Linear Dynamical Systems
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Week - 05
State Feedback Controller Design
Lecture – 30
Regulation and Traking

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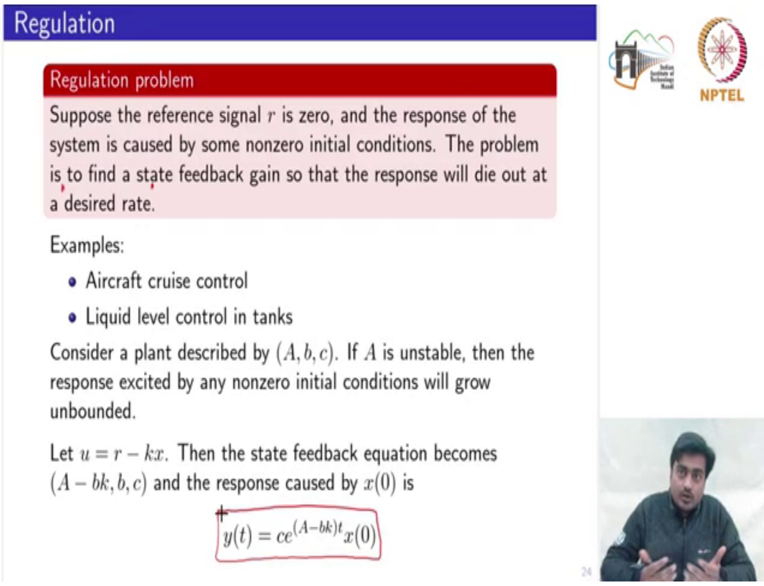

So, far we have discussed about the State Feedback Control Design problem for a generic case, there we have assign the eigen values to some desired location. The further classification of that generic state feedback control design problem are two forces the regulation problem and second is the tracking problem which we will discuss later.

So, the regulation problem deals with the effect that suppose the reference signal r is 0 and the response of the system is caused by some nonzero initial conditions. The problem is to find a state feedback gain, so that the response will die at a desired rate.

Now, this computation of the desired rate is basically consists of defining the objectives. That control objective which you can define in terms of the eigen values with where you want your eigen values of the closed loop system to be placed.

So, there are many examples, so the first one the aircraft cruise control or the liquid level control in tends ah. So, if the so considered the plant described by the pair A b c. If the system A is already a stable, then it is pretty much clear that for the zero reference signal and for nonzero initial conditions, the response of the plan would be a stable it would tend to it is 0. But in that case you won't be able to achieve certain desired rate.

So, we need to design a state feedback controller k, so that we could place the eigen values to some desired location. Now, if the f A is the unstable then for any non zero initial condition the response will grow and bounded. Now, to first of all we need to make the closed loop stable which we have done so far by using u the control signal s r minus k times x, where k is the feedback controller which we need to design such that the response caused by the nonzero initial condition is given by this.

So, we have seen this equation number of times and this is only the response due to the initial conditions. But the response due to the external signal reference are would be 0, because the reference signal is 0. So, this problem could be pretty much dealt in a similar way what we have done so far for designing the state feedback controller design.

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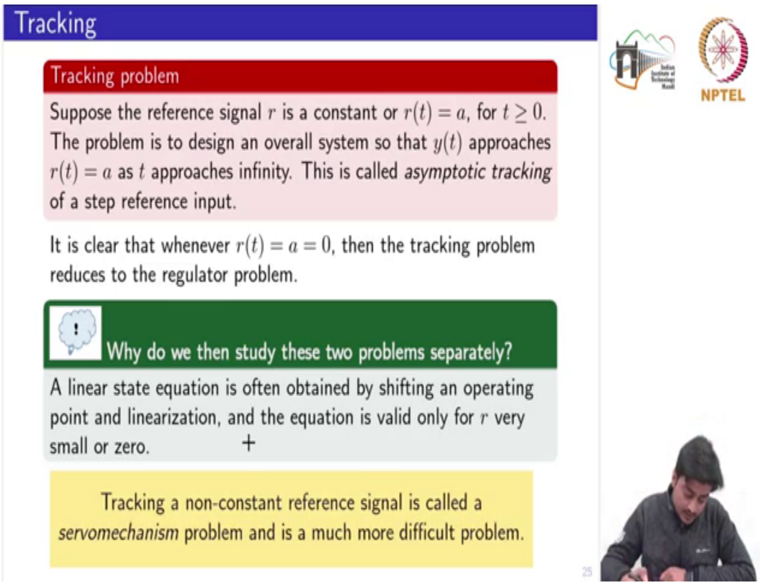

The more involved problem is the tracking problem, where so suppose the reference signal r is a constant or r of t is some or let us say the magnitude is a for t greater than equal to 0. The problem is to design an overall system that is the closed loop system. So, that the response y of t approaches the reference signal at as t approaches infinity. So, this is called the asymptotic tracking of a step preference input.

Now, between these two classification of the state feedback control design problem. Now, if you suppose if we put r of t as for a is equal to 0, then the tracking problem would reduces to the regulator problem meaning to say. That for so the first implication of this that for any given a, if we design the controller as a regulator as a regulator, then would it be possible that the tracking would always happen for any non zero constant reference signal. So, the answer for this would be certainly not, because it would depend on the d c k of the closed loop system of also. So, if the first question raises then why do we then study these two problems separately.

So, the first; so the state forward answer to this one that in most of the linear systems. What we are discussing derived basically by linearization of a non-linear system around some operating point. Now, this operating point may change from time to time depending on the operating conditions. So, whenever the operating condition changes the reference signals would also change. So, we need to ensure that for the same system, when we do the linearization around different operating points resulting into different reference signals that all those signals should be trackable.

Now there is so suppose now if the reference signal is no longer a constant signal or it is a time bearing signal. Let us say assign a residual signal then this that problem is called a servomechanism problem; in that problem is a much more difficult problem. So, we won't be addressing the servomechanism problem, but along the solution of the tracking problem, those problems would also be dealt with. So now, we will see their how you can achieve the tracking for any constant reference signal.

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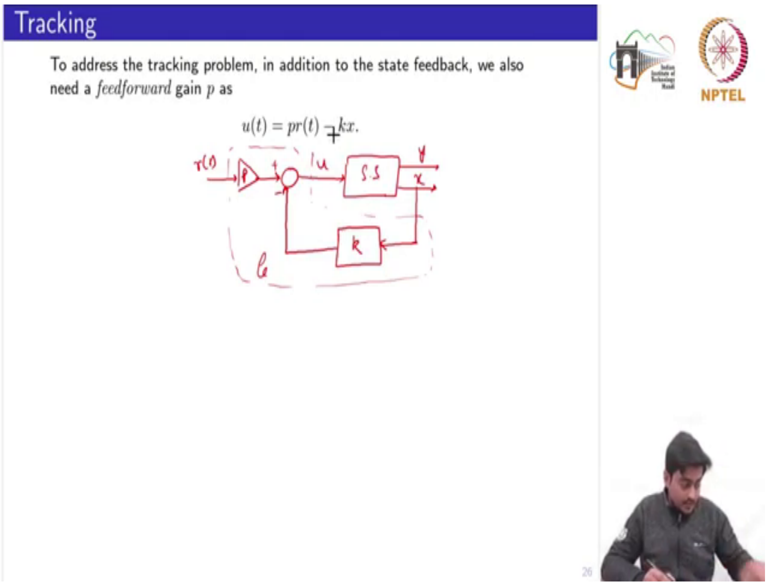

So, to address the tracking problem in addition to the state feedback we also need a feed forward gain p defined by this. So now, if you see a structure of the state feedback system, this is the state feedback system where we have two or could let us say y and so suppose we also have the measurement of there a state and this is u. So, so far we have considered the design of the state feedback controller by considering k taking the feedback from x and coating it back to the state space feedback system. Let us say this is zero or so for example let us put it r of t.

So this could be plus and minus, so a u would become r of t minus k times x. But here we have also introduced another degree of freedom by p which would turn into another gain p by reference signal r ok. So now, we have two degree of freedom for the designing of their controller one is the state feedback gain k and another is this p. So, this is the control structure.

So, this complete part we will call the controller and this is nothing but this equation u t is equal to p r minus k x. Now, here we need to design two gains; one is p and k, such that the y becomes equal to the reference signal or to a constant value a.

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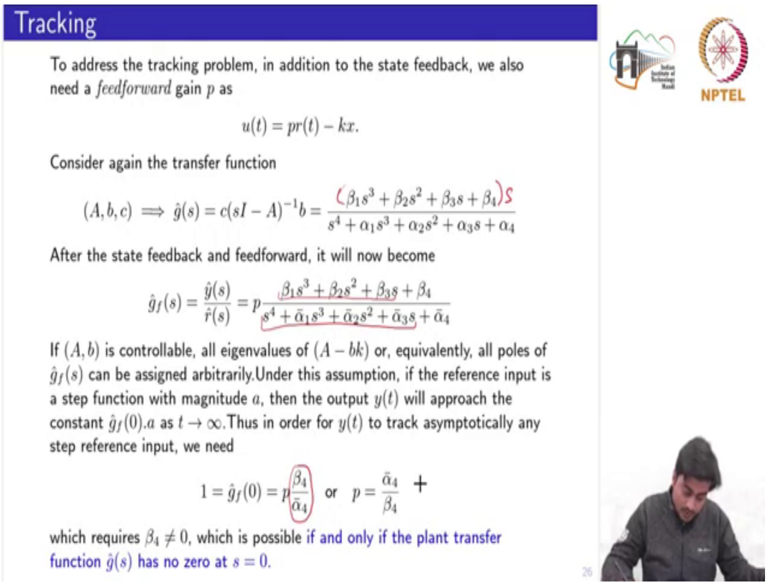

Consider once again the transfer functions what we had seen earlier what the pair A b c denoted by g at s. So, this transfer function is written for n is equal to 4 now. So, we have seen again all these beta i s and alpha i s of the plant.

Now, after applying this is a state feedback and the feed forward the overall transfer function of the closed loop system would become from y hat over r hat. So, that transfer so that denominator would contains all the eigenvalues all the desired eigenvalues denoted by written in terms of a characteristic polynomial and their coefficients with this alpha i bars and then numerator won't change has we had seen earlier.

Now, in addition to this overall transfer function we have additional parameter p associated to the overall transfer function. So, if A b is controllable if the original A b pair is controllable, then all the eigenvalues of the state feedback of the new state feedback state matrix a including the state feedback a or equivalently all poles of this g f hat can be assigned arbitrary hm. This is one of the result we had studied earlier. Now under this assumption if the reference input is a step function with magnitude a then the output y of the t will approach the constant g hat f at 0 dot a as t tends to infinity hm.

So, we basically we are interested in computing what would happen to the y in time domain as t tends to infinity, which is equivalent to computing the dc gain of this transfer function also. And the dc again can be computed by putting s is equal to 0 in this overall transfer function.

So, if I put s is equal to 0 all this part would go away and similarly all this part would go away and we would have y hat of 0 is equal to g hat f 0 times A. Because A is the dc value, dc value of the reference signal. Now, as all this numerator and the denominator part will go to 0, the remaining part is beta four by alpha 4 bar and we want this g hat f of zero is equal to 1. In that case only we would have y hat is equal to r hat in the steady state.

So, on the right inside we would have p times the ratio of beta 4 by alpha 4 bar. So, if we substitute p as the inverse of this factor, which requires that beta 4 should not be equal to 0. That tracking would be possible if and only if the plant transfer function g hat s has no 0 at s is equal to 0 right because if there is an s, if there is a 0 available at the origin then we know that the numerator cannot be changed, the i the roots of the numerator polynomial cannot be changed.

So, this s would stay as it is, if this is s would stay as it is then the dc value would definitely be 0. Now, in that case we cannot track any reference signal, because the response would always be 0. Since the denominator is always a stable (Refer Time: 11:15) polynomial right.

So, if we compute p by this, then the so there are two design parameters the state feedback in k there first we have computed, so that we could place the eigenvalues at the desired location. The second design parameter has been computed such that the tracking to a constant signal is possible which is computed in this way. So, this is one of the important result.