Linear Dynamical Systems
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Tutorial on State Feedback: Part I
Lecture – 31
Tutorial - 5

So, now we will see the tutorial on the State Feedback Controller Design. So, this tutorial is divided into two parts. Today we will become covering part I, covering first half of the overall module state feedback controller design.

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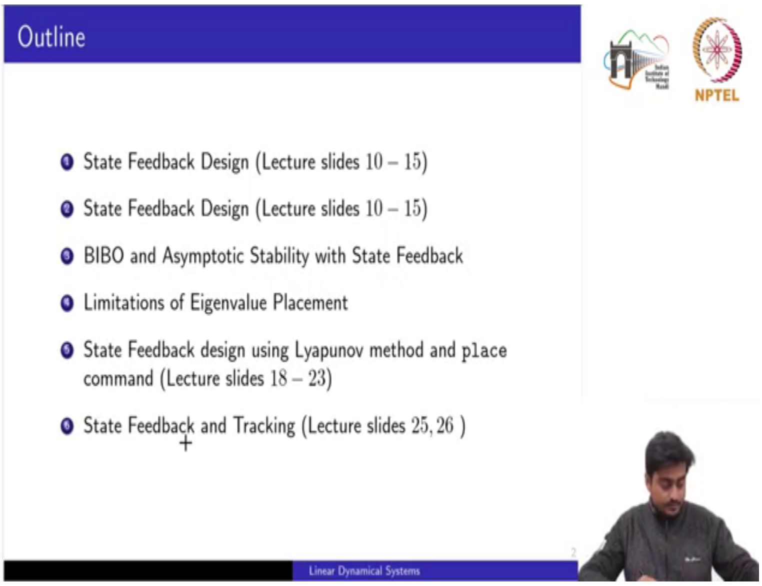

So, 6 problems we would considered to cover the modules. So, the first two problem deals with the different aspects of the state feedback controller design and these notes you can take from the lecture slides from 10 to 15. Third problem we will deal with the BIBO and asymptotic stability with the state feedback. So, this BIBO and asymptotic stability concepts were introduced in the week 2 of stability. We will see that how they are affected or they are influenced under the effect of state feedback. Forth we will see the limitations of eigenvalue placement that what eigenvalues we can place arbitrarily.

The fifth problem deals with the state feedback design using two methods, we have introduced during the theory classes. First using the Lyapunov method and another we will directly show with the place command. So, this place command is basically equivalent to the eigenvalue assignment method we have covered during the lecture slides number 18 to 23. The last problem deals with the state feedback and the tracking problem.

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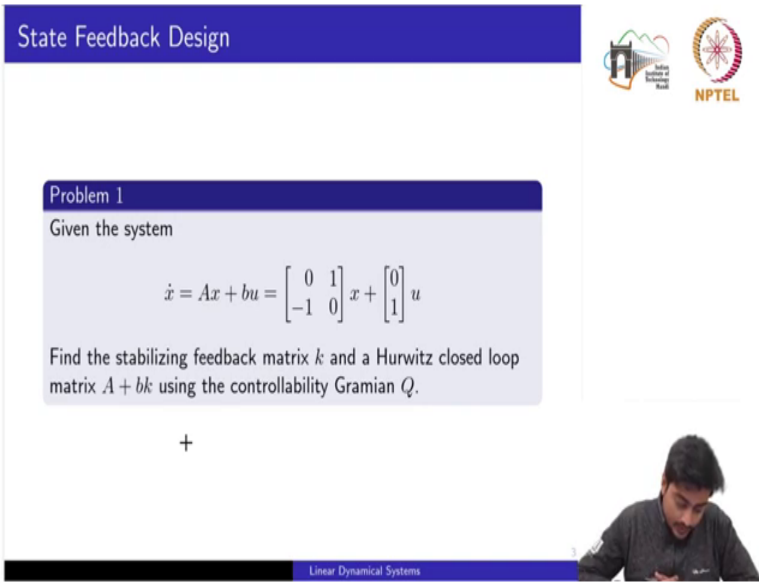

So, in the problem one, we have LTI system given to us where A b matrices where u is a single input variable and we have two-dimensional system. We need to find the stabilizing feedback matrix or the feedback gain vector k and Hurwitz closed loop matrix A plus bk using the controllability Gramian Q. So, this problem we have introduced in the tutorial problems because this gets new viewpoint of designing a controller using the controllability Gramian.

In the lecture slides, we had seen the eigenvalue placement by either using the to the Lyapunov method or the place command. So, here we would design the gain vector k by using the controllability Gramian u.

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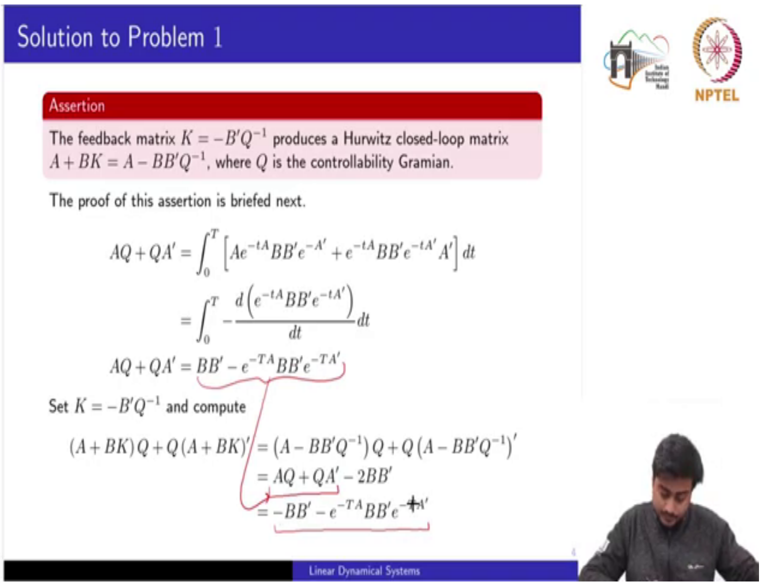

So, first we will introduced the method that how you could use this method to compute the state feedback gain. So, the result says that the feedback matrix k given by is minus B transpose Q inverse produces a Hurwitz closed loop matrix A plus BK which is equivalent to this one, if I substitute k from here to here where Q is the controllability Gramian, ok. So, in the controllability week we had seen that how to compute the controllability Gramian. So, once we computed at Gramian the B matrix is already known to us. So, by using this formula I can compute the state feedback gain, ok. So, let us see the proof of this one.

So, we know on to the left hand side is basically similar to what we had seen the is a Lyapunov equation during the stability week, since we need to prove or we need to show that the closed loop matrix is Hurwitz. So, we will take the help of the Lyapunov equation to show this result. So, if we recall the left side it is the left side of the Lyapunov equation AQ plus QA transpose, the controllability Gramian is computed using this method where we have this A transpose t, ok. So, A times Q plus Q times A transpose in the overall integral.

So, here we are using the integral from 0 to capital time T, otherwise we could take it also from 0 to infinity. Now, if we see this integrand this integrand we can express this as the derivative of this quadratic form, which is been integrated from 0 to T. Now, solving this definite integral B obtain this on to the right hand side, ok. But this only shows this only shows the stability of the plant itself and we need to show the stability of the closed loop matrix.

So, if I apply the same result, so the closed loop matrix is this one and now replacing this A matrix by this A matrix, we can write the left hand side and setting k is equal to minus B transpose Q inverse because we want to show that for this state feedback matrix the closed loop matrix, the closed loop state matrix is stay. So, simplifying this expression on to the left hand side we get this one which for the simplifies to AQ plus QA transpose minus 2 BB transpose.

Now, this part we have already seen from the above, so if I substitute this one to here and simplify further we obtain minus BB transpose minus e to the power minus T into A BB transpose e to the power minus TA transpose, ok. So, we need to show that for the stability if you recall the Lyapunov stability theorem for the LTI system, Q is already a positive definite matrix, and this is the matrix of which we want to show the stability. So, we need to only show that the right hand side is negative definite, ok.

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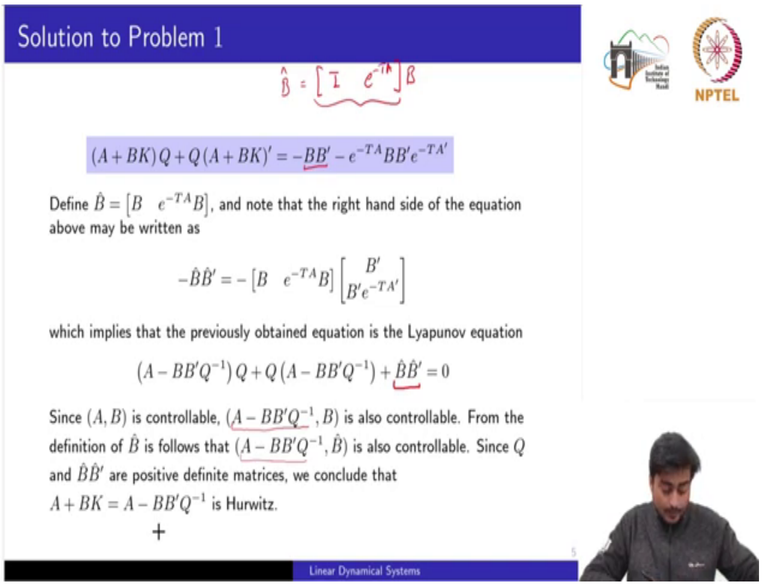

Basically, so we will deal with this equation now. We define another matrix let us say B hat define as containing two block matrices B and exponential matrix times B, and note that right hand side of the equation about may be written as minus B hat times B hat transpose because if I simplify this expression I will get this one which implies that the previously obtained equation is basically the Lyapunov equation and finally, we obtain this.

Now, we only need to visualize whether this matrices positive definite or not. So, since the original pair A comma B is controllable, this new or closed loop state matrix with or paired with the matrix B would also be controllable. This was one of the results. Now, from the definition of the B hat it follows that this pair would also be controllable um. Because why? I can express this B hat as I and e to the power minus TA times B, ok.

Now, this matrix would be a nonsingular matrix definitely. So, pre-multiplying with the nonsingular matrix could not change much of the properties of the B, the rank would remain the same. So, we need to say this closed loop state matrix paired with another B matrix which is B hat would also be controllable.

Now, since Q and B hat B hat transpose are positive definite matrices we conclude that this pair would definitely be Hurwitz. Now, this is easy to visualize that why this matrix would be positive definite matrix. So, see this one. So, B into B transpose would be positive definite or would less or at most sorry at least it would be positive semi definite it cannot be negative, right. Now, this part because of the exponential matrices it would always be positive definite. So, if I at a positive semi definite matrix to a positive definite matrix the resultant would always be a positive definite, right because of this we conclude that this matrix would always be a Hurwitz matrix.

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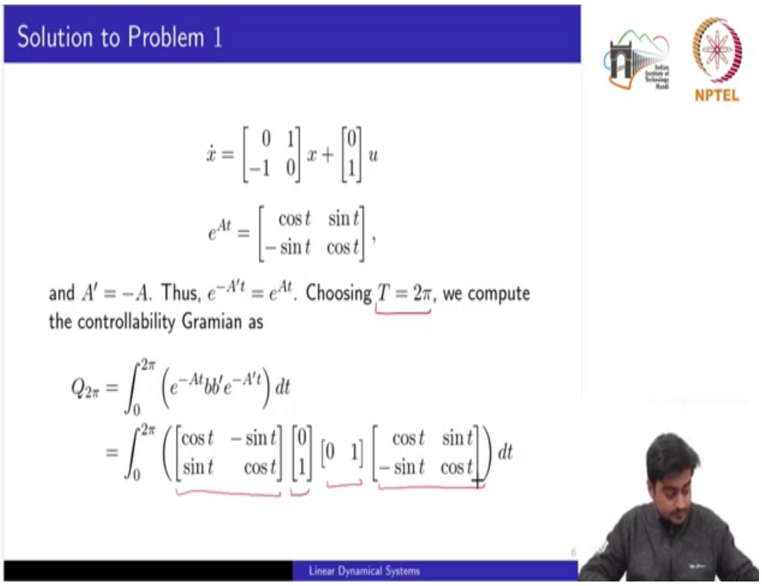

Now, coming onto the problem we want to design the controller we have this AB matrix. So, for computing the controllability Gramian we need the exponential matrix of the A matrix. So, once we compute we get these exponential dumps and at the same times we notice that A transpose is equal to minus of A. Thus we can we can also write this one that e to the power minus A transpose T would be equal to e to the power AT, ok.

Now, if you recall that we need to compute the controllability Gramian over sin over sometime T or to infinity. Now, all these terms are the sine cosine terms. So, it would be if I integrate them from 0 to infinity it would be infinite, ok. But we also know that these they are periodic in nature. Every term is periodic in nature, so I can compute over 0 to 2 pi and finally, write their result in terms of or by choosing T is equal to 2 pi, ok. So, computing this matrix, so we substituted all these parts this is the exponential term BB transpose and again the exponential term.

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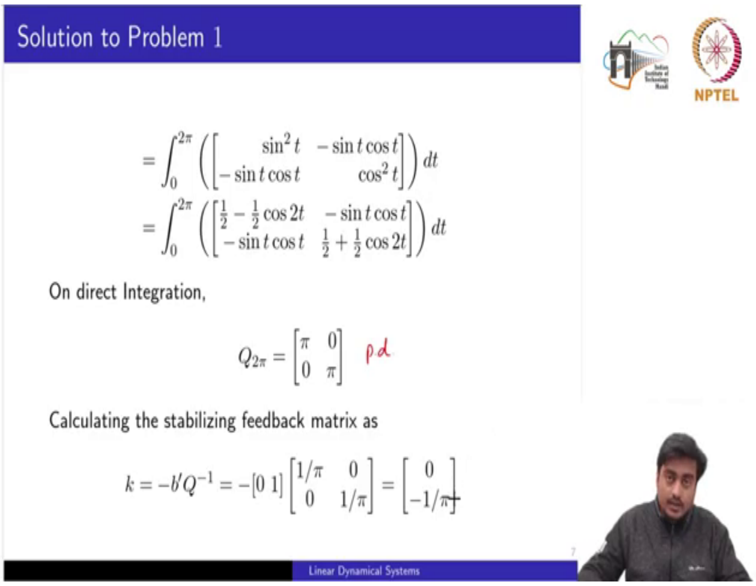

So, solving this definite integral on direct integration we obtain the controllability Gramian is this matrix. So, these are pi, right, so this would definitely we have positive definite matrix, ok. Now, calculating the stabilizing feedback matrix as k is equal to minus B transpose Q inverse, we finally obtained this state feedback gain vector.

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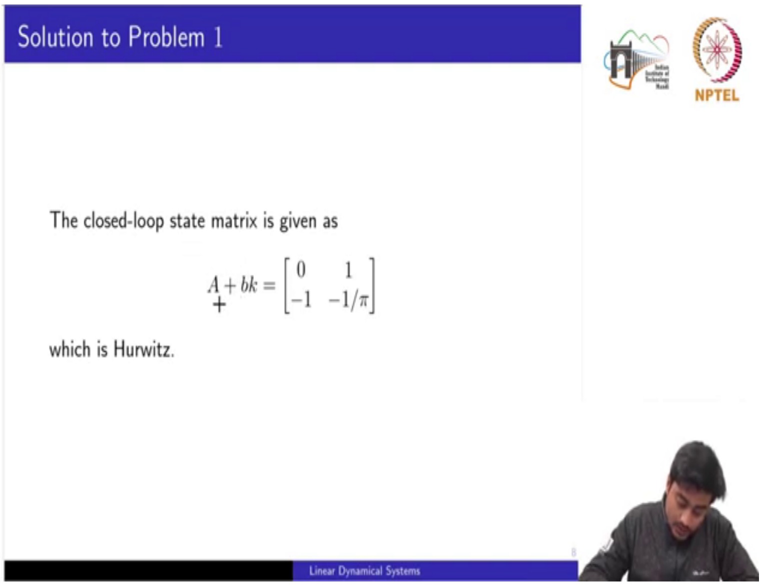

Now, next you can verify the closed loop state matrix whether it is Hurwitz or not. So, by computing the eigenvalue of this matrix you would see that this matrix is Hurwitz. So, here we are presented another way of computing the state feedback matrix by using the controllability Gramian.

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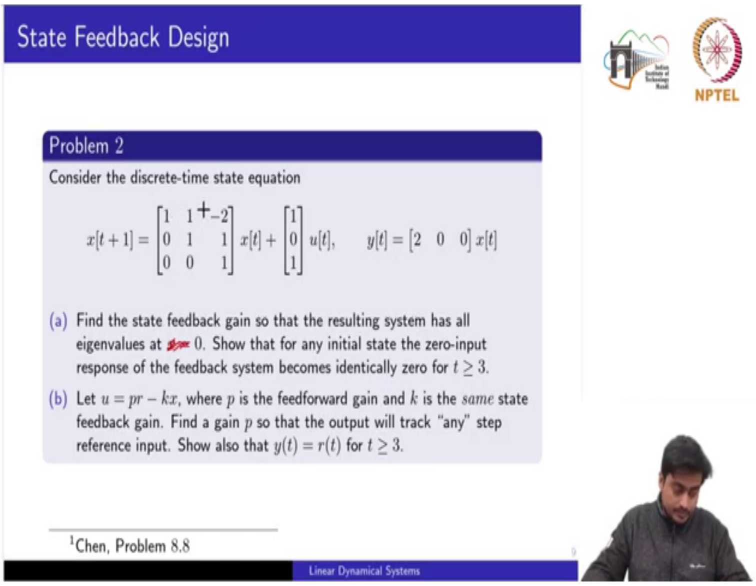

In the problem 2, we discuss the discrete time controller design where we are given the AB matrices of a discrete time system and also the c matrix. The first part a deals with finding the state feedback gain so that the resulting system or the closed loop system has all the eigenvalues at x is equal to 0. Now, do not confuse here. So, x is the state basically. So, all the eigenvalues should be located as 0. So, should not be x is equal to 0.

Note that you should not confuse here that we are not in the continuous time domain because it we are in the continuous time domain and locating all the eigenvalues at 0 would result into an unstable closed loop system. But in the discrete time placing the eigenvalues are 0 meaning that all the eigenvalues are inside the unit circle, ok. So, we need to design the state feedback where we are placing all the eigenvalues at the origin. The next part we want to show that for any initial state the zero input response of the feedback system becomes identically zero for t greater than equal to 3.

So, this is important part that here once we go into the proof of this part, it is obvious that for any initial state, we can have the response of the feedback system identity equal to 0, right. So, we will come on to this part once we see the solution.

Now, the b part we are introducing another feedback which is given by u is equal to pr minus kx, where p is the feed forward gain and k is the same state feedback gain what we have computed in the part a. Now, here we want to find again p, so that the output will track any step reference input and the next part say that we want to show that y of t is equal to r of t for all t greater than equal to 3. So, first we will take up the first part for placing the eigenvalues at 0. Note that here you can compute the eigenvalues of the A matrix since it is in upper triangular matrix, so all the eigenvalues are located at 1 and we know that it is an unstable system because the, it is a discrete time system.

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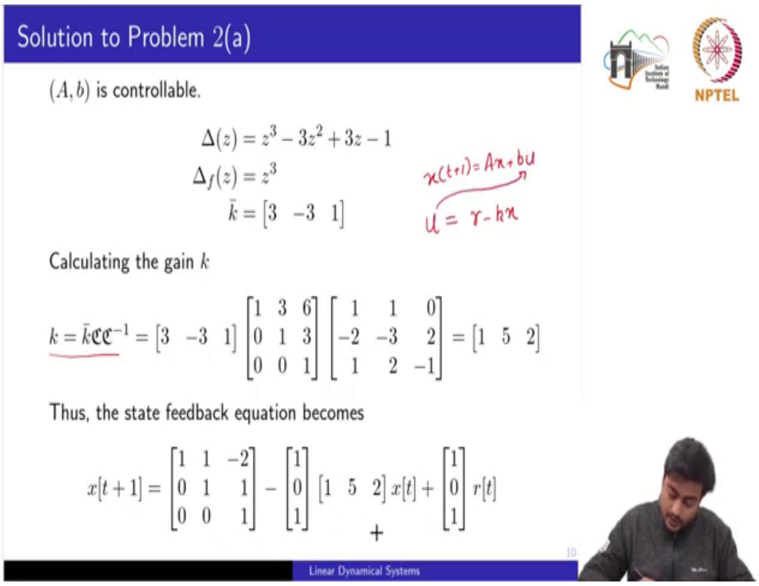

Now, first of all we need to check the controllability of the pair A comma B which you can do by computing the controllability matrix and then checking the rank of it. So, here we will carry out the design using the eigenvalue assignment. This delta z is the characteristic polynomial of the plant and this characteristic polynomial is the desired characteristic polynomial.

So, because if we compute the roots of this patriotic polynomial it could be multiple roots located at 0. So, first we compute the k bar which is computed by the differences of these coefficients of the z having same powers. Since all these coefficients are 0, so these k bar can be form directly with the coefficients of the characteristic polynomial of the plant.

Now, the gain k can be computed by using this formula, where k bar we have assigned by heading the information of the characteristic polynomial of the plant and the feedback loop and then the controllability matrix multiplied by its inverse. So, using this equation we can compute this k matrix, k vector.

Now, the second part if you recall that we want to show that for any initial state the 0 input response of the feedback system becomes identically 0 for t greater than equal to 3. So, we will compute first of all the response of this system by putting this gain k or by substituting or by using this equation u is equal to r minus k x into the plant, substituting this u we obtained directly this one.

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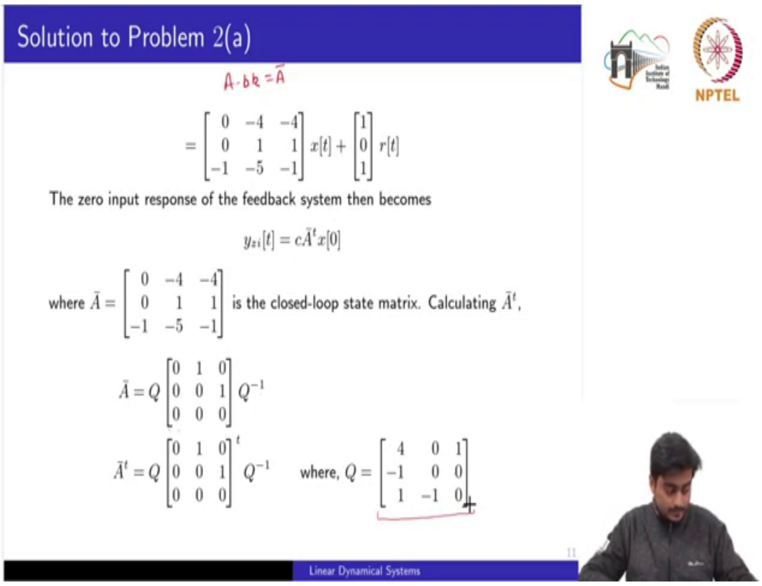

And finally, the closed loop matrix or the state matrix is given by this one which is we as assigning is A bar, ok. So, this could be the closed loop state matrix and this is the distribution matrix with respect to the reference signal. Now, the zero input response meaning to say that we are substitute r is equal to 0 and if we see directly the equation of the solution of the discrete time system or the homogeny system is given by this one. The c matrix multiplied by A bar to the power t, where t is in discrete in nature times x of 0, ok, where A bar matrix is the closed loop state matrix. Now, for calculating this A bar to the power t we can use different methods.

So, one method we had discussed during the week 1 is using the Cayley Hamilton theorem. The second method is by computing the first of all, the similarity transformation matrix. So, this is what we have done here by computing or by expressing A bar is equal to Q times this matrix because they should be equivalent to the closed loop state feedback matrix. And we know at the outside that the closed loop state feedback matrix should contain all the eigenvalue at 0 and these are repeated and it would contain only one Jordon block, ok. So, this is only one Jordon block times Q inverse.

Now, expressing the powers of A bar is these matrices will not change because these are the similarity transformation matrix. So, we can directly take the power of this matrix where if you want to compute Q this you will get.

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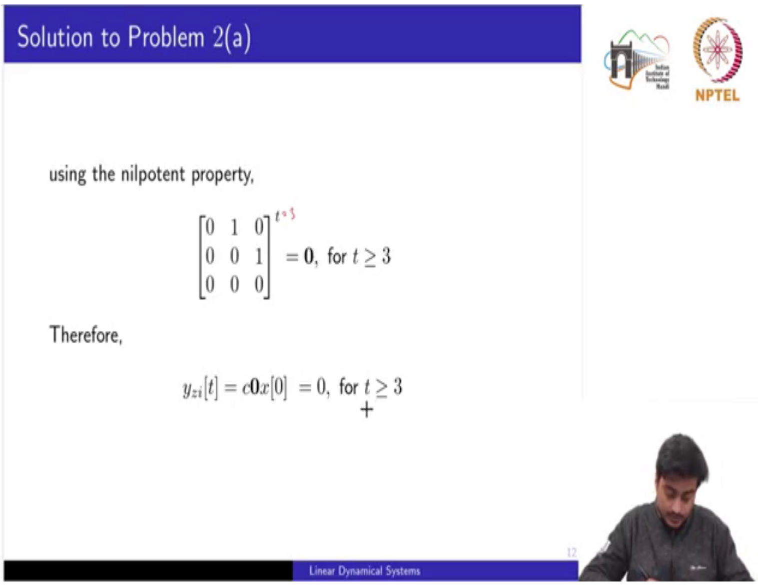

Now, we will start computing thus powers of this matrix for t, 1 starting from 0, 1, 2, 3 and we will see that once you reach to t is equal to 3 or higher this matrix should always be a 0 matrix. This is the reason that is the 0 input response of the system is always 0 for t greater than equal to 3, ok.

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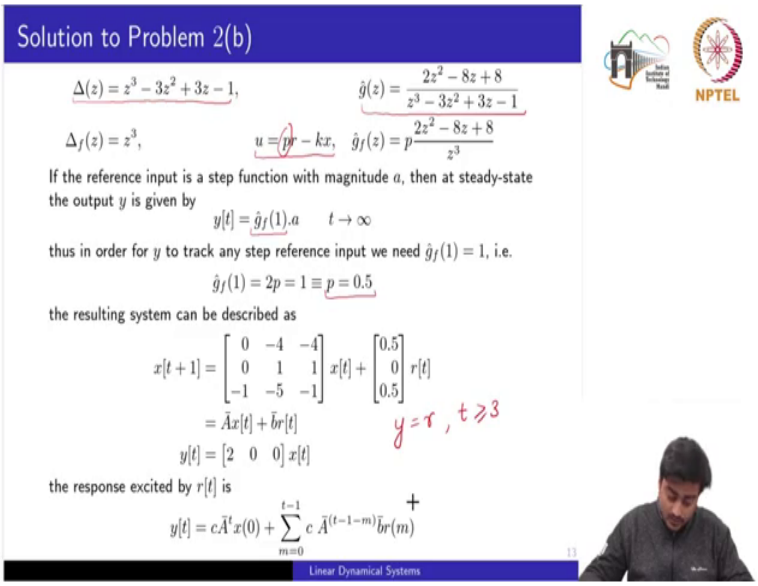

Now, in the second part we have introduced, we want to track any step reference input. So, this characteristic polynomial is the, is of the plant and this is the transfer function of the open loop system, right. As the property of the feedback controller design we know that the numerator polynomial would be affected, only the denominator polynomial would be affected because all the eigenvalues were shifted from 1 to 0, ok. And this is the characteristic polynomial of the feedback loop. This is the controller we are now using where we have introduced this p because we want to track the reference signal, and the feedback the transfer function of the feedback loop is now given by this where because of inclusion of this p we would get the overall transfer function with the denominator polynomial change multiplied by variable p, ok.

Now, if the reference input is a step input where magnitude a, then it steady state the output y would be given by this one g hat f at 1 times a as t tends to infinity, where t is discrete in nature. Thus in order for y to track any step reference input we need this vector is equal to 1, that is if I substitute or if I compute g hat f of 1 we will get twice of p from here and if I substitute is equal to 1 we get p is equal to 0.5, ok.

So, the state feedback gain matrix or the vector would remain same as you have computed in the part a. Now, to track any reference input we need to compute this p and this p is supposed to be 0.5. So, the resulting system can now be described as this where this is the closed loop matrix in the B bar matrix will also change and by of it is given by this. Now, the response excited by the reference signal is given by this. We are computing the response because we want to show that y is equal to r for all t greater than equal to 3, ok. So, we can express the response of the closed loop system with input r by using the variation of constant formula or by using the iterations for computing the powers of a.

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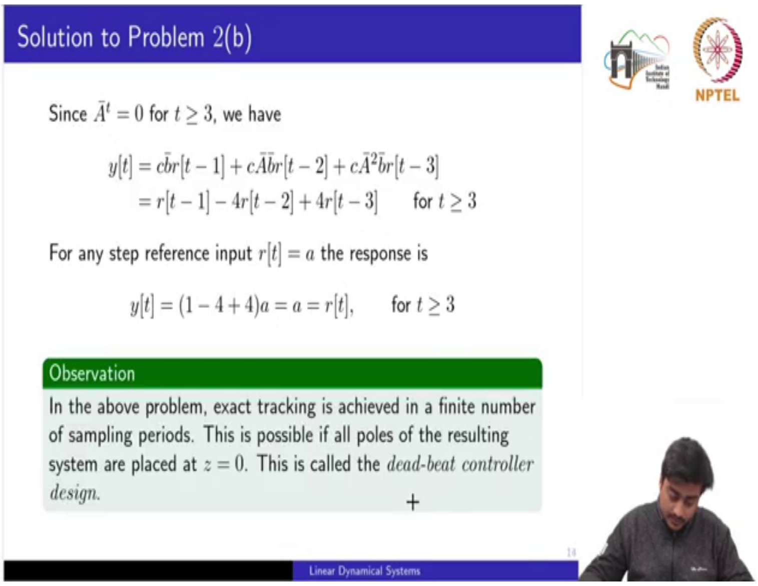

So, since we know that A bar of A bar to the power t is equal to 0 for all t greater than equal to 3, so, we can compute only for finite sample times, t 1, 2, 3 and afterwards we know we would get A bar to the power 3 is equal to 0, ok. Substituting the c in the b vectors we finally obtain this one.

Now, we know that this reference signal is a step input of magnitude a, so I can replace all these reference signals which are the shifted version because they would remain the same being a step input, so we have replaced by 1 minus 4 plus 4 times a, ok. So, finally, we with get this a, which is nothing but is equal to r of t for all T greater than equal to 3, ok.

So, in the above problem you would notice that the exact tracking is achieved in a finite number of sampling periods and this was possible if all the poles of the resulting systems are placed at z is equal to 0. So, we in the theory classes we have discussed tracking a reference signal asymptotically, when t tends to infinity, but here the advantage we got by placing the eigenvalues at the origin is that we are now able to track the reference only in finite sample periods and this procedure or this design is also called the deadbeat controller design, ok.

So, we try to cover this concept by using the tutorial problem instead of going into the theory because the design of the state feedback controller remains the same. It is just the aspect of tracking a reference signal infinite sample periods, ok.

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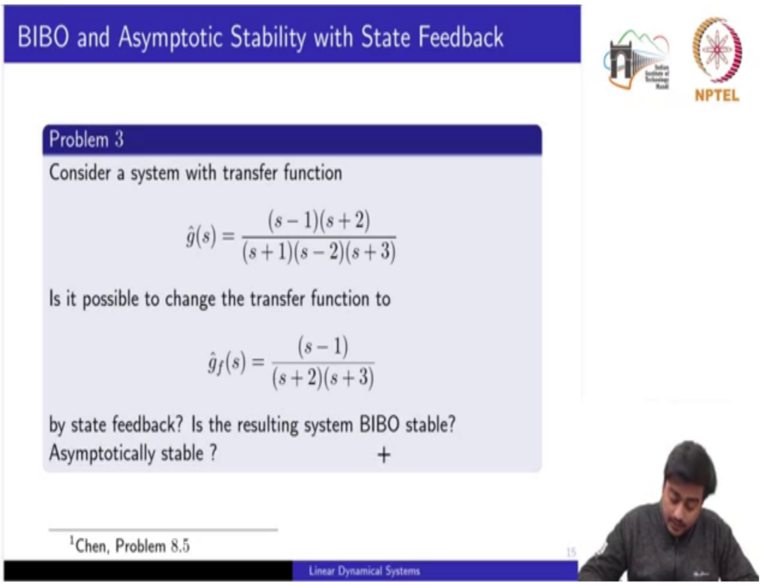

In the problem 3, consider a system with transfer function. So, here we have taken the single input single output system, where one 0 that is unknown minimum phase system because one 0 is located on to the right hand side and at the same time which is also a unstable system. Because one eigenvalue or one pole of this transfer function you will also located on to the right hand side.

Now, the question says is it possible to change the transfer function to this one, ok, by the state feedback. So, if you recall in the lecture slides we had said that the controllability property is invariant under the state feedback and also all the eigenvalues are or let us say the numerator polynomial is unaffected, by the state feedback. But here you would notice readily that one of the 0 located at minus 2 is not available in the closed loop transfer function, ok.

So, this aspect we had said that we would discussed during the next week we will be discuss more about the observability. So, first of all we will deal with this problem that whether we can change this transfer function to this transfer function by using the state feedback. We also need to comment about the stability of the resulting system in terms of BIBO stability and asymptotic stability, ok. The question is this question is significant from the view point of that if you recall from the stability week we had discussed on to one important aspect that whenever the system is asymptotic stable then it implies that the system would also be BIBO stable, but if the system is BIBO stable we cannot say that the system is asymptotically stable, ok.

So, let us see the solution of this problem.

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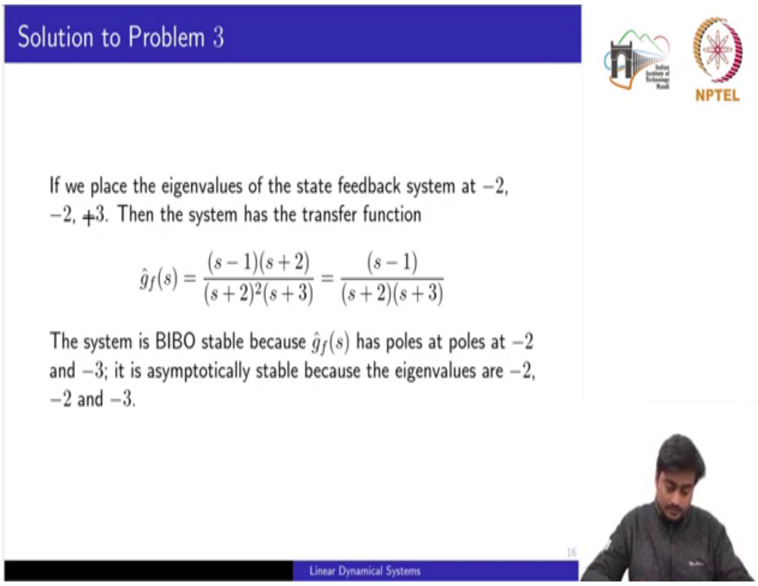

Now, if we place the eigenvalues of the state feedback system at minus 2, minus 2 and minus 3 then the system would have the transfer function given by this one, right. The numerator polynomial would remain as it is. We can change the denominator polynomial by placing the eigenvalues here, where two eigenvalues are located at minus 2 and another eigenvalues is located at minus 3, ok. Now, since s plus 2 factor is common this would cancel out and finally, we would have this transfer functions, ok.

Now, if you see that the system is BIBO stable because all the poles of this transfer functions or on to the left hand side located at minus 2 and minus 3, ok. Now, at the outset we also know that by the state feedback that for the obtaining the state feedback system we have deliberately place the eigenvalues on to the left hand side.

So, this system would definitely be asymptotically stable because the closed loop system implicitly would have all these eigenvalues on to the left hand side, ok. So, since there is some cancellation, so some property or the some dynamical properties of the systems are lost and this what are these properties how are they lost we would cover during the observability week.

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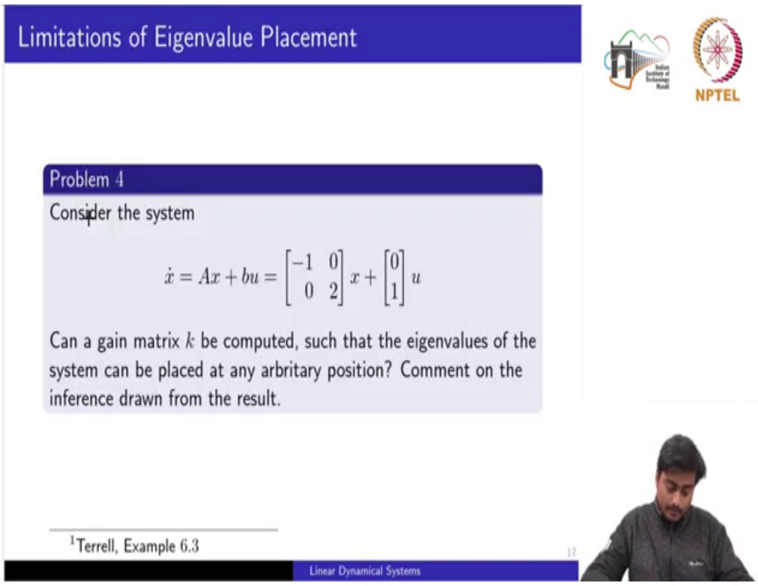

The next problem deals with the limitations of eigenvalue placement. So, considered the continuous time system with AB matrices has given as this. So, the first question say that can again matrix k be computed such that the eigenvalues of the system can be placed at any arbitrary position.

The next part we need in the next part we need to comment on the inference drawn from the results. So, this is very important result which we had discussed during the lecture slides. So, here if you see that we only need to show the controllability whether the system is controllable because if the system is controllable then we can place the eigenvalues of the system at any arbitrary position. Now, this here we are showing numerically that what eigenvalues could be shifted or could be placed arbitrarily,.

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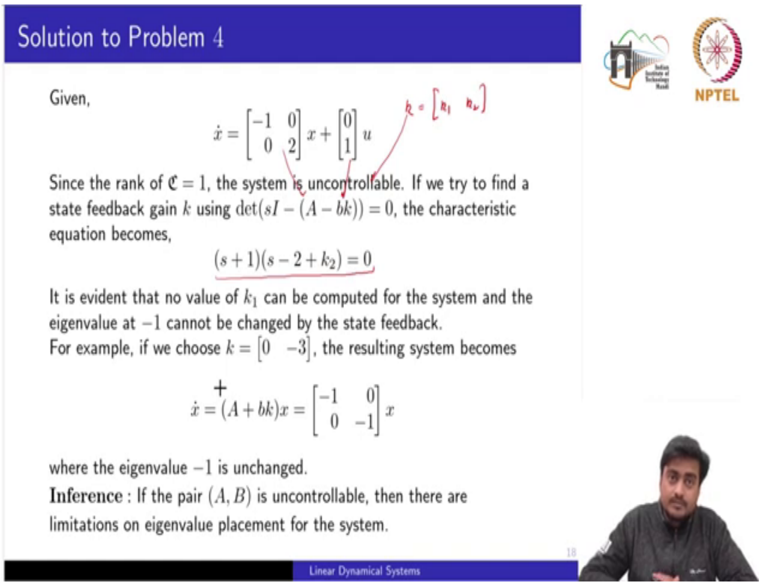

So, let us see this is sum. Once we compute the rank the rank of the controllability matrix as 1 and not equal to 2, so we know with the system is uncontrollable and we cannot play place the eigenvalues at are any arbitrarily equation.

Now, if we try to find the state feedback gain using this formula determinant of sI minus A minus bk by putting it equal to 0, the characteristics equation becomes this, ok. Now, here k we have chosen as k 1 and k 2, ok. Now, once we substitute k here and the A matrix form here the B matrix here and solve this equation for open up this equation we obtain this one. One important thing you would see that k in k or one of the elements of the gain vector k is not available here, which is k 1.

Meaning to say that, we can place or we can use any value of k 2 to place the one of only one of the eigenvalues. This eigenvalue would remain as it is, that is one of the poles located it minus one it cannot be change, but the another eigenvalue which is located at 2, meaning to say that the system is an unstable system we can place it anywhere in this way, ok.

So, the inference what we have drawn from the from solving this problem that if the pair A B is control uncontrollable then there are limitations on eigenvalue placement for the system. It also shows their depending on the rank of the controllability matrix that only one of the eigenvalues can be place anywhere. And this inference you can also obtain by using the decomposition by the using the uncontrollable decomposition and then expecting the controllable part and the uncontrollable part. So, the controllable part would be this one and the uncontrollable part would be this one, and since the uncontrollable part is having the eigenvalue onto the left hand side, meaning to say that this system would at least be stabilizable, ok.

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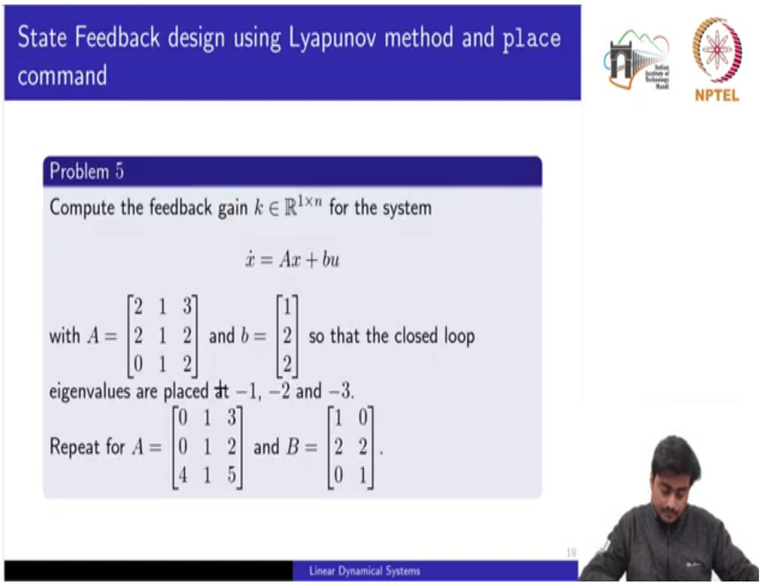

So, in the problem 5, we will see the design of the state feedback gain by using two methods. One is the Lyapunov method, another is the eigenvalue method. The objective here is that we want to place, so for these matrices A and B we want to place their eigenvalue or the eigenvalues of the closed loop at minus 1, minus 2, minus 3, ok.

Here we want to see numerically that if we use two different methods for computing the state feedback gain to place the eigenvalues at the same location whether that vector or whether that gain or those gains would be same or different, ok. So, this problem we could see in with the single input variable case and also with the multi input variable case, because the dimension of the B matrix or it contains two number of columns.

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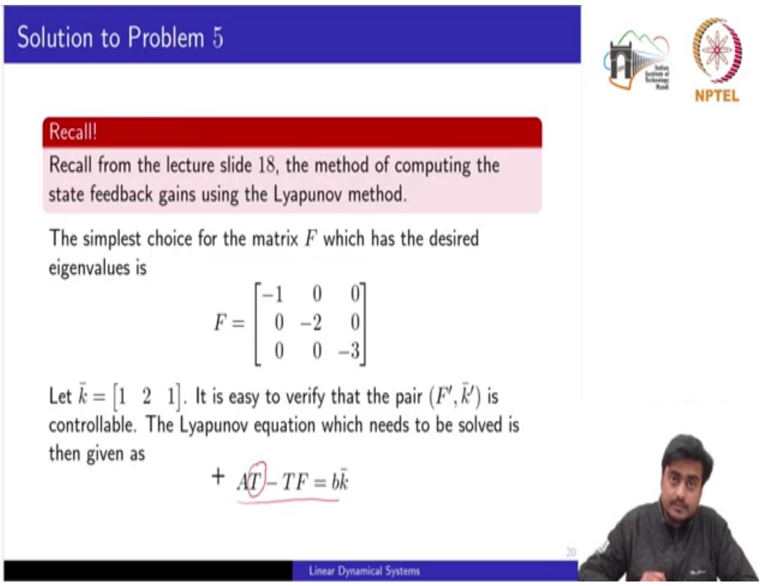

So, first we will solve the single input variable case. So, recall from the lecture slide 18 the method of computing the state feedback gains using the Lyapunov method. So, first we form a matrix F so that the eigenvalues of the matrix F would be at the desired location. So, here we are using diagonal matrix so that the eighenvalues would readily be visible at minus 1, minus 2 and minus 3.

So, the second step if we recall that we choose k bar an arbitrary k bar such that the pair F transpose k bar transpose is controllable and then we solve the Lyapunov equation like this one to finally, compute this T matrix, ok.

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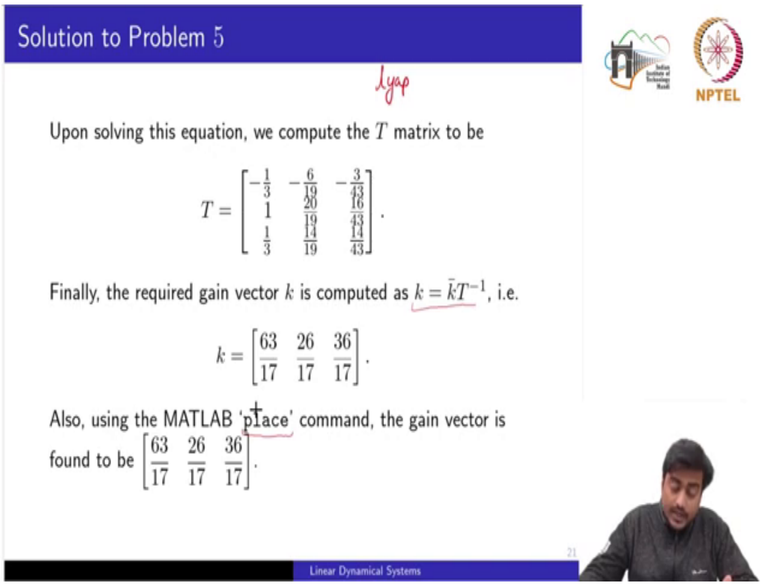

Now, solving this equation you can use the lyap command, that is to say this is a MATLAB function lyap to compute this matrix T. So, we compute this matrix T and we have also commented during the lecture slide that this T matrix should be nonsingular otherwise you cannot compute the state feedback gain using this equation which involves the inverse of the T matrix, ok. So, from there we compute k is equal to this one.

Now, if we use the MATLAB place command which is equivalent to using the eigenvalue assignment method then we also computed the same gain vector, ok. So, either we use the Lyapunov method or we use the eigenvalue assignment method for the single input variable case we obtain the same feedback gain vector, ok. So, now, we would verify that whether this equivalents also holds for the multi input variable case.

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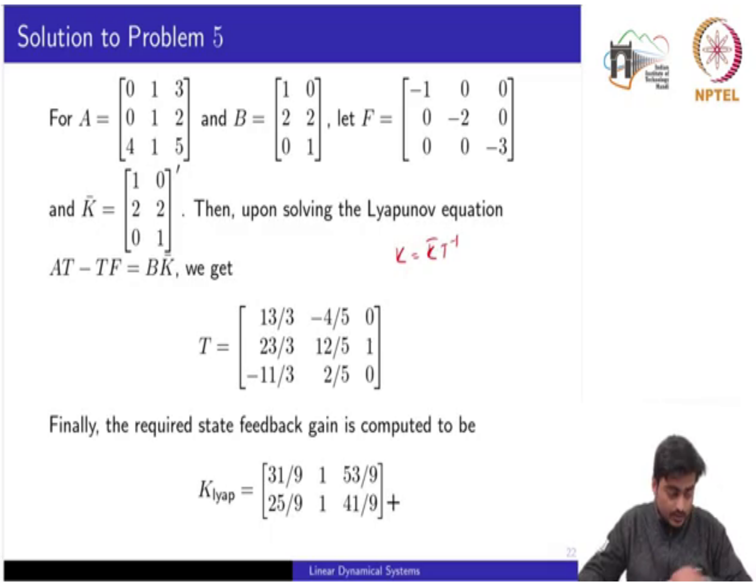

So, these are the AB matrices given to us we need to place the eigenvalues. So, if we form the F matrix containing all the eigenvalues at minus 1, minus 2 and minus 3, ok.

So, first of all again the first step we choose K bar and arbitrary K bar such that f transpose and K bar transpose is controllable, ok. Now, using this Lyapunov equation we solve for T. Now, if you recall that in the multi input variable case we cannot ensure the non-singularity of the T matrix always, but here fortunately we have chosen this K bar in such a way that the T matrix becomes a nonsingular, ok. So, now, using the equation that is K is equal to K bar T inverse be compute this state feedback matrix and we put a subscript as lyap so that we can identify that this matrix had been competed by using the Lyapunov equation method, ok.

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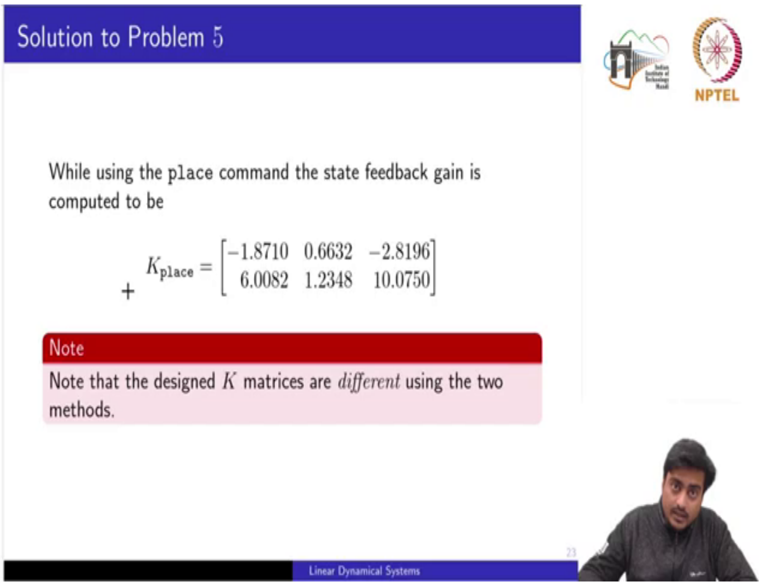

Now, in the MATLAB we can use the place command to compute another state feedback matrix which is given by this one. Now, if you see this equation and this a state feedback gain they both are different, that both the design K matrices, they both are different. Now, it could be verify that if we use either of the state feedback gain matrix the eigenvalues would definitely be placed at the same location minus 1, minus 2 and minus 3.

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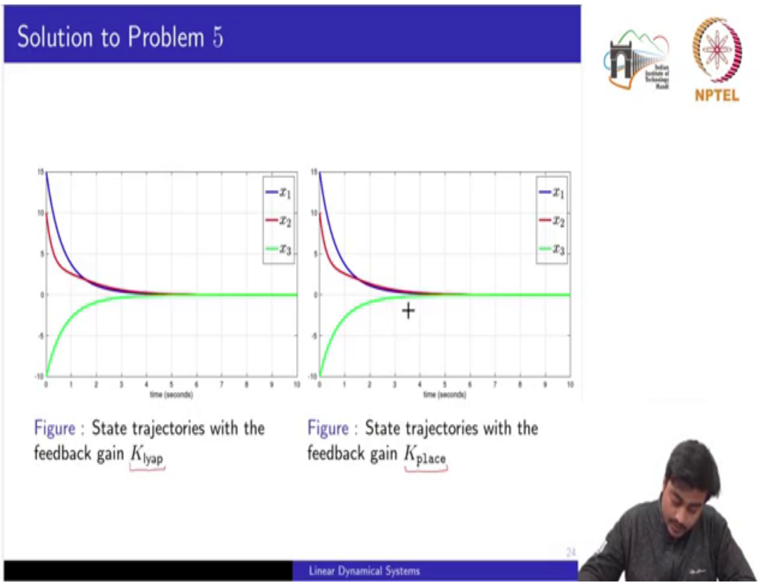

So, let us see the response of the system by putting this two different state feedback gain. So, one we have the K lyap and another we have the K place. So, you would see that we obtain the same state trajectories when we apply these two different state feedback gain.

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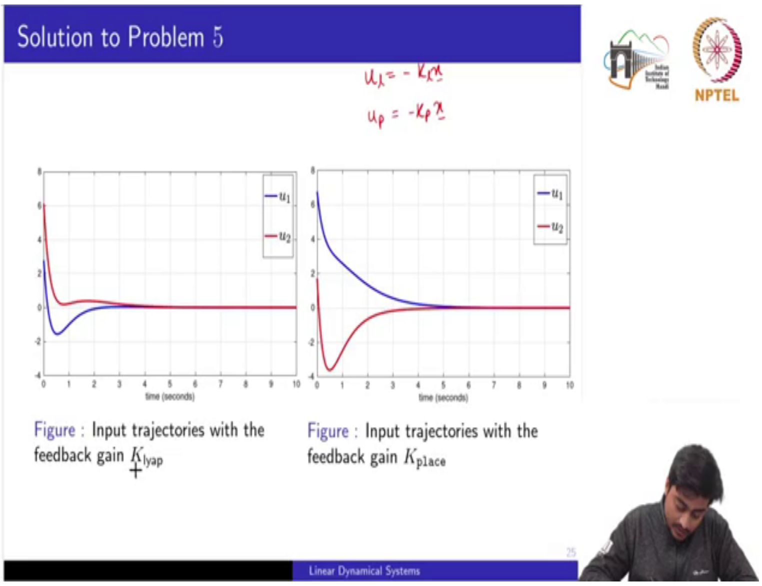

So, for the Lyapunov case we have the less control energy if you want to compute the control energy you can compute by using the formulas, introducing the earlier weeks, but by using the place command we are getting a higher energy or the as it appears at least in the signal u 2, ok while here the u 1 trajectory is more shuffle then the u 1 trajectory for the first feedback loop.

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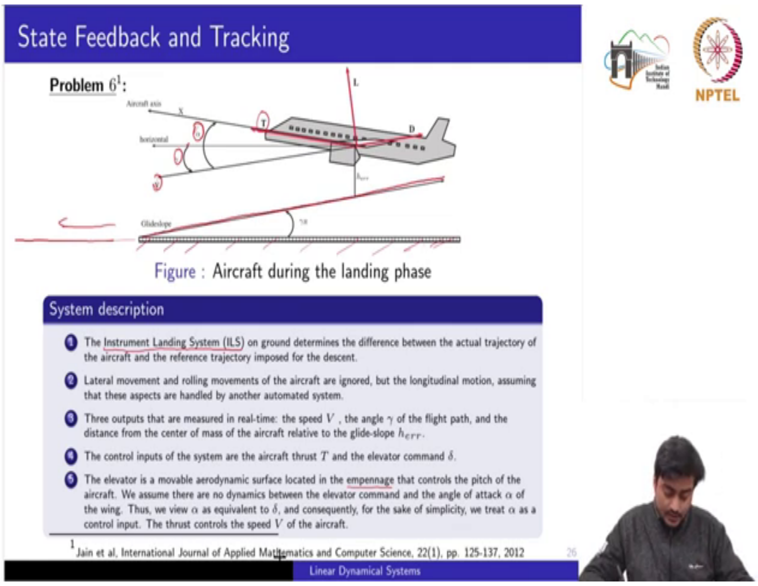

Now, the last problem deals with the state feedback and tracking. So, this is a more practical example where we have considered an aircraft which is in fact, during the landing phase where the aircraft. So, the, so ok, let us recall this system description. So, this is the ground. Now, there is some a glide slope, there we want to learn this aircraft along this trajectory and afterwards it will go onto the runway, right.

Now, and practical setups we generally have the instrument landing system what we also call called the ILS on the ground which basically determines the difference between the actual trajectory of the aircraft and also the reference trajectory imposed for the this end. Now, the reference trajectory is basically this one given by the glide slope, ok. That the error what whatever the height of the aircraft is from the center of mass and it should descend along this glide slope to finally, a run on to the runway to lowered speech.

Now, in this system description we have not taken the lateral and the rolling movements of the aircraft, but we are only concerned with the longitudinal motion assuming that the lateral movement and the rolling movements are being handled by another auto automated system. So, 3 outputs that we are measured in real time is the speed V. So, the speed V of the aircraft along this glide slope, the angle of attack gamma of the flight path from the horizontal exists. So, this one is the angle of attack and the distance from the center of mass of the aircraft related to the glide slope; so, this h error, ok.

Now, the control inputs of the system are the aircraft thrust applied into the aircraft excess which is defined by this variable T and the elevator command delta. So, if you if you have some information about the aircraft, so these elevators are basically located at the back side or onto the wings or which we also see the empennage, that basically controls the pitch of the aircraft. Now, have you assume that there are no dynamics between the elevator command and the angle of attack alpha.

This angle of attack is basically the angle between the aircraft axis and the axis along the glide scope, basically this one, angle of attack, alpha of the wing. Thus we view alpha as equivalent to delta and consequently for the sake of simplicity we treat alpha as the control input instead of the elevator command delta, ok. So, the thrust controls the speed of the aircraft. So, if you want to get to know more about the system description of this aircraft during the landing phase, I would suggest you to go through this reference which is given in the footnote.

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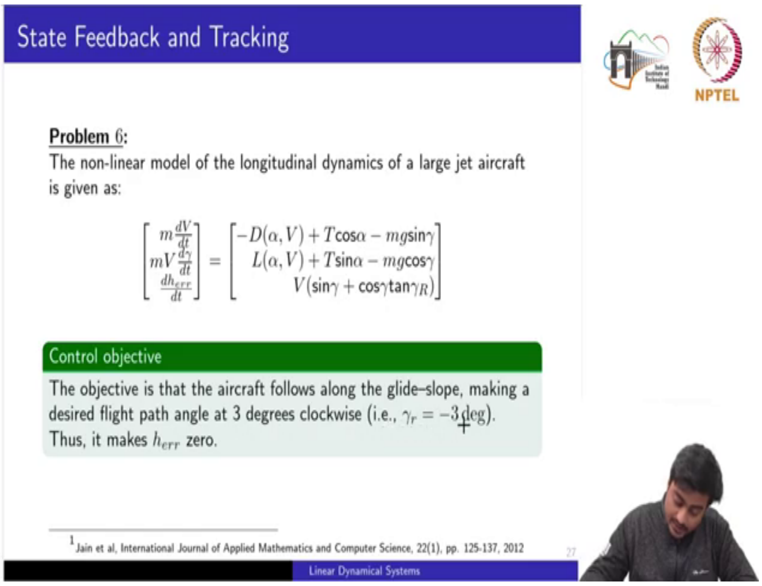

Now, the non-linear model of only the longitudinal dynamics of a jet aircraft is basically given by this, where we have the differential equations in terms of the velocities the gamma which is the flight path, angle of the flight path and also the distance between the center of mass of the aircraft and the glide slope. And the objective is that that the aircraft follows along the glide slope making a desired flight path angle at 3 degrees in the clockwise direction, ok. So, since we are taking the anticlockwise directions is a positive. So, for the clock wise we would have minus 3 degree, thus it would make h error equal to 0, ok.

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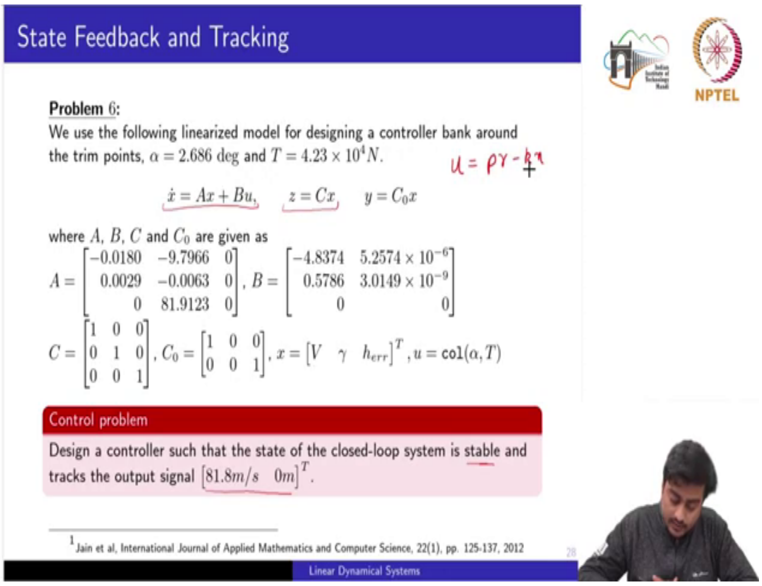

Now, here we are using the linearized model to design the controller where that trim conditions are taken it alpha is equal to 2.686 degrees and the thrust has been taken as 4.23 into 10 to the power 4. So, by using the MTALAB commands work you can compute that trim points of the non-linear model and around those trim points you can compute the a linear (Refer Time: 42:18) equation, basically, the AB matrices for these trim points, ok. So, this is the nominal state space equation.

The second one we are using we are expressing a z is equal to C times x, where C is taken as identity. So, meaning to say z would be equal to x. We have taken y is equal to C naught x because we only have the we want to control only two of the outputs. If we see C naught is then among the x which is taken as the velocity the angle of the flight path and the h error we only want to track the velocity and the h error, ok, by using the control inputs the angular of attack and the thrust.

So, now, the control problem becomes that we want to design a controller such that the state of the closed loop system is a stable, this is the first objective, and tracks the output signal given by this, ok. So, we have two objectives the stability of the closed loop system and another we want to track it. So, one of the method we had discussed during the lecture slide by is by either using the feed forward gain. So, we have expressed the controls signal is u is equal to pr minus k times x, ok. So, in this problem we would use another method of designing the state feedback controller which is a stable and tracks the output signal, ok.

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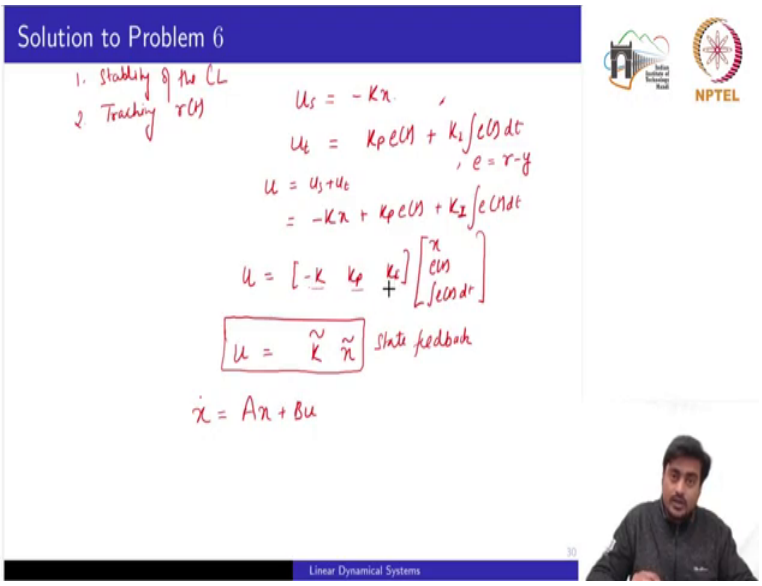

So, if you recall let us write it. So, first is the stable, stability of the closed loop, second is the tracking r of t, ok. So, for the stability we can design a state feedback system by using let us say u s minus of K times x, ok. Now, for tracking here we would design a multi variable pi controller because if you recall from your UG control plus that for tracking a reference signal pi controller gives a better performance. So, the let us say we say u t as a multivariable pi controller in terms of K p error plus K i integral of e t, ok.

So, if I combine both these input as the actual input that is u s plus u t, I would have minus K times x plus K p e of t plus k 2 integral e of d t, ok, where my error signal is r minus y of t, r is the reference trajectory and y is the output of the system. Now, here we will show that we can achieve both the objective by using the method we have introduced during the lecture slide. So, I will parameterize all the system as a as a designing of the state feedback gain. So, this is K i.

So, let us see I can express my u also as minus K, K p, and K i, where my this vector would be x e of t in the integral of e of p dt, which again can be expresses some K tilde and x tilde, ok. So, my overall controller is now given as where my output of the controller in the same as u which is equivalent to K tilde and x tilde and it is nothing, but the state feedback. So, here we would see that by using the principle of the designing a controller using state feedback I can infect design a pi controller which is multi variable in nature, ok.

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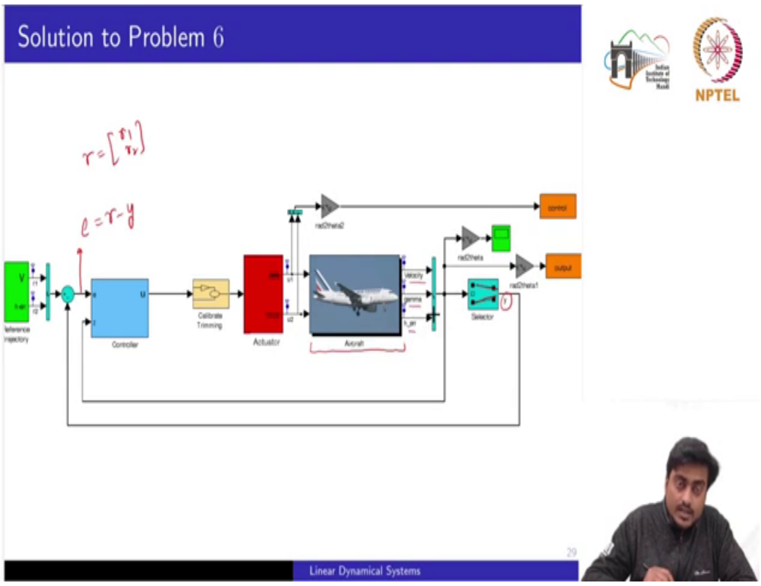

So, if we see the closed loop system this is how it would look like that these are the aircraft dynamics the non-linear system around some trim conditions which is the actuators. So, delta and thrust would be two output of these two input to the system. Here it just the trimming block which we use in the assembling and these are 3 states, velocity, gamma and h error, ok. And here we can place this entire controller as it is where if you see that we require information of two things, one is the state the original state of the plan and also the error signal which is given by r minus 1, ok.

So, here we compute this error signal, this is the error signal r minus y, where this r are the reference trajectories r 1, r 2 and here we are taking the y by taking only the first output and the third output, ok. So, from here we compute this error signal and this input to the controller is z which is nothing, but equal to x, because c is the identity matrix. Now, since this is a state feedback and if we see that the our original plan is given by this one and I and we need the state as x tilde instead of x to finally, design this K tilde matrix, ok. So, we will parameterize our plan into this new state variable which we are defining as x tilde, ok. So, let us see. So, let us write a couple of equations.

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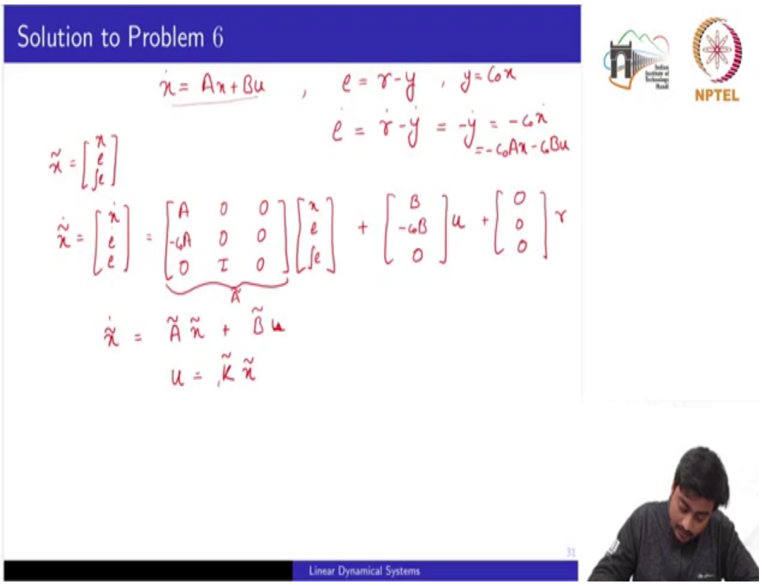

So, we would have the first x dot is equal to Ax plus bu. e we have defined as r minus y. So, if I take the derivative of this error I would get r dot minus y dot, ok. Since, r is a constant signal the derivative of r would become equal to 0 and we would have minus y dot and y is nothing is equal to Cx. So, we have y is equal to Co times x, so I can write as minus Co x dot, ok. So, I would get e dot is equal to minus Co or C naught x dot, ok. So, let us write the state vector as x tilde x e and integral of e, ok.

So, for writing a parameterize state equation I would take the derivative of this x tilde which would be x dot, e dot and e, ok. I need to form this matrix there we have this x tilde as x e and integral of e plus some matrix into u, we do not have the reference signal r more possibly we could write in see whether we get any distribution matrix for the reference signal.

So, if I use this equation first equation to form this state matrix I would get A, I do not have any relationship between e and e 0, so these components would become 0. The B matrix would come and simply we do not have any relationship with r into the first equation. The second equation e dot, I can express as the derivative of x dot which is again I can write as minus C naught Ax plus C naught Bu, ok, by putting x dot from here to here. So, if I write the equation of e dot we would get minus C naught A and we do not have any relationship with e and integral of B with respect to u. Excuse me, this would be a negative sign. So, here we would have minus C naught B and again zero ok.

Now, here if we see e, we already have e here in the state, so I can write as this one, ok. So, we do not have any distribution matrix with respect to one. So, I can write this new state space system as x tilde dot is equal to A tilde, where I have this A tilde x tilde plus mu B tilde and u, ok. Now, using this equation, using this new state space equation I have the exists to x tilde and I can design my state feedback matrix as this one, ok. The dimension of this x would depend on x e. So, if we have x as 3 dimensional, e is 2, again integral of e is 2, so in total we would have 7 dimensional or 7 dimension or 7 variables of this x tilde, ok.

So, this is how we can compute this K tilde and once we compute this K tilde I can expect this K, K p and K i from this K tilde. Now, implementing those controller into the feedback loop we can see that how the response of the system comes.

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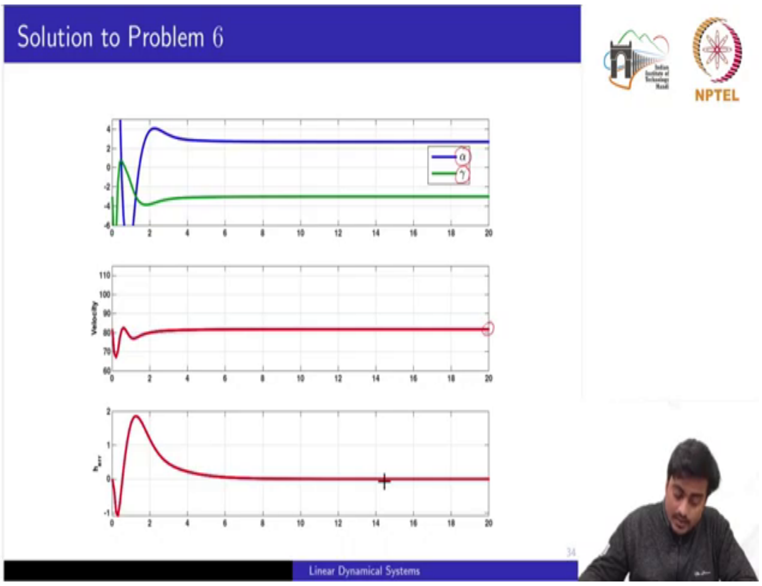

So, this is the overall response of the closed loop system if I synthesize the k matrix with that parameterize system. This is the input this is one of the state. So, gamma should stay it should stay at minus 3. So, it is clearly visible that this gamma is tracking the minus 3. The velocity should be should track the value 81.8 which is it also tracking and h error should be equal to 0. So, all the state variables or all the outputs are tracking the reference trajectories, all the signals are stable. So, using the principle of the state feedback design we have in fact designed the pi controller which tracks and also stabilizes the system at the same time.