Linear Dynamical Systems
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Week - 05
State Feedback Controller Design
Lecture – 27
State Feedback – I

So, today we will be starting with the week 5 of the course Linear Dynamical System.

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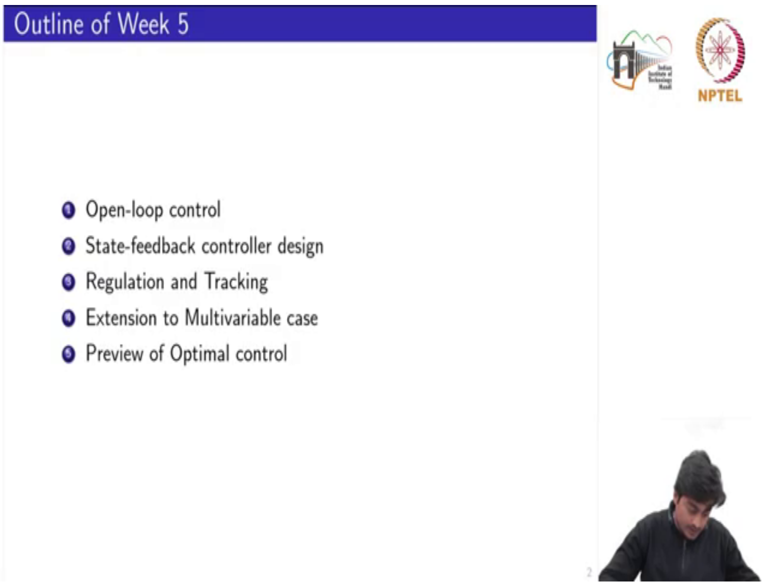

So, this week in this week we will discuss about the different techniques for designing the feedback controller. This will be the outline of the overall week where first we will start with the open loop control. So, here we would recall the some of the concepts which we have discussed during the controllability week. Second we will discuss different techniques for the design of the state feedback controller and we will also motivate the problem of the feedback controller that is the regulation in tracking.

So, there are two different problems or control problems one is the regulation and another is the tracking. So, we will discuss the solution of both the problems. So, the first three parts we mostly we would be discussing into the single variable case, meaning to say we would have single input. So, whatever the results we would obtain for the first three points we would extend it for the multi variable case as well. And the last sub module is the where we discussed the optimal control problem.

So, we would see only the we would say the preview and how the optimal control problem can be formulated in our general framework of the defining a control problem. So, starting with the control problem we want to. So, basically we need three ingredients to formulate the control problem.

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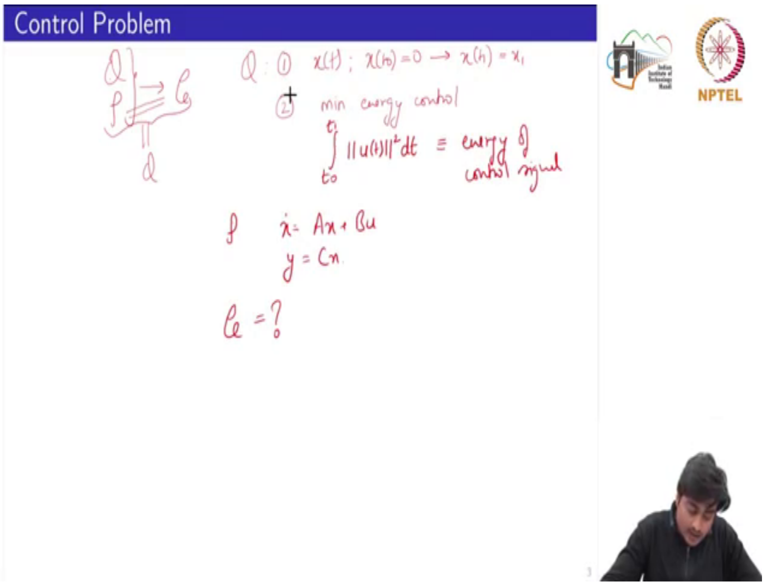

First is the control objective another is the model of the plant and by using these two information we want to discuss we want to design a controller. So, that once we connect the controller with the plant, then this overall closed loop will give us the desired objective what we have specified in the beginning.

So, if you recall during the controllability week, when we were discussing about the characterization of the controllable and the reachable subspaces. We not only we have given the conditions for the controllability and the reachability, but at the same time we have computed the controller as well. So for example, if we take if we try to formulate a control problem using the theorem which we have introduced for characterizing the controllable and the reachable subspaces. Say for example, we have two control objectives the first control objective is that we want to steer the trajectory.

Let us say the trajectory x of t from x of t naught which is the origin to x of t 1 let us say x 1. So, this concept we have studied about the reachability analysis, that whether the system is reachable or whatever the reachable subspaces are. The second objective is we want to steer the state trajectory from the origin to some non zero value x 1 by keeping the minimum energy control. Meaning to say that the then we compute the energy of the control signal it should be up the minimum value and if you recall in some of the tutorial problems we have also computed the control energy.

So, the control energy if u is a vector if u is a vector, then if I want to compute the energy consumed in the control signal from time t naught to t 1. We have specified by taking the norm of the signal u t square d t. So, this is the energy in the control signal energy of control signal ok. Now if u is a; u is a scalar then instead of taking the norms we take we consider only the magnitude.

And the plant given to us is let us say a plant is x dot is equal to Ax plus Bu and y is equal to Cx ok. Now we need to design the controller, such that when we connect that controller with the plant both the objectives are achieved ok. So, we will try to find the solution to this problem along the same line along the same line of the controllable controllability and the reachability subspaces.

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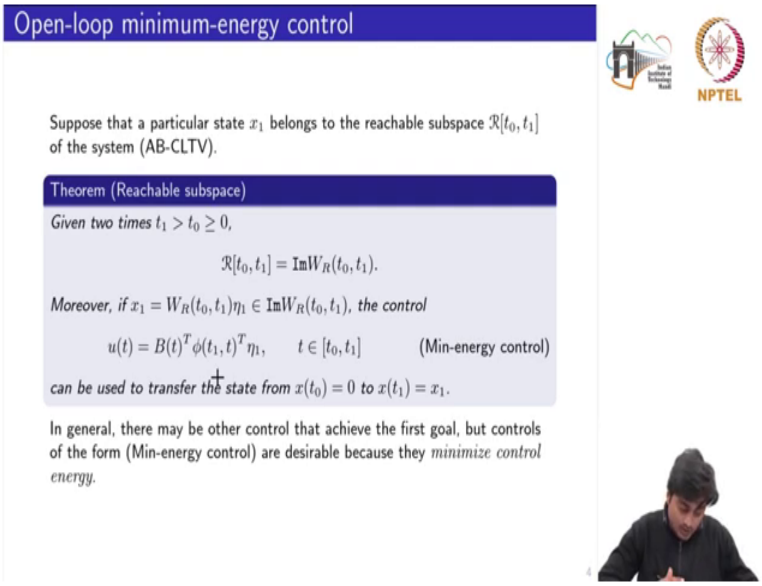

So, Let us say suppose that a particular state x 1 belongs to the reachable subspace which is defined by this of the system AB continuous time CLTV system linear time varying system. So, we want to steer the trajectory from the origin to x 1 and suppose this x 1 belongs to this reachable subspace.

So, if you recall that this was one of the main results where we have characterized the subspaces in the reachable subspace and terms of the reachable grunion and we have also specified that. If x 1 belongs to this reachable subspace meaning to say that there exists an eta 1. Such that this equation is satisfied then this control then this control can be used to transfer the state from the origin to x 1. Now here you would notice one point that we have already pre specified that whatever the control signal or the control law, Let us say we have determined it is already a minimum energy control.

So, we want to compute the control which would have a minimum energy, such that both the objectives are achieved. Now the first objective was automatically clear, expect it came it comes from the definition of the reachable subspace that if I apply this control signals. And in fact we had seen the detailed proof that applying this control signal could steer the trajectory from the origin to x 1 right.

So, at the same time we also know there to take the trajectory from x naught to x 1 there could exist many control signals and this is one of the control inputs which could take from x naught to x 1. Now we need so this would achieve at least the first objective that this control would take from 0 to x 1, the rest is to prove or to show that this control signal is a minimum energy control.

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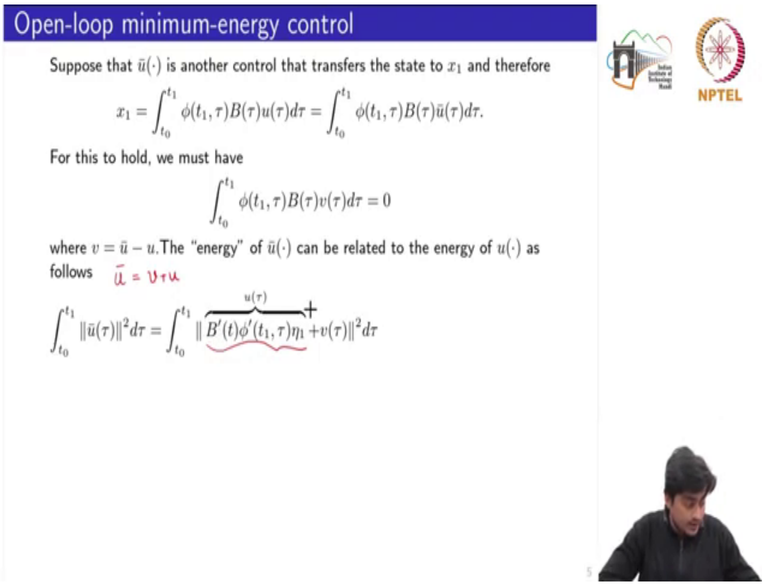

So, to observe this is suppose that there is another control signal denoted by u bar that transfers the state to x 1. So, meaning to say now there are two different control signals one u which was specified in the result, another we are saying u bar which again x 2 x 1 ok. So, this is the simplification of the solution of the state equation where we have specified x naught to be 0 ok.

So, now there are two control signals u and u bar. So, for this to hold we must have phi t 1 comma tau B tau v tau is equal to 0, meaning to say. So, if I take this part on to the left hand side I put have u minus u bar. So, that signal we have denoted by v. So, v is either u bar minus u or u minus u bar.

Now let us compute the energy of the signal u bar the energy of the signal u bar can be compute by the formula which we had specified earlier. Now writing the u bar here. So, u bar would be from this equation v v plus u and u is the signal what was specified in the theorem. So, this part was there which is denoted by u plus v and the norm square ok.

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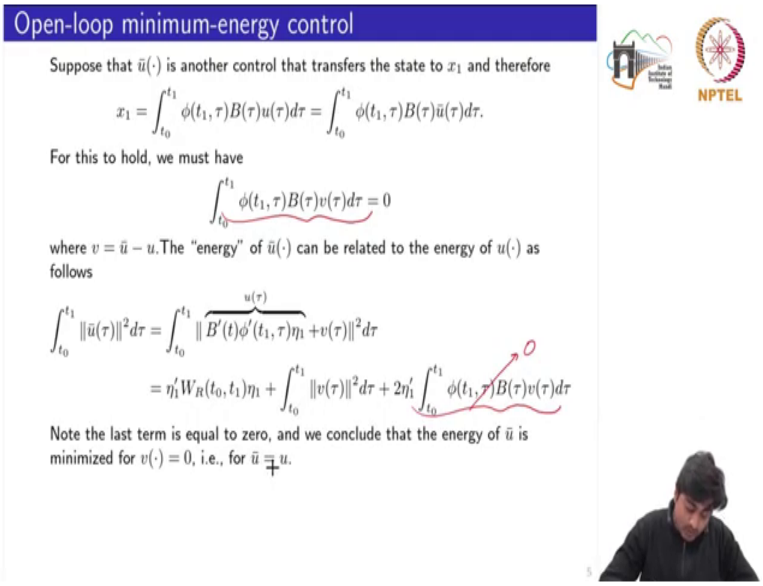

Now, if we open this norm we would have the square of the first term, the norm square of the first term norm u square plus norm of v norm square of v plus twice of u into v by keeping the vectors notations into mind. So, if we specify this one norm u square fed me represented by u transpose into u and u transpose into u would be eta one transpose phi b b transpose phi transpose eta 1 ok.

So, we would have eta one transpose and eta one at the end and the rest of the remaining part would be the reachable gramian matrix, which we are specified by WR. The norm v square integral would stay as it is similarly plus twice of eta 1 transpose phi b and p ok. Now note that here that this term would be 0 from here and the remaining part is the energy in the control signal u bar would be minimum depend on only if this v itself is 0.

Because the norm the using the property of the norm which it could be either 0 or positive. So, it needs to be v needs to be 0 to have the minimum energy of the u bar signal.

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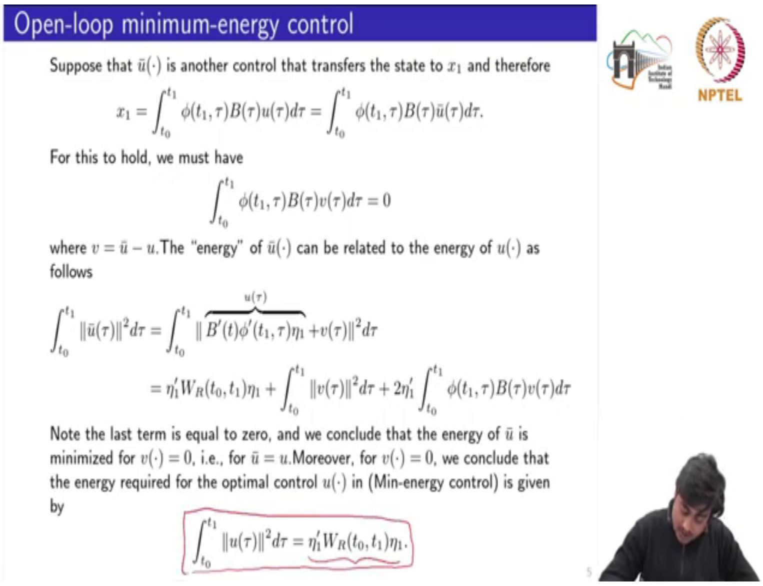

So, it would happen where u bar becomes equal to u. Now moreover for v equal 0 we conclude that the energy required for the optimal control u in the control signal, what we had specified in the theorem is given by this one. So, this is the minimum energy and in fact, we have computed that how much energy would be utilized in the control signal.

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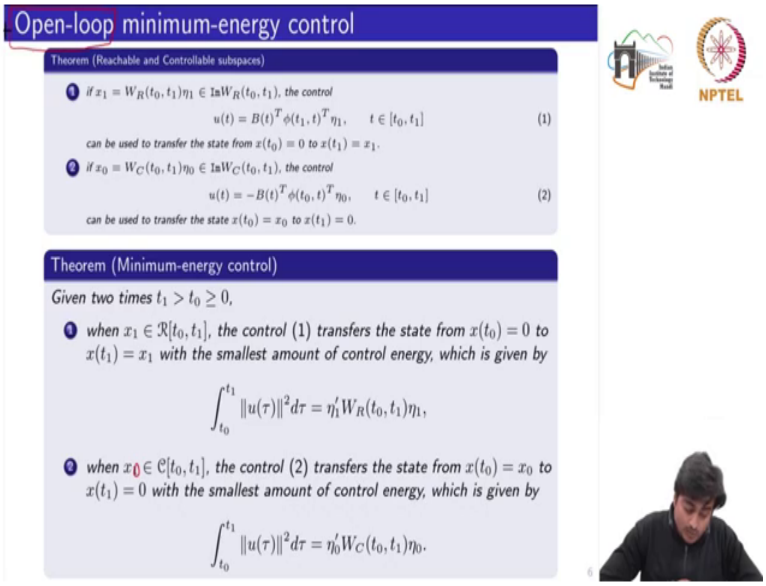

So, if we summarize this overall result we would have the so these two points are basically report from the controllability v, where we have done the characterization for the reachable in the controllable subspaces and we have also computed the control signal. This control signal is for taking origin or steering the trajectory from the origin to x 1 and this control signal is from x naught to 0 in t 1 amount of time.

Now, based on the analysis what we had done in the previous slide. So, this theorem states the minimum energy control that when x 1 belongs to the reachable subspace, the control one transfers the state from the origin to x 1 with the smallest amount of control energy which is given by this. And we had seen the detailed derivation of this energy.

Now when x 1 so in fact it should be when x naught. So, when x naught belongs to the controllable subspace the control do transverse the state from x naught to the origin with the smallest amount of control energy which is given by this. So, this control says this minimum energy you can compute again by using the same equations, what we had done for the reachable subspaces.

Now, the important point to note here is and in fact we have also specified during the controllability v that we this is nothing but an open loop control signal, because we by open loop we mean to say. So, Let us see so whatever the control signal we have obtained we have a plant.

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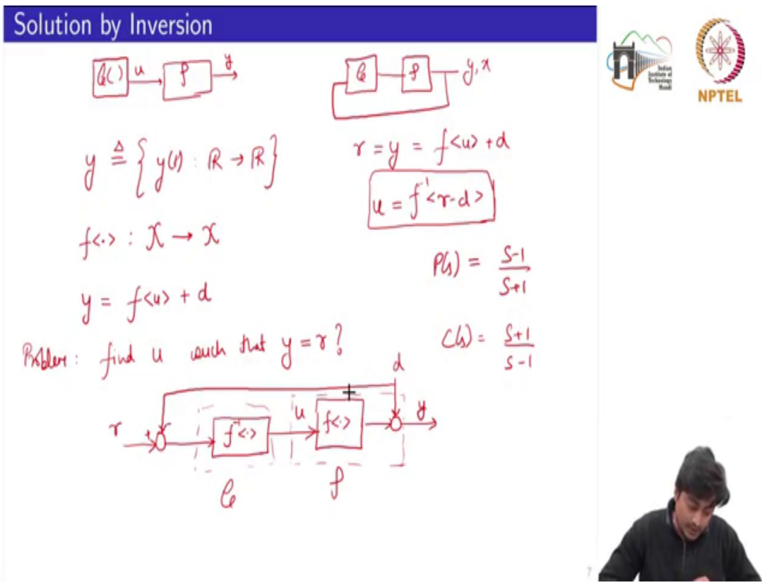

Let us say we have a plant and which is interacting with the signals u and y ok. Now the controller what we have specified is taking some constant parameters or let say it is a controller or some of some constant parameters. So, this is an open loop control, that the controller which we have design is not taking any information from the plant when it takes information from the plant.

We use the architecture of the closed loop system as that it is taking some feedback from the plant. What feedback is taking we will discuss it later. So, whenever this controller is taking any feedback the input signal to this controller subsystem would either be y or x, because the output of the plant we can take either y or x. Where x is an internal variable and y is the external output signal. So, if controller is using any information from the output of the plant we say it is a feedback system and if the controller is not using an information from the output of the plant it would be an open loop system.

So, if you pay attention here to both the control equations either for the reachable subspace or the controllable subspace, u is not a function of the output of the plant of either y and x. That is why we call it up open loop minimum energy control ok. Now there are other ways of computing the open loop solution of the control problem. So, Let us say so we denote a element of a function space let us say y which is defined by some element of y t, such that which maps from some real valued space to real valued space ok.

Now, we defined function let us say f which is an operator map from one function space to another function space let us say from x to x. Now assume that the output this y can be expressed as this operator as a function of the control signal u plus some disturbance variable which is we are defining by d.

So, the control problem we have the problem find you such that the output of the plant or output of that model is equal to some reference signal which we are denoting it by r. So, we want to compute u such that y becomes equal to r, where the model of the plant is given by this. So, this is the objective we want to attain y is equal to r and the model of the plant is defined by a generate model of the plant is defined by this equation.

Now, the straightaway approach to by taking this equation, if we would have y is equal to f u plus d and though we need. So, we need the y is equal to r I would write y as equal to r directly and compute the control signal from here, which would be r minus d the function inverse ok. So, this function is and so this function is an inverse of the function f and is a function of the signal r minus d. In a similar way or it was a function of u right. Now if we try to implement this control signal we would have the reference signal, the controller function of the input signals u. So, this is overall y and right.

So, this one so this dotted box is our plant, if you see y becomes equal to function of u plus d and this is our controller. So, a naive a so this is a very naive approach to compute an open loop control. So, there are some important points to be noticed that this control strategy first of all is not a good control strategy from various viewpoints. So, first of all if you see that for computing the controller subsystem, we need to take the inverse of the function f. Now so, suppose if the Let us say the plant is represented by some transfer function plant is represented by some transfer function say suppose S minus 1 upon S plus 1 ok.

Now, if I want to design a controller, that controller would be the inverse of this transfer function would be S plus 1 over S minus 1. So, this is clear that this controller would be an unstable controller and with this the with this controller you would not be able to achieve y is equal to r right. So, this is the major problem here that the controller subsystem requires the inverse of the plant itself. Second that for computing the output of the controller it requires the information of the disturbance variable, because the disturbance variable is being feedback also to compute the control signal.

So, in usual practice we do not have the availability of this disturbance variable. So, if the disturbance is signal is not available to us, if it is quite difficult to compute the control signal. So, because of these two major reasons you cannot we generally do not prefer to use the open loop control strategy. Although it has been in practice in many industrial processes as well, so this is what we say the solution by inversion. Now from here we were going to motivate that why do we need the feedback.

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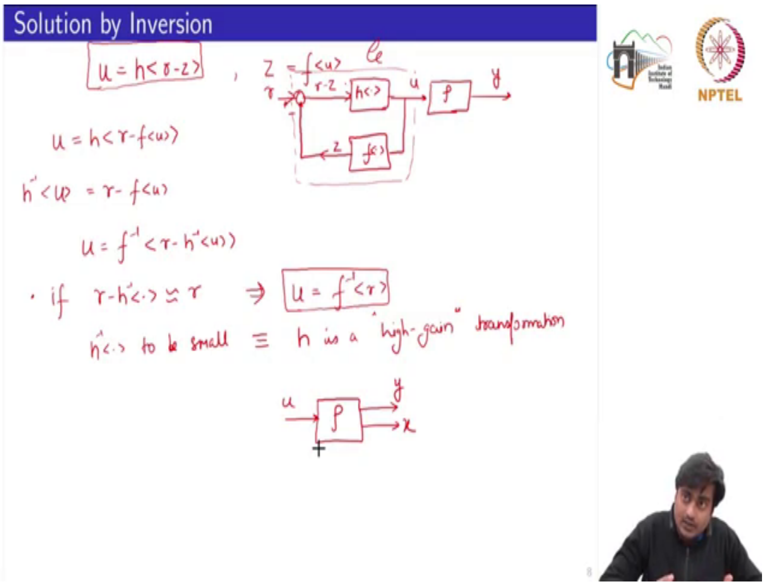

Say for example, now I define the control signal let us say u is equal to some function h which is again an operator defined as a function of r minus z and z is we defining as the function itself ok. So, the idea what we are going to pursue here, let us say this is our plant the function of y and this is u the controller u would be a function of the signal or could be a function h of the input signal.

So, this is our reference signal and we define a representation of the plant which we is represented by this function f and this part is z and it is being fed to here, so here we have plus minus h. So, if you see that you now becomes equal to h which is a function of r minus z. So, here we would have r minus z and z is representation of this actual plant given by the function h.

So, now substitute z from here to here we would have u is equal to h r minus f of u. Now from here we were going to compute the control signal which is given by u, taking this h onto the left hand side we would have h inverse is equal to r minus f of u. Now from here we would compute u is again f inverse r minus h inverse of u ok. So, u was specified here now all these inverses we have done to compute this internal u.

So, first we would have the h inverse or minus h inverse of u and then finally the f inverse ok. Now suppose if this r minus h inverse of some function becomes equal to r. This is a very decent assumption why because this function h is defined by the user, now user can defined the function h such that r minus h inverse of some function becomes equal into r. Then in that case it implies that you would have u is equal to f inverse of r, here we assume d is equal to 0 for the moment because the major problem in the previous solution was the inverse of the function itself.

Now, what do we require what first of all you should understand what is the physical meaning of the inverse of this function h, then of this assumption as well. So, we require this h inverse to be small that is or equivalently. We can say that h is a or some high gain transformation h is some high gain transformation. That if I take the inverse of this high gain it becomes a very small value in that case r minus h inverse would become almost equal to 1.

So, important point to note here that no where we have taken the inverse of the function for the controller, because this now becomes the controller. This is our controller and this is the plant. Now in the previous solution we had seen that to compute the control signal u we need to take explicitly the inverse of this function f.

Now because of this feedback mechanism though we are taking the feedback from the control signal to this summer, because of this feedback mechanism we have not taken any explicit inverse of the function f. Although at the same time we have implemented this one. So, this technique is also called the approximation of the inverse by using the feedback mechanism.

Now this principle, so this is how this is the importance of the feedback. Now this approximation of the inverses the same concept or the same idea behind this construction of the controller we would going to use for the feedback of the states that. Let us say now going into the signals the control signals of or the signals which are u. And so far we have discussed that the output would be the external output the system. Second if we have access to the states of the system we can take it also as an output to feedback.

So in fact, whatever the control schemes you would see we need to we need to use the inversion. Now this inversion we are going to implement by using the feedback mechanism and explicitly we would use the feedback from the states. So, these concept we would going to so we were going to discuss various tools that how we can design the seed feedback mechanism.