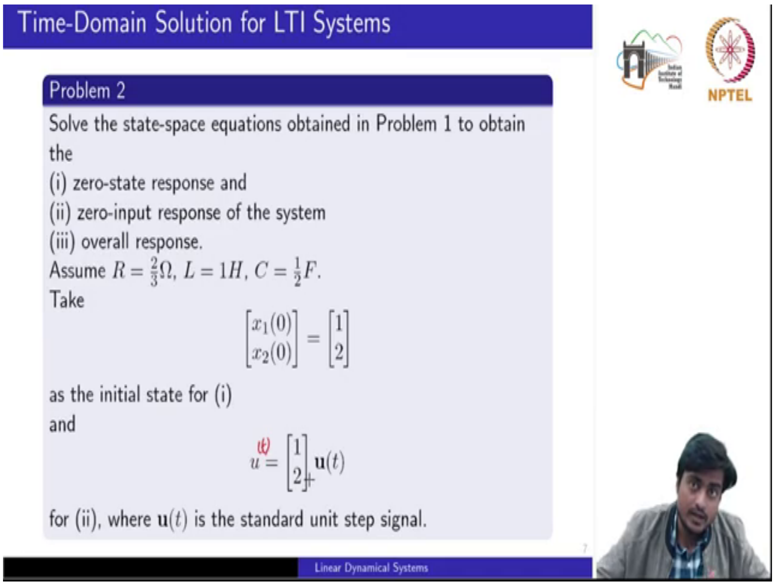

So after obtaining the state space representation of the problem 1; now our interest in computing the time domain solution of the LTI system; so for the computing the overall solution of the LTI system, we had studied that the overall solution can be represented as a summation of the zero state response and the zero input response. So, here we will compute three different parts individually first; we will compute the zero state response; where the initial conditions are taken as 0 and we want to see the output which is only influenced by the; some certain input.

The zero input response of the system in which the initial conditions are in fact, where the output the input in fact is 0; the overall response would be the summation of the first two responses. We have taken these numerical values to compute the solute; compute the numerical solution and the initial conditions are taken as 1 and 2.

We are supplying a specific input to the system where the input is given by 1 and 2 by multiplied by a standard unit steps signal which meaning to say that this input in fact, on the left hand side; we should have u t also. So, meaning to say that this input is applicable starting from t greater than equal to 0. So, before that; before that the input is 0 ok.

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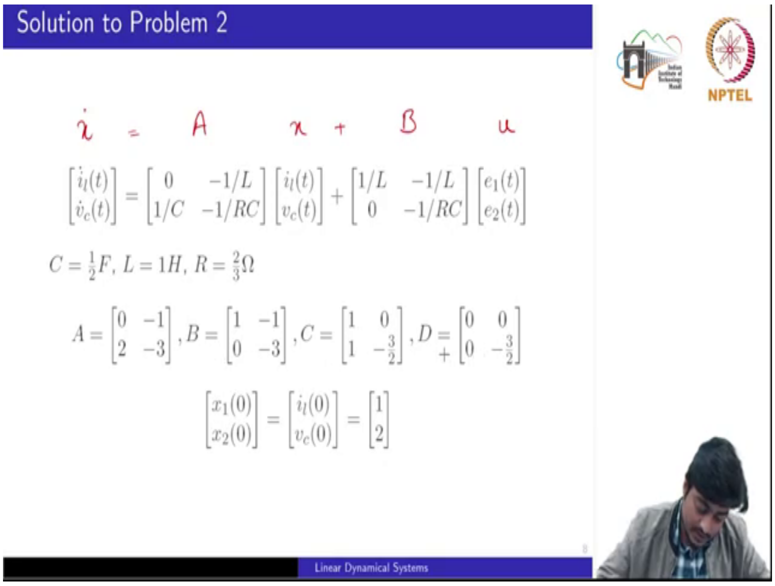

So, again writing the state space equation into a similar form where you have this x dot; this A matrix x plus B u and putting these numerical values what we have specified in the problem itself; then we obtained A, B, C, D matrices as these and the initial conditions we are all it also provide to us.

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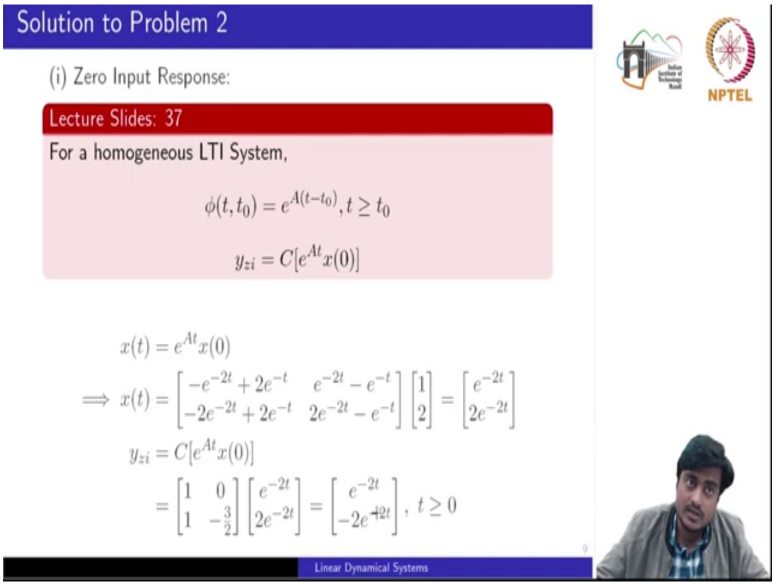

So, if you recall from the lecture slide number 37; view the zero input response can be computed by computing the state transition matrix. Now, for the LTI systems; we had specified that the state transition matrix is nothing, but an exponential matrix defined by e to the power A into t minus t naught; where t naught is the initial time, but here t naught we have taken is 0. So, the response which we would be concerned with only e to the power A t; then once we have obtained a state transition matrix, we can compute the zero input response which is merely the multiplication of the state transition matrix by the initial condition and pre multiplied by the output matrix.

So, this is the response of the homogeneous system x t is equal to e to the power A t into x naught. So, after solve; after pertain this e to the power A t and x naught; we can simplify it to this one. Now, if you have any difficulties in computing the matrix exponential matrix. So, there are a couple of ways to compute this matrix the first method is to compute using the Cayley Hamilton theorem.

The second way you can compute since we are dealing with the LTI systems. So, you can use the Laplace transforms to compute this e to the power A t. So, first we will compute the Laplace transform of this time domain function and then again to obtain the time domain solution; we take the a Laplace inverse ok. Then putting the C matrix; the exponential matrix and the initial condition we obtained finally, this solution. So, this is the zero input response for the given initial conditions.

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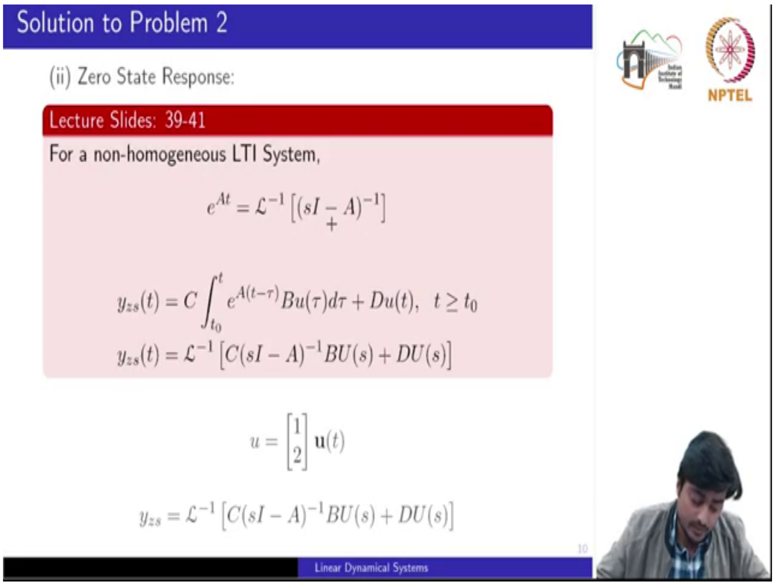

Computing the zero state response, you can refer to the slides number 39 to 41 where we have considered a non homogeneous LTI system. Here, we have shown explicitly that how you can compute this exponential matrix by using the Laplace transforms. So, if you recall that the zero state response is basically given by this equation where you need to compute the integral of this exponential matrix which is a bit complicate procedure.

So, as an alternative again you can simplify it by using the Laplace transform. So, first compute the Laplace transforms and then compute the Laplace inverse to finally, obtain the solution into that time domain. So, this input is already specified to us 1 and 2 with a unit step signal. C matrix has given to us; A, B the; it is the Laplace transform of this input; the D matrix and finally, the U matrix.

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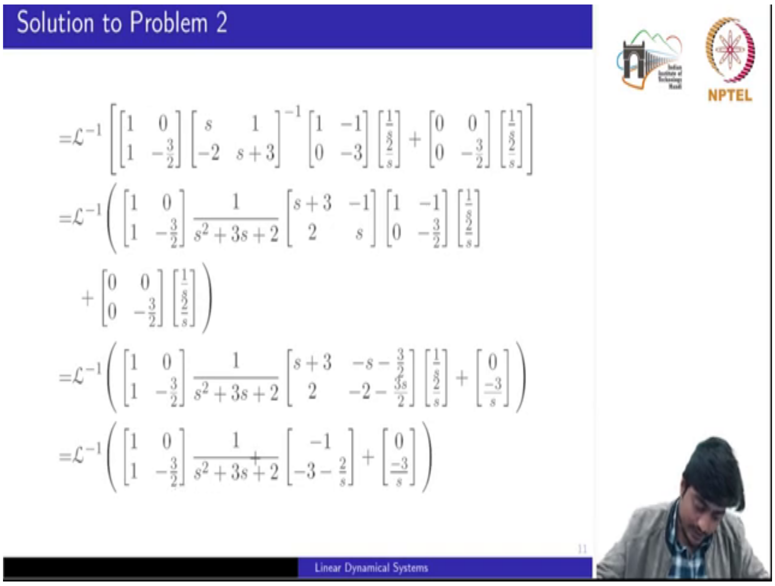

So, after putting all these matrices into this equation; so a detailed solution is been given here; so you can verify by yourself with there are all those matrices rightly placed. So we will not go through the detailed of all the solution; just the simplification by putting all those matrices into the last equation ok.

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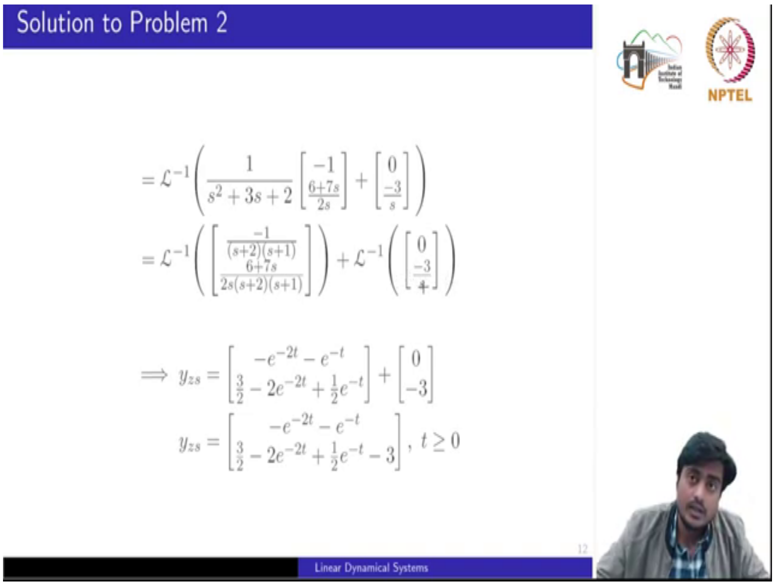

And finally, we would end up to this simplified version of the Laplace inverse of the s domain matrices. So, once we compute the Laplace inverse; this is the solution we would obtain which is valid only for t greater than or equal to 0 ok.

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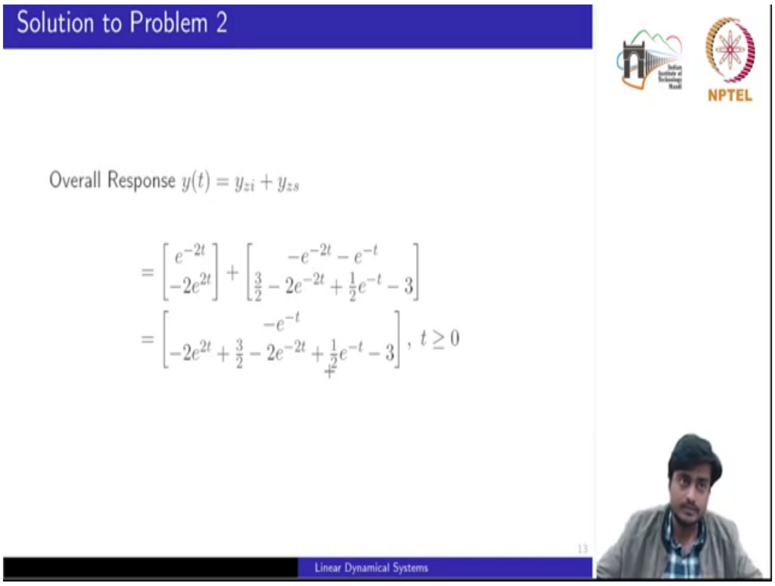

Now, as mentioned earlier the overall response is given by the summation of the two independent responses we have computed in the first and second part. So, after summing those the zero input response and the zero state response; this would be the overall response of the circuit, given the initial conditions and also with respect to this specific input ok. Now, similarly if some other initial conditions and some other input is given to you. So, here we have taken the step type input only where the first input was having a magnitude of 1 and the second input was having a magnitude of 2.

Now, instead of those the step inputs there are number of inputs by which you can compute the responses. So, either you could use the ramp inputs, the sinusoidal inputs all the impulse inputs as well ok. So, with respect to each and every input; the response would behave differently, but with respect to initial conditions; you can parameterized and it would appear more or less the same except the change in the numerical values.

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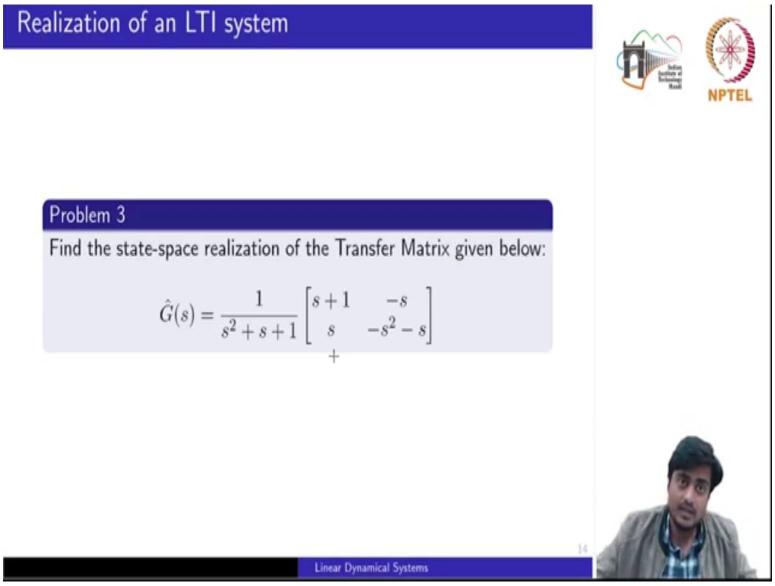

So the problem 3 deals with the state space realization of the transfer matrix. So, here we have specified some transfer function matrix which is a 2 by 2 systems meaning to say we have two inputs; in two outputs and the realization problem deals with that given a transfer function; we need to see whether there exists this A, B, C, D matrices. So, if you recall the theoretical results; we have investigate, we have basically supplied the conditions that under what conditions we can say a transfer function or a transfer function matrix is realizable.

And particularly for the LTI system we have used some specific representation which we have also called that canonical representation and that canonical representation was nothing, but the controllable canonical controllable representation. So, we will see that how you cannot obtain that specific representation. But at the same time we should not forget that the transfer function is a unique representation, but the state space representation is not unique. So, we will obtain one specific representation; otherwise you can, otherwise there exist in infect infinite number of representations ok.

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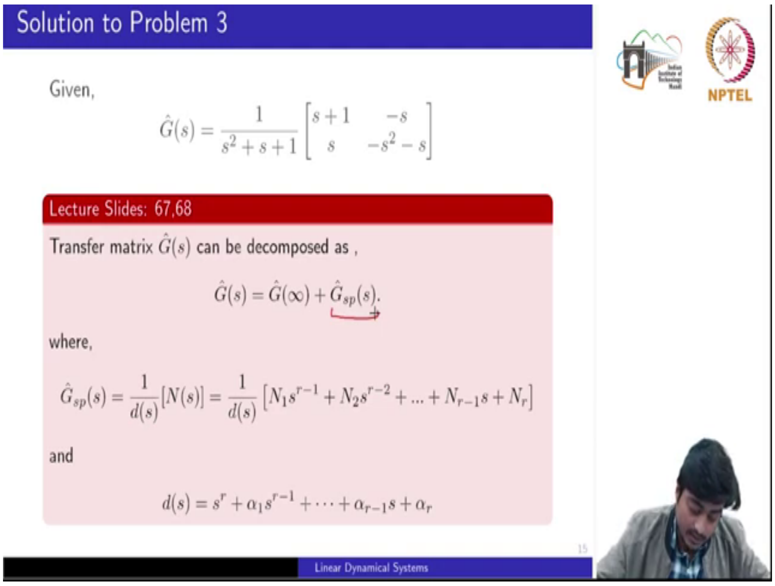

So the results which we have discussed on to the slide number 67 and 68 of the lecture slides; we would going to use those results. So, the first equation says that we need to decompose the complete transfer function into two parts. First part which is or let us say the second part which is the strictly proper part meaning to say the order of the numerator is less than or strict or definitely less than the order of the or the degree of the denominator polynomial and G hat infinity is when we have computed the value of the transfer function when s approaches to infinity.

So, the strictly proper part was given by this multiplication of this matrix which is composed of the numerator; of the numerator polynomial of the matrix multiply by 1 by d of s, where d is the some polynomial of rth order given by this one ok. And once we have this matrix N of s, we have represented again as a summation of some constant matrices multiplied by the power of s ok. So, we will see in detail that how we can obtain these matrices.

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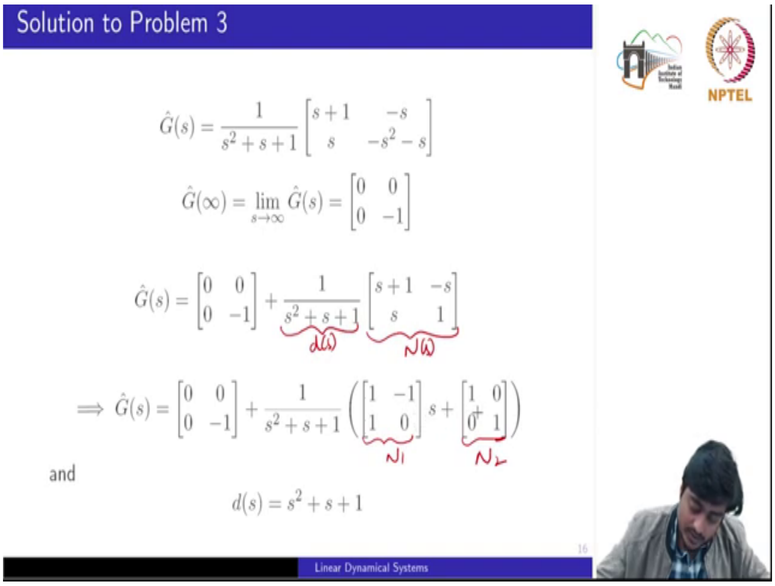

So, G hat of s is given to us this one. So, the first part that what happens at when s tends to infinity; we can compute and given by 0, 0, 0, minus 1. Now, the strictly proper part is pretty much straightforward here; the matrix is already given into that form where this part is directly you can take it as the denominator polynomial.

The numerator polynomial we would take as this one because if you see this first, second and third element all those elements are having the degree less than the degree of this denominator polynomial; while this polynomial is having the degree equal to this polynomial. But we saw that the degree of this polynomial should be less than this polynomial that is why we have extracted out this G hat infinity part. So, after extracting this or decomposing this G hat infinity part; we can write this one as this one because once you sum them up; you would finally, obtain this equation ok.

So, this is you can see as N of s and this part which is in the denominator is d of s and this is a function of s; now this the maximum power here is s to the power 1. So, first of all we will represent this constant matrix which is N 1 and this one is N 2, N 1 into s plus N 2. Again we have t decomposed this complete polynomial into the summation of two polynomials with respect to the power of s ok. So, basically it is to say that once you simplify this equation; simplify this terms inside the brackets, you would finally, obtain this N of s.

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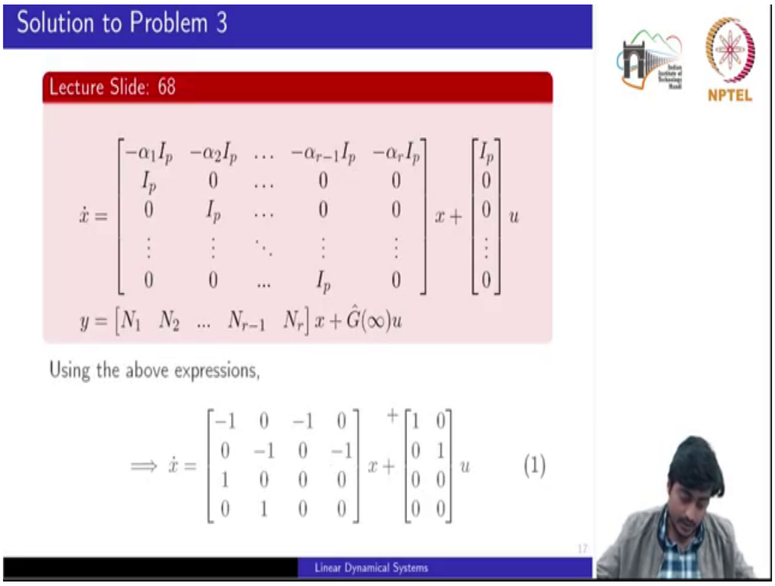

Now, after obtaining all those matrices and the polynomials and the coefficients as well; this is the adapt representation or the canonical representation we have specify, where these alpha 1 to alpha r are the coefficients of the denominator polynomial. These matrices we have obtained already G hat infinity and this is nothing, but only the identity matrix.

So, this p here is basically p is equal to 2. So, here we would have a I p would be a identity matrix of dimension 2; so this part is your; I p and the rest is 0. Similarly here we are having the scalar multiplied by the identity metro the same dimension. So, here this is part is minus alpha 1 p and this part we have this minus alpha 2 p. This one is this I p matrix because p being equal to 2; we will have only this part and this is just the representation of this one.

And the this one is a block matrix; a 0 block matrix or dimension 2 ok. So, this is the A and B matrices of the given transfer function matrix. So, again you should not forget it at the same time that this is only one specific representation. Now there could be another representations from where you can compute the same transfer function ok.

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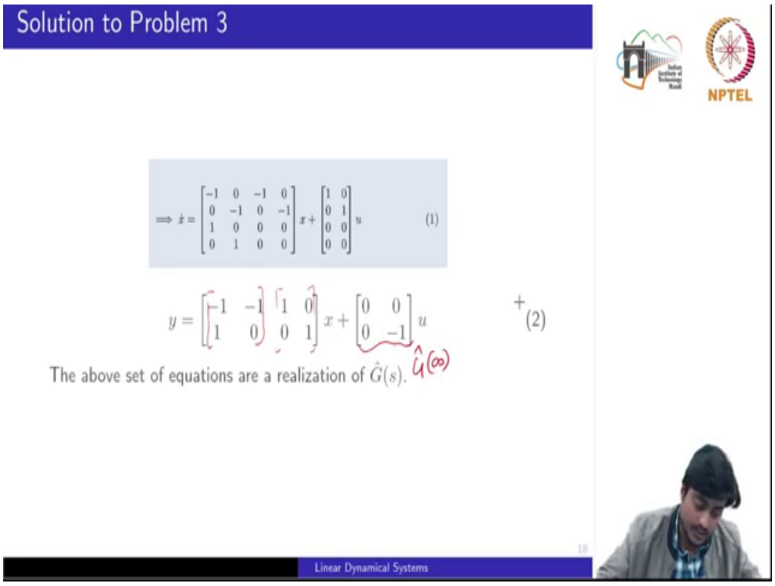

And the output matrix being the p is equal to 2. So, this one is N 1 matrix and this one is your N 2 matrix and this one is d which is nothing, but equal to your G hat infinity part ok; which is the basically the feed forward term into the state space equation. So, above equations 1 and 2 are called the realization and the transfer functions. So, this solves this problem number 3.

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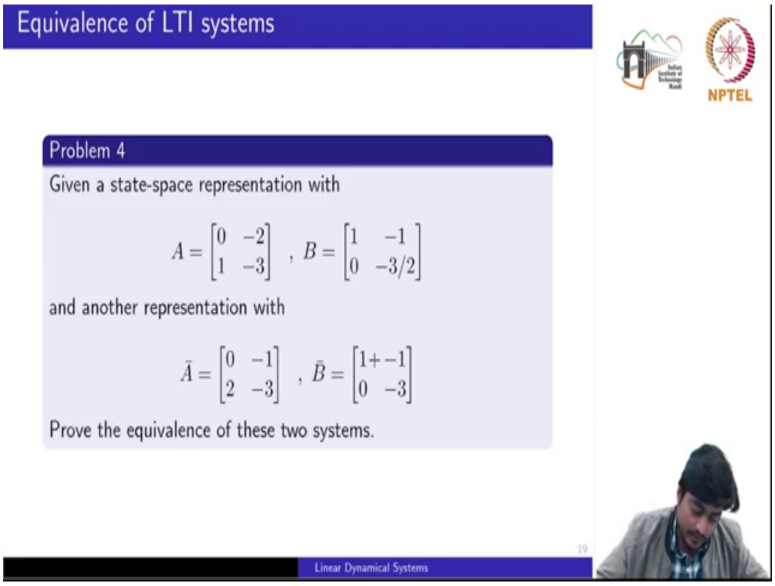

Now, the problem number 4 deals with the equivalence of the LTI systems; by equivalence we mean to say that given a pair of A, B matrices; we want to see that whether these two pairs denotes the same system ok. So, first pair is this one with the numerical values and another pair is with A bar, B bar with different set of numerical values. So, we want to verify whether these two pairs actually represent at the same system or not.

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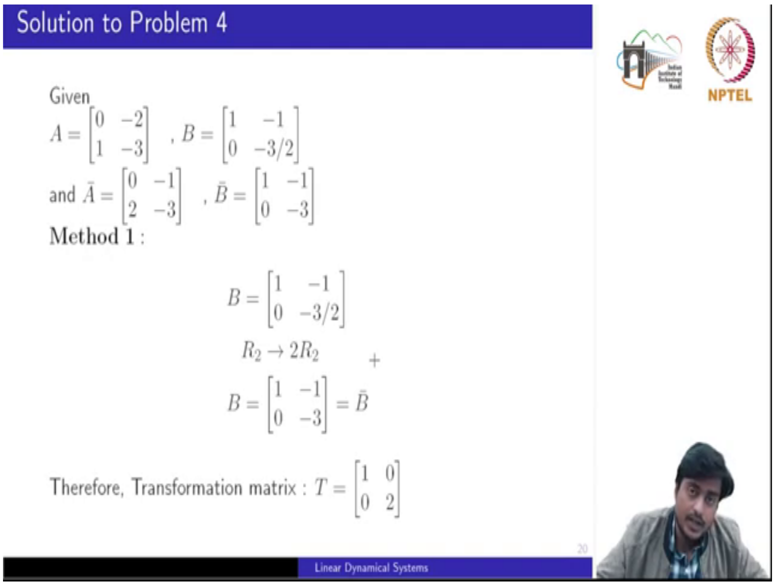

So, here we have listed two methods to find out whether these two pairs are equivalent or not. So, in the theoretical results you would recall the base the key ingredient is to find the transformation matrix T or P so that the these A, B pairs are related by that transformation. So, those transformation matrix could be obtained in a number of base say for example, if I take this B matrix starting with this B matrix and I do this operations that the row 2 is replaced by the scaled value of this row.

So, I would obtain this one B matrix and this B matrix; if you see that it is nothing, but your B bar matrix. By doing simply this row transformation or this operation onto the second row of the B matrix, we obtain this B bar. So, in that case this transformation matrix is simply given by this one ok. You can also verify whether the transformation relation between the A and A bar matrices are also satisfied for this transformation matrix.

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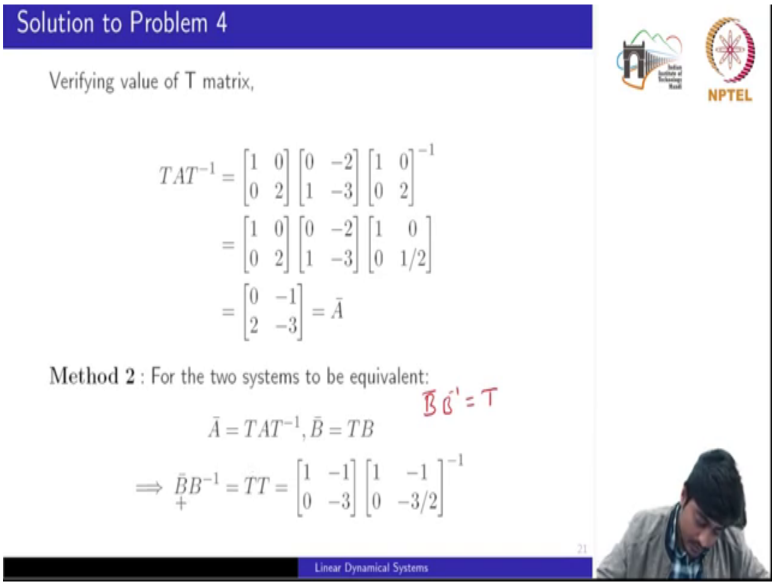

So, this is done in this slide here; where we are taken the same T matrix multiplied by A; T inverse should be equal to A bar. So, after simplifying all these inverses and the multiplication; we could actually verify that this transformation is verified. So, with this T matrix or this transformation matrix we saw that these two pairs basically represent the same system or they both are equivalent. Now, we here we have an advantage because if you recall this equation; B bar is equal to B bar is basically given by T into B; if there exist any T matrix by which B bar and B are related.

At the same time if A bar and A are related using this equation we say that T is a transformation matrix. Now, here we have an advantage that B is a square matrix and on the top of that B is infact non singular. So, if I pre multiply by B inverse into this equation, I can obtain the T matrix directly say what I mean to say that this equation is given to us; if I pre multiply by B inverse; I would have B bar, B inverse is equal to T.

So, I can obtain directly the transformation matrix by the multiplication of the B bar and the inverse of the B matrix and this is what we have done here. So, this T is extra; so it should be B bar B inverse is equal to T.

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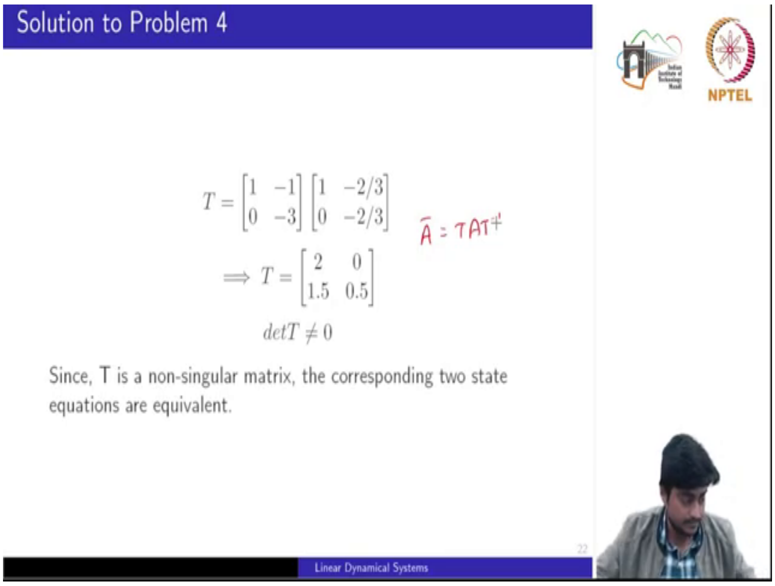

And after simplifying we obtain this T matrix ok. Now, for the singularity you can verify with the determinant of this T matrix is not equal to 0 meaning to say that it is non singular. So this was in fact, the condition that if there exists are non singular T matrices by which these two relations; first between the A bar and A, second between the B bar and B are related; then those two pairs are equivalent.

So, we have computed in fact, two different non singular transformation matrices by which those two pairs are equivalent. Now, you can verify by yourself that for this T matrix; the first relation that is A bar is equal to TAT inverse should also be verified ok.

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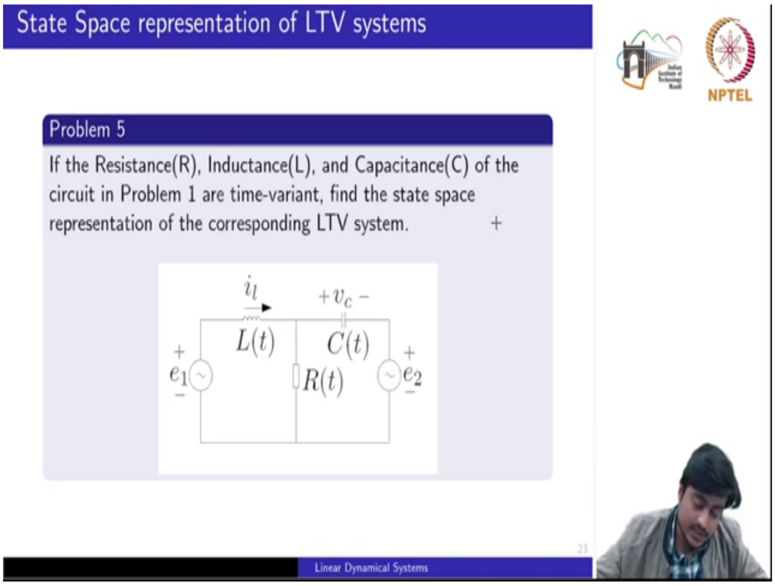

The problem 5 deals with the state space representation of the LTV system. So, here for the LTV system; we have assumed that all those parameters pairs of parameters which were having some constant value in the first part of the; in the problem 1, we are assuming that all those parameters are now time dependent parameters ok; the inputs would remain as it is.

So, now if you apply the superposition principle on this circuit you would see that it satisfies the linearity, but it is time variant because of the parameters being time in time dependent.

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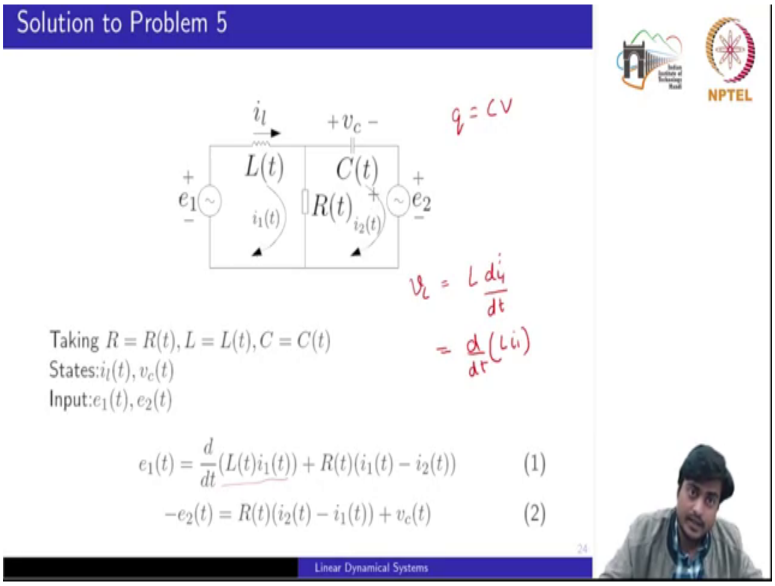

So, the idea would remain the same that we have the two loops and just one point to note here that once we write the equation of the first loop; since L was a constant parameter in the problem 1; we have directly represented the voltage across the inductor as L di y 1 by dt ok. But now here we have L as a time dependent parameter; so we should have the derivative of the flux around this inductor which is L i 1. So, this part we have used here.

So, be careful while writing the loop equations when we have the time dependent parameters in contrast to the time independent parameters. So, writing the loop equation for these two different loops and doing the simplifications.

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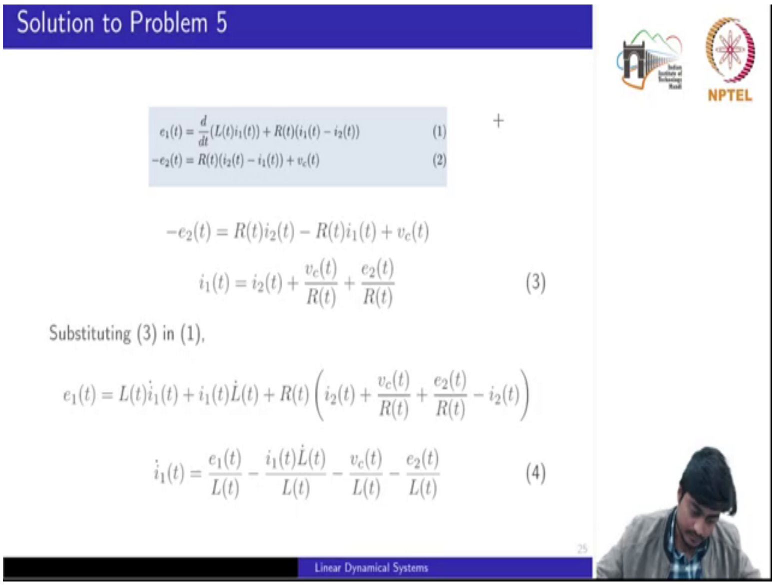

And so we want to discuss through all these simplifications you can see more closely whether you are also able to obtain these simplification. The idea is that on the left hand side we need the positive derivative of the two states and here the states variables remains the same to the problem 1 and on the right hand side we need some function which is a function of the state and the input right.

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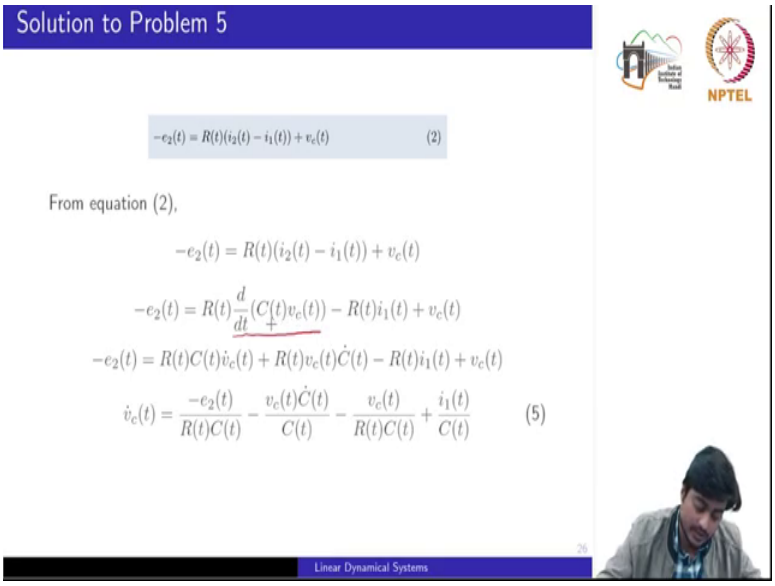

So, from equation 2; we get this. So, another point that here also once you start writing the equation for this capacitor or this loop equation. So, here q is basically given by the C v and we are taking the current and the voltages. So, we need to take care when you take the and derivative or the integral because here now c is not a time independent right.

So, this is taken care here; once we write the voltage equation for that loop that the current we are taking is the derivative of the charge store on the capacitor and charges C into v.

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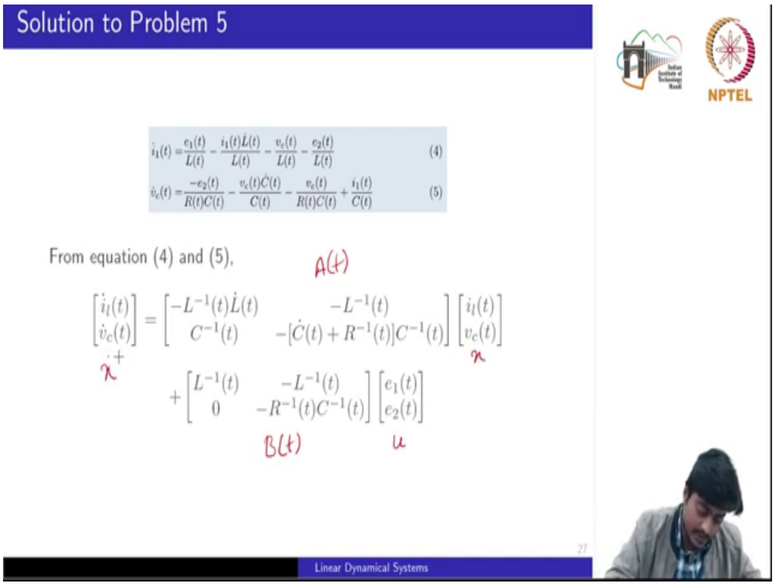

So, after simplifying these two equations; this is the state space representations representation we have obtained where this is the state variable, this is the A of t; the time dependent A matrix. And similarly we would have time dependent B matrix and this is u, this is x and this is x dot ok.

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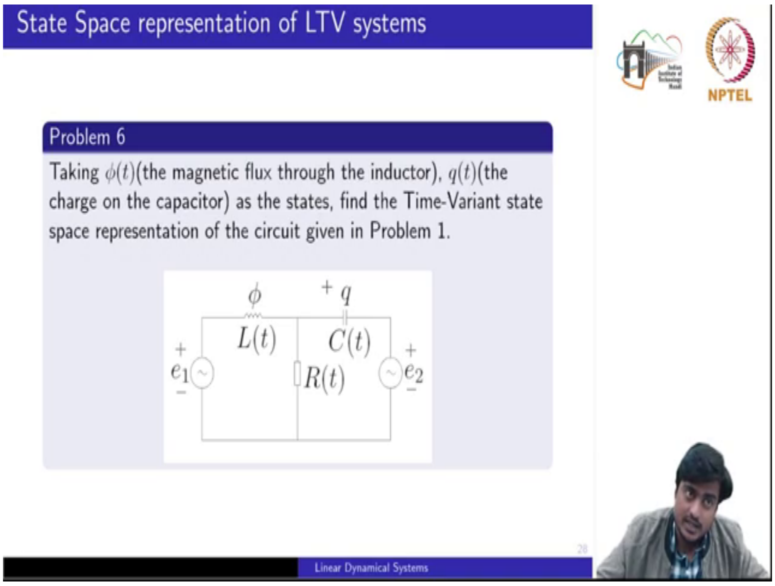

Now, for this problem 6 also deals with the states space representation of the similar circuit, but here we have chosen different state variables. So, in the problem 5; the state variables were chosen as the current flowing through the inductor and the voltage develop across the capacitor. Now, here we have chosen another state variable and we want to see whether with respect to those state variables; we can again obtained similar kind of state space representation right.

Now, we also know at the outside if the states are different and since the system is known to us already. So, we definitely we would going to have some representation, but in that case the A and B matrices would entirely be different.

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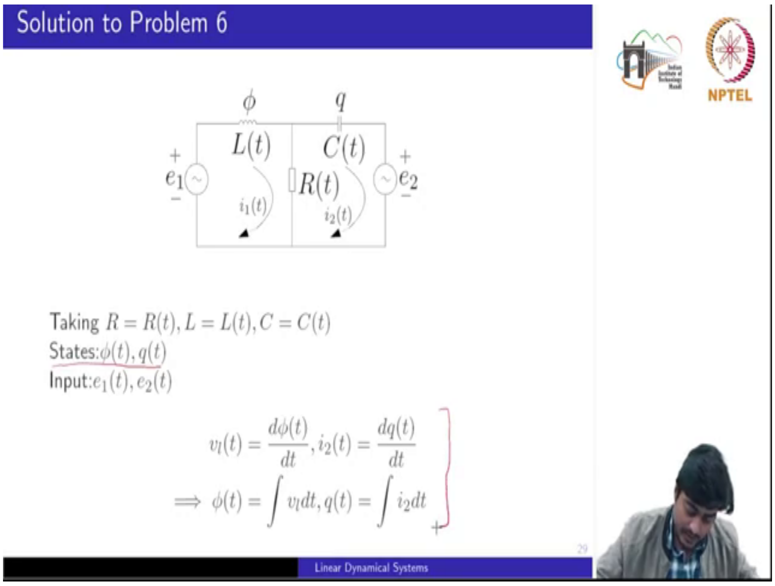

So, here we have chosen these states the flux across through the inductor and the charge across the capacitor. The inputs are already pre specified as e 1 and e 2. So, these relations we basically know from our plus two physics.

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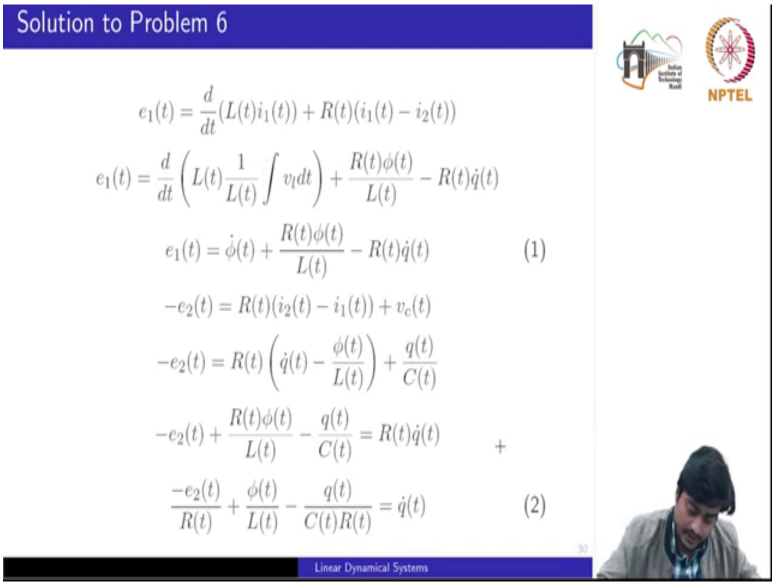

So, again writing the loop equation for both the loops; you can go through in more detailed simplification of those two loops in terms of the 5 and the charge.

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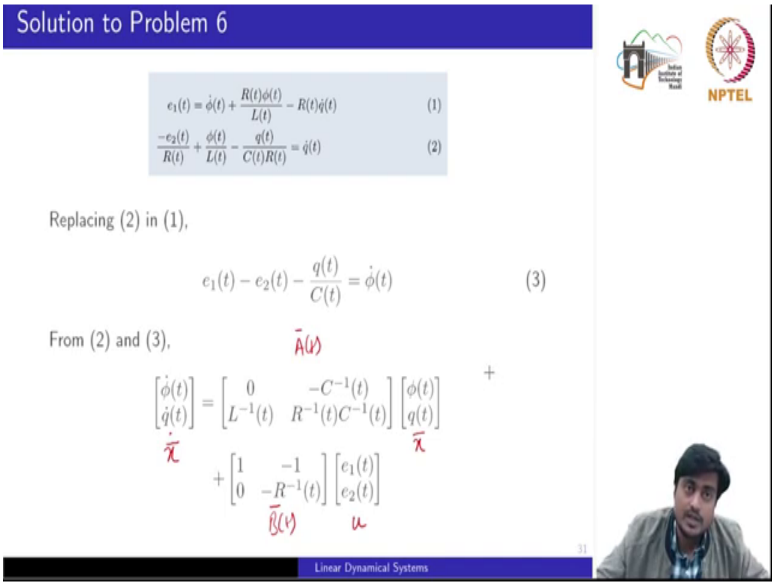

So, doing further simplification; we obtained this equation where again we have this A of t and B of t. And this let us say I would say another state variable x bar and here we have this x bar and the u remain the same. So, if you match those A, B matrices with the solution of the problem 5; you would notice that these two matrices. So, it is better to represent these matrices by bars as well so that we are not confused whether the A and B matrices remains the same ok.

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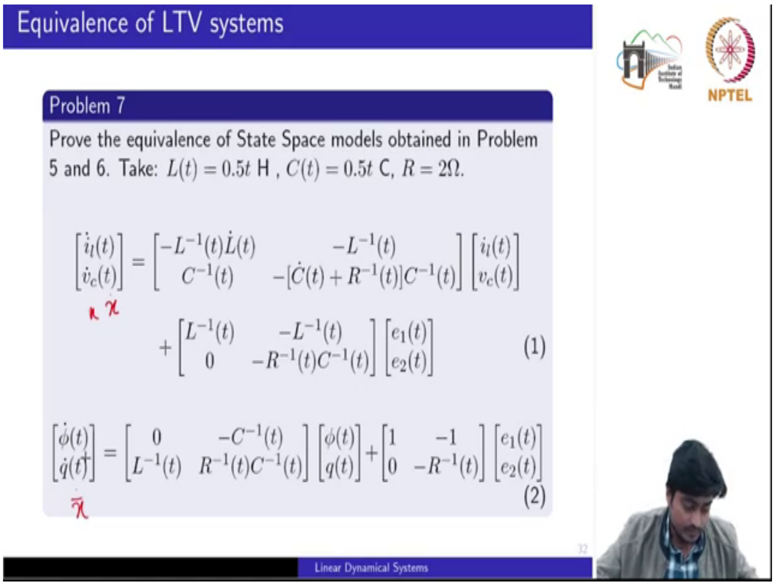

Now, if we want to see the equivalence between these two different representation. So, this problem is in fact trivial why? Because generally we are given with the A, B pair at the outset and from that given A, B pair; we want to see that whether there exist and equivalence without knowing the system.

But here the system is already known to us and for that system we have two different state space representation; now if we want to see the equivalence between those two state space representation that would definitely with their because those two representation belongs to the same system. So, we just want to verify that the application of the result we had seen for the LTV case.

So, these are the two representations one in terms of the state variables; let us denote this state variable as x; excuse me x, i l and v c and another we have x bar which is phi and q ok.

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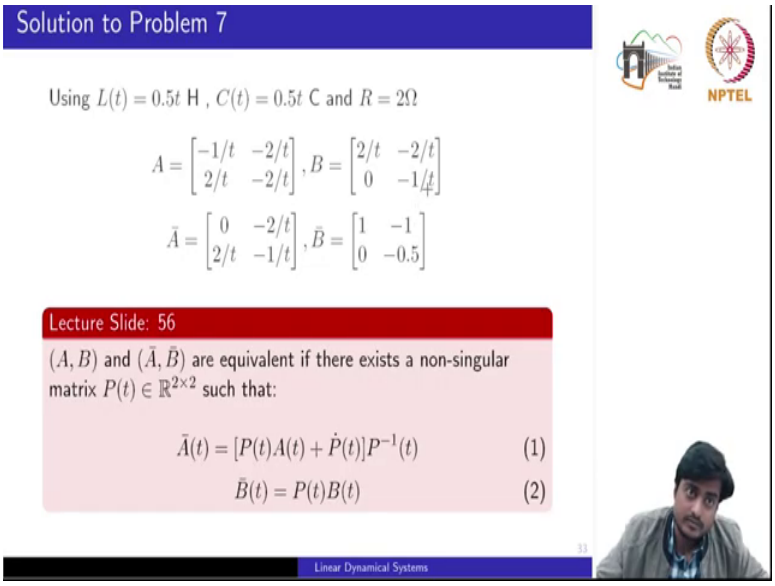

So, if you recall the lecture slide number 56; this is the a condition we have given that the two pairs; time dependent pairs are equivalent whenever there exists a non singular matrix P of t. So, here we have specified already a dimension because the dimension of the state matrix is 2 cross 2. So, we are looking for that matrix P, which is a square matrix or dimension 2; such that these two relationships are satisfies between these two pairs ok. So, we need to find a P matrix; if there exist a P matrix these two matrices are equivalent ok.

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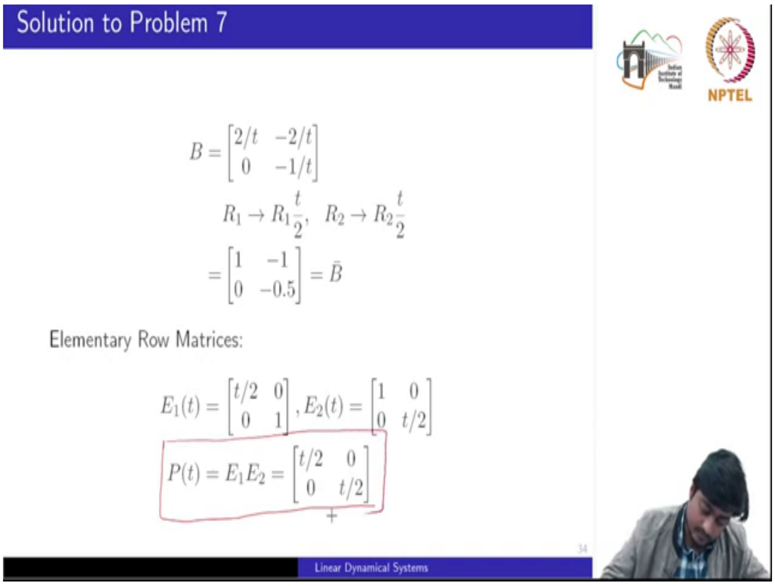

So, there are two method one second. So, one is by doing the row operations or the column operations by which we want to see the equivalence between first the B and P matrices. And after and obtaining the transformation matrix through those operations, we want to verify that the first relationship is also verified because at the same time we should know that P of t should be a continuous matrix ok. So this is the first method by doing the elementary row operations and this is the final P of t matrix we have obtained.

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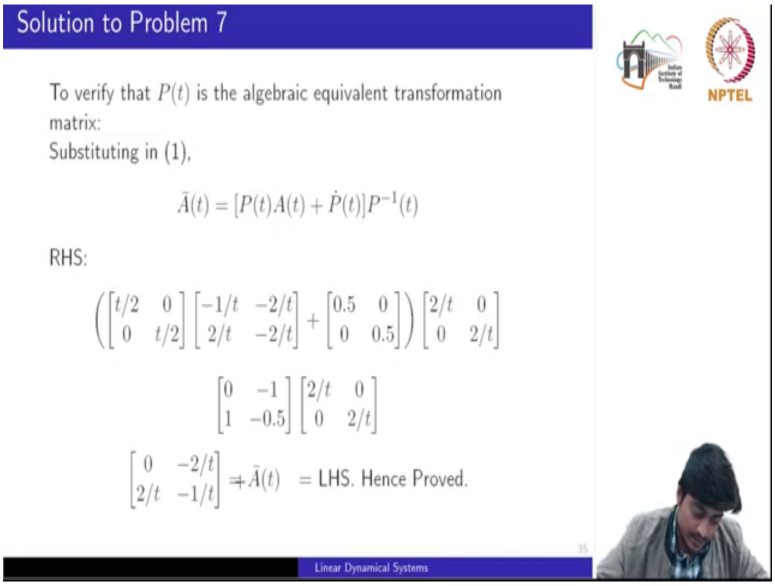

Now, to verify that P of t is the algebraic equivalent transformation matrix a substituting in 1; we obtained this A bar as this one ok; P of t; A of t, this is the derivative of P and multiplied by the P inverse. So, after doing the simplification we finally, obtain this A bar; meaning to say that P matrix is basically a transformation matrix for the time varying system.

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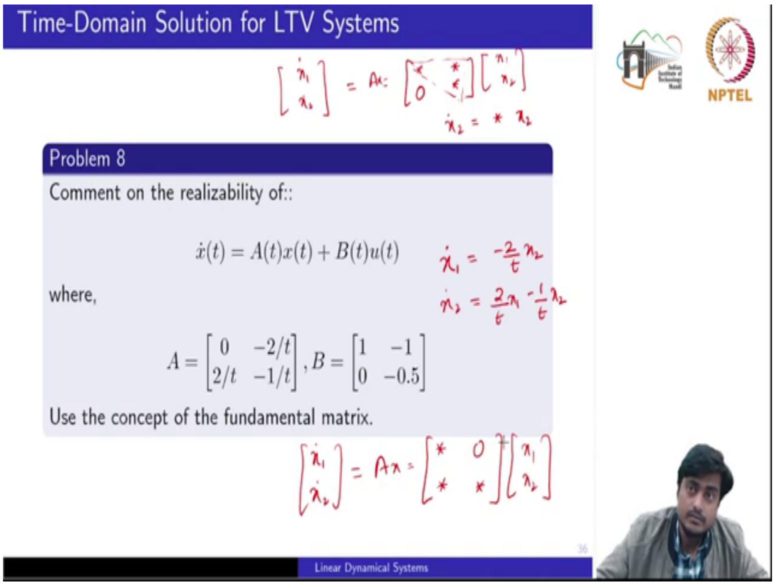

Going to the time domain solution for that LTV system; so here we are taking the A, B matrices as this one. So, if you see that these A, B matrices are possibly the matrices which we have written for the second part considering this phi and this charge has the state variables. So, there are two points we want to solve here; first we want to comment on to the realizability and the second point is the we want to compute the fundamental matrix ok.

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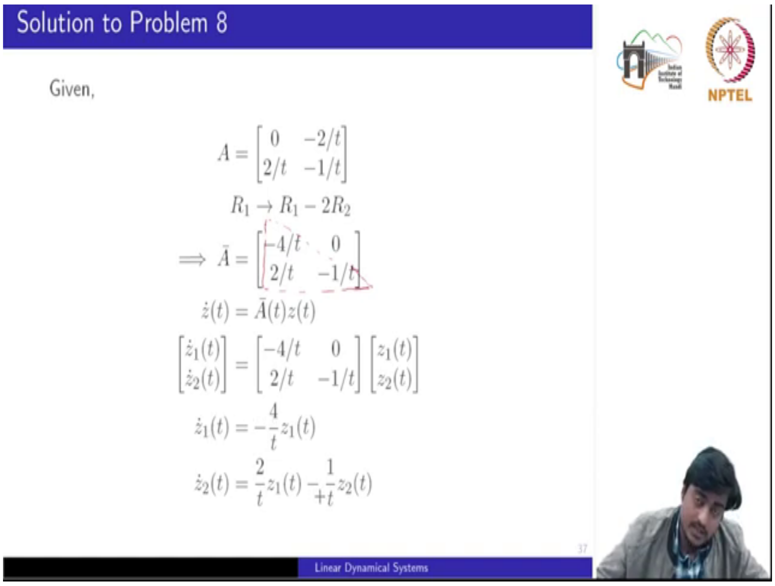

So, note that here first of all once you see this equation that the computing the in fact, the state transition matrix for an LTV system is pretty much difficult. And we have discussed once a scenario that if the matrix; if the state matrix considering the two dimension, it is given as let us say into this form; into this upper triangular form, where the element inside this triangular are non zero or could be not zero, but this one should definitely be 0 right.

Now, if we have an A matrix into this form in fact, we need to see. So, we need excuse me; so we need the A matrix; in fact, it should not be this one. So the A matrix should be they could be non zero element, but this one should be 0 right ok. So, this triangular upper triangular we should have; why we do this?.

So, that if we have the state variable x 1 dot, x 2 dot equal to s 1; x; x 1 and x 2, we should be able to write that x 2 dot is some element x 2 ok. So, it becomes automatically a decoupled system; at least the second state is decoupled and we can solve it separately. And once we have the solution for this x 2; we can put it into the equation of the first and then we can obtain the solution of x 1 and then using the concept of the fundamental matrix; we can finally compute the state transition matrix.

But here if we see the this A matrix it is not into this form because if I write the two equations; we would have x 1 dot is equal 2 minus 2 by t; x 2 by first equation this is only the homogeneous part I am writing, I am not considered or not concerned with the B matrix for the moment. And the x 2 dot I would have 2 by t x 1 minus 1 by t; x 2 right. So, now I cannot solve any of the questions separately to compute the solution of the state matrix; state variable. So, this is a pretty much difficult.

So, to address this issue; first of all we will do some, we will try to see whether given this A matrix; are we able to obtain either this matrix into this form, where the x 2 state can be computed separately or either the x 1 state can be computed separately. In that case, if we want to say that the x 1 state can be computed separately; we should have and another upper triangular form, it should be some non zero and this should be 0 and this could be non zero element ok. So, that if I write the equation of the first equation, I would have x 1 dot is equal to some non zero function of time into x 1 ok.

So, first we will convert that A matrix into the form so that we can compute the fundamental solution. So, by doing this row operation onto this A matrix; we obtain this A bar matrix, where it is into that perfect form by using which we can compute the fundamental matrix and thus finally, the state transition matrix. So, since we have performed the row operation and this state matrix A bar could be in terms of another state variable.

So, we have represented that state variable by z and these z state variable could be any variables or could have any physical meaning on to that electrical circuit. So solving these equation; we can separately solve this z 1 and then putting the solution of this z 1 into this second equation, we can solve separately this z 2 in the solution for z 1 is given by this one and the solution for z 2 is given by this one.

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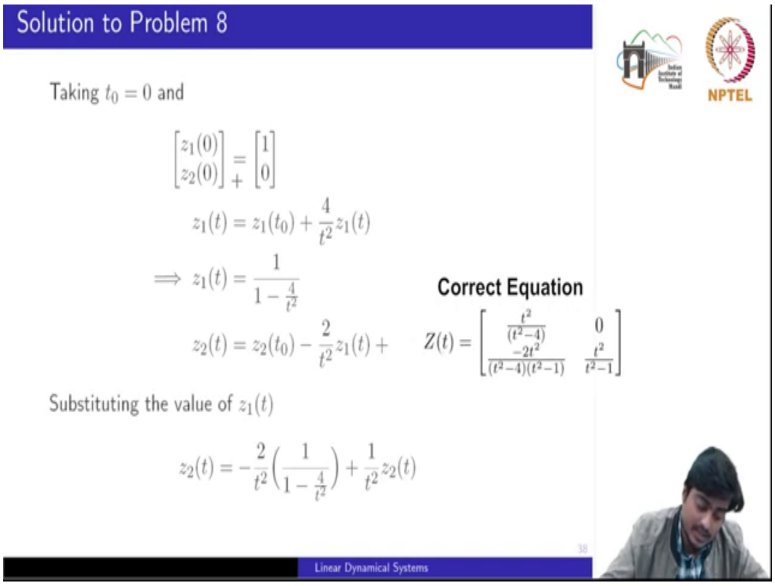

Now, these two solutions are specific to these initial condition. Now, if you recall the computation of the fundamental matrix we specify we need at least two vectors which are linearly independent so that once you compute the fundamental matrix that fundamental matrix is non singular. So, first initial condition we have specified this one and another initial condition; we need such that both these vectors are linearly independent. So, another natural choice of the initial condition would be 0, 1.

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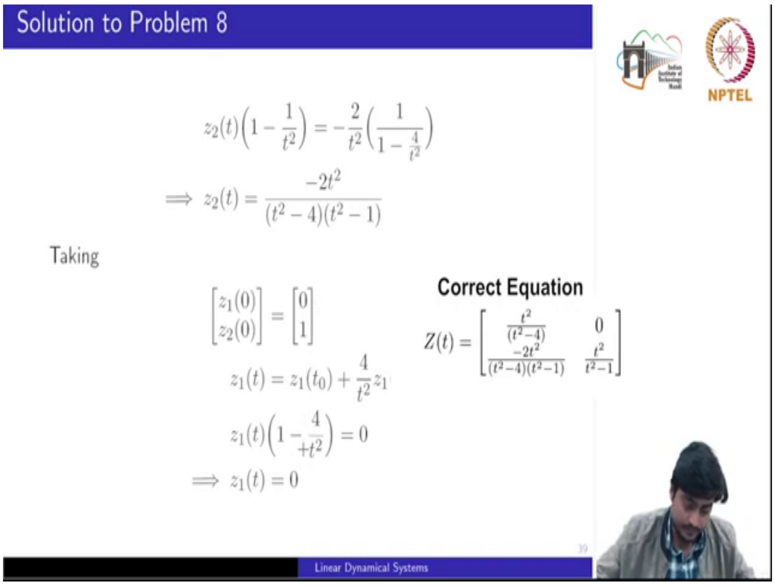

So, we choose this 0, 1 as another set of initial conditions and with respect to these initial conditions we solve for z 1 and we solve for z 2.

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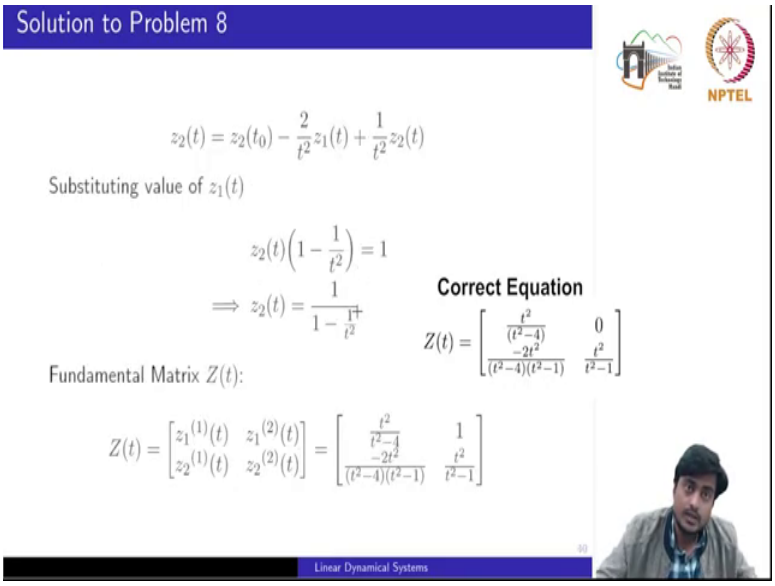

Now, with respect to those initial condition and whatever the solution we have obtained; we compute that fundamental matrix we call it z t by capital Z t which is the fundamental matrix with respect to the new state variables z 1 and z 2.

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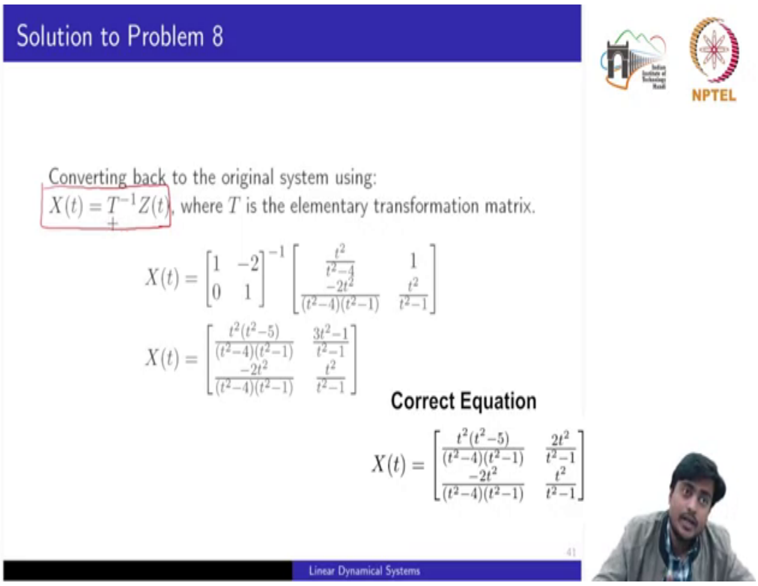

So, this is the capital Z of t; now since these two fundamental matrix. So, this is also one of the result we had studied that if the two pairs are algebraically equivalent, their fundamental matrix would also be equivalent or it would also be related by using this transformation matrix.

And this transformation matrix; we have obtained by from the elementary row operations we have performed initially. So, using this t inverse and capital Z; we obtain the fundamental matrix for the original state variables. Now, using this state this matrix; we can compute the a state transition matrix and finally, the solution on the or the time domain solution.

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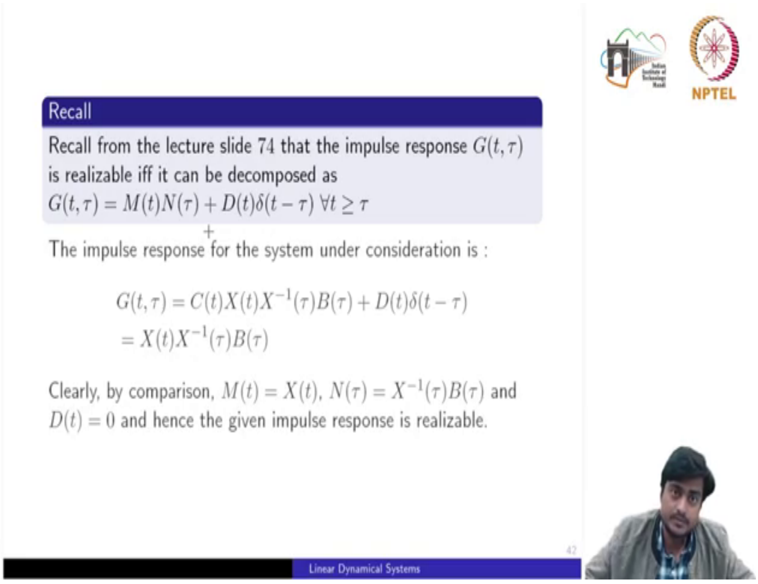

Now, talking about the realization; so this was one of the important result which was given on the slide number 74; that if the impulse response of the system is given to us as G t comma tau we say that this impulse response is realizable; if and only if that impulse response can be decomposed into this form, where this M and N matrices were parameterize in terms of the fundamental matrix.

So, for this system; if I write the impulse response I can write this one where the C matrix, you can assume it as an identity matrix. The D matrix can be assumed as the 0 matrix; so the rest part remains the same. So, if we compare these two terms; we see that your empty is nothing, but equal to X of t; the fundamental matrix itself.

And N of tau is basically your X inverse tau and B to tau meaning to say so we are able to find such two matrices M and N such that if any impulse response is given to us that we can say that this impulse response is realizable ok. So, by realizability we actually mean the physical implementation of that impulse response.

Now, this impulse response since we have computed from a practical electrical circuit; so, it was trivial that impulse response would be implementable, but since you want to show the application of those theoretical results. So, just for the sake of that we have shown that how these M N matrices can be computed ok.