Linear Dynamical Systems
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Week - 2
Stability
Lecture - 12
BIBO vs Lyapunov Stability

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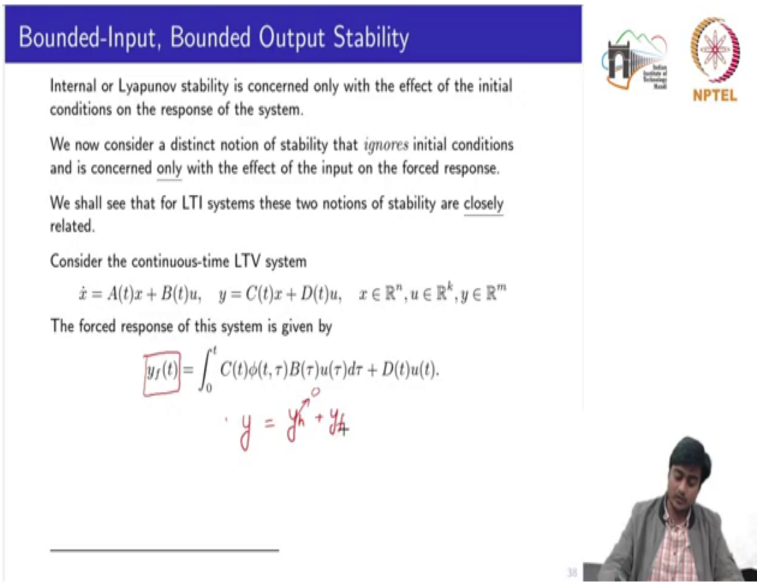

So, now we will start with another definition of the stability that is Bounded Input, Bounded Output Stability. So, so far we have seen the definitions and some of the key results in the sense of Lyapunov or the internal stability, which is mainly concerned with the effect of initial conditions. So, we had not seen with respect to the control input u. So, this particular concept of stability deals with that when the input is also included in the definition.

So, we now consider a distinct notion of stability that ignores the initial conditions and is concerned only with the effect of the input on the forced response. So the, so far what we had seen about the internal and the Lyapunov stability, we have considered the homogeneous systems. So, here we will consider that the initial conditions as zero and the output or the state is only affected by the input given to the system.

So we shall see that for LTI systems that these two notions of stability are closely related, that is the bounded input bounded output stability and also the Lyapunov stability. So, we will start with the continuous time linear time varying systems, where all these matrices A B C D are the time varying matrices. So, first we will start with the continuous time. So, we had discussed on the first week that, the forced response of the system is given by this equation. So, here we have specified y with a subscript f which denotes the forced response.

And we also know that the total response of the system y is one component comes from the homogeneous plus component comes from the forced response. So, this homogeneous component is because of the initial condition x naught and this forced component is because of the external input u. So, now, we will concerned or we will see the definition of the stability when we have the initial condition equal to zero; meaning to say we would not see this response, we will only concerned with the forced response.

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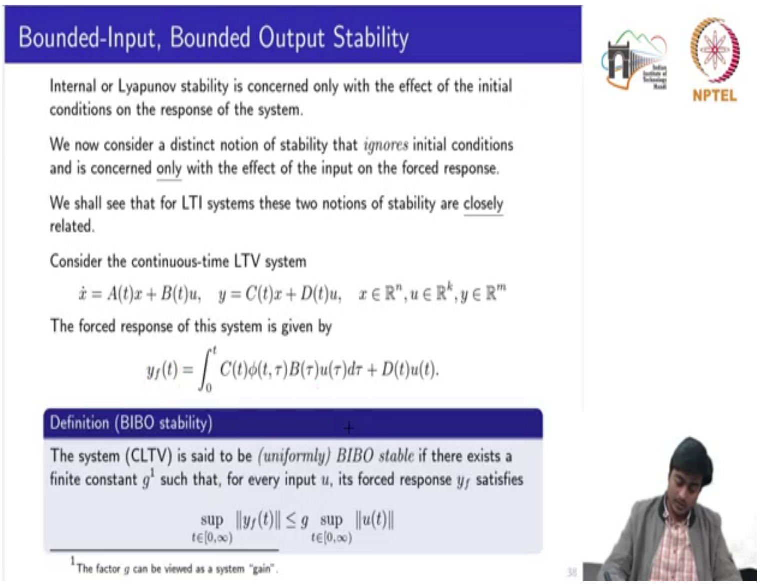

So, the definition of the BIBO stability is given here, that the system continuous time linear time varying system is set to be uniformly bounded input bounded output stable; if there exists a finite constant g. So, here you can consider the factor g as a system gain; such that for every input u it is forced response y of f satisfies this inequality. That is to say that the supremum of the norm of the signal y of f for over t for or let us say for all t greater than or equal to 0, is less than equal to the gain g multiplied by the supremum of the norm of the signal u of t over all t greater than equal to 0.

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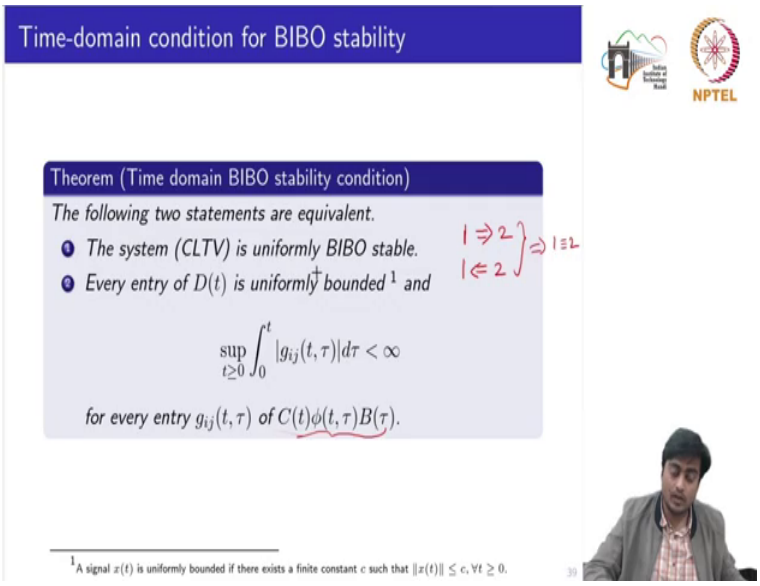

So, this is the key result for the bounded input bounded output stability for the LTV systems. So, these two following statements are equivalent, that the system is uniformly BIBO stable. Second that every entry of the matrix D of t is uniformly bounded and the supremum of the integral over t greater than equal to 0 of this integrand is less than infinity: for every entry g i j t comma tau of this matrix, ok.

So, again we will see the proof in two parts. So, this is the, when we say that these two statements are equivalent; we actually want to show that 1 implies 2 and 1 is implied by 2. So, once we have proved both ways implication meaning to say, that 1 is actually equivalent to 2 and this is this key result speaks about.

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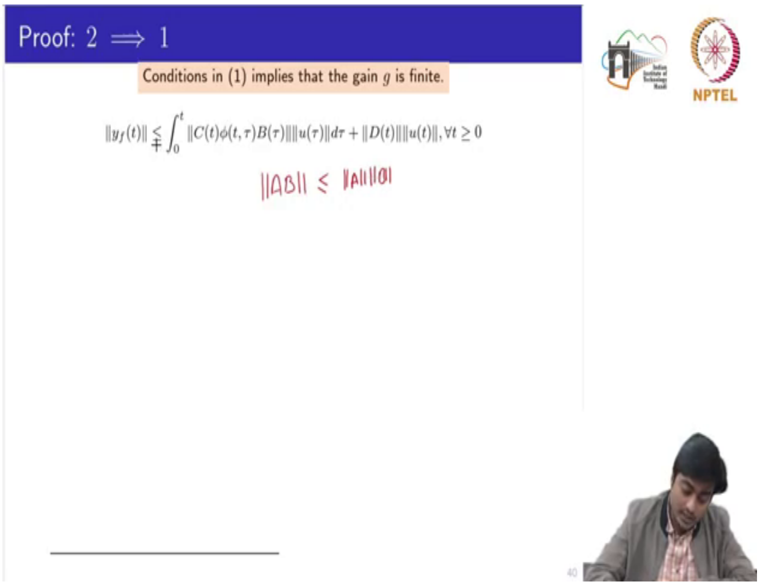

So first we will see 2 implies 1 meaning to say; that if every entry of D of t is uniformly bounded and this condition is satisfied, then it implies that the system is uniformly bounded input bounded output stable. Now see that the conditions in 1 implies that the gain g is finite. If we go back to the definition of the uniformly BIBO stable, so this we say that the system is uniformly BIBO stable; whenever there exists some finite gain constant or finite constant g such that this equation is satisfied, ok. So, either saying that the system is BIBO stable or there exist a g is equivalent. So, we actually going to prove that there exists some gain g which is finite ok.

So, let us start with the response, the forced response of the system which is given by this equation. If we take it is norm both side, so we can say that the norm of the signal y f is less than equal to the individual norm of these two signals. And this comes from the property that the multiplication of two matrices or the norm of the multiplication of two matrices is less than equal to the multiplication of their individual norms, ok. So this inequality comes from there and similarly we apply this property on to the second part ok, for all t greater than equal to 0.

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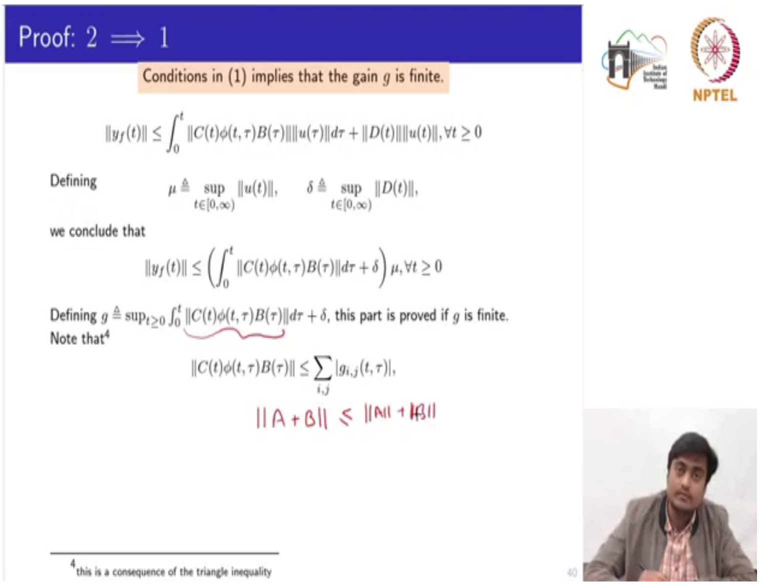

Now, let us define two signal or two scalars mu and delta; this mu is given by when we take the supremum over t belonging to 0 to infinity for the norm of the signal u of t and delta as the supremum of the norm of that D of t for all t greater than equal to 0. So, by defining these two scalars mu and delta, we can rewrite this equation like this. Let us say, so the norm of the signal y f is less than equal to the integral, this part would come as it is.

Now if I take the supremum of this one is mu. So, this signal, the norm of the signal y of f would also be less the mu and delta. So, this part comes here delta and I have taken the mu common out of this complete equation, right. So, now, we define the gain g by this part, which is the supremum of over for all t greater than or equal to 0 and this part. So, what we need to prove that, if g is finite or not; to basically to prove this implication 2 implies 1, we only need to show that this g which we have defined by this is finite.

So, note that, the norm of this complete entity is less than equal to the summation of the absolute value of g i j t comma tau; and this is basically comes from the triangular inequality. This triangular inequality says that, the norm of the summation of two matrices is basically less than or equal to the summation of their individual norms, right. So, this is effect.

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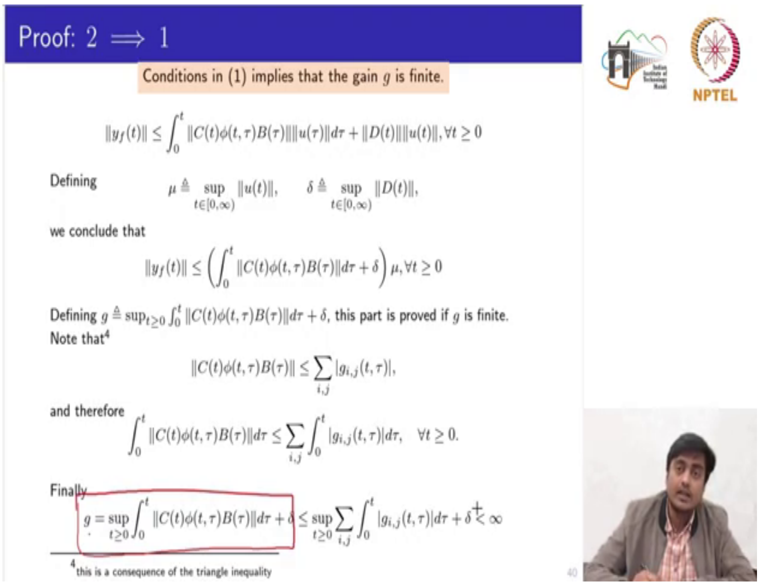

Now using this effect, we can rewrite this inequality by this by taking the integral both sides; because it would not change, because both the quantities are positive. So, I have added the integral both sides. So, from 0 to t and 0 to t, again now taking the supremum both sides this part you would see. Once I write the supremum over here, this part is basically what we have defined initially by the summation of the delta, ok. So, from here to here I have done two things; I have added the delta both side first of all and then took the supremum.

So, if I see the first part, basically the left hand side it is nothing, but what we had defined initially the gain vector g. Now this g is less than this part, which we know is already a finite. So, this proves the this implication 2 implies 1; because we only need to prove that there exist a gain g which is finite. And by using the properties of the norms, we have actually concluded with the effect that this g is basically finite, ok.

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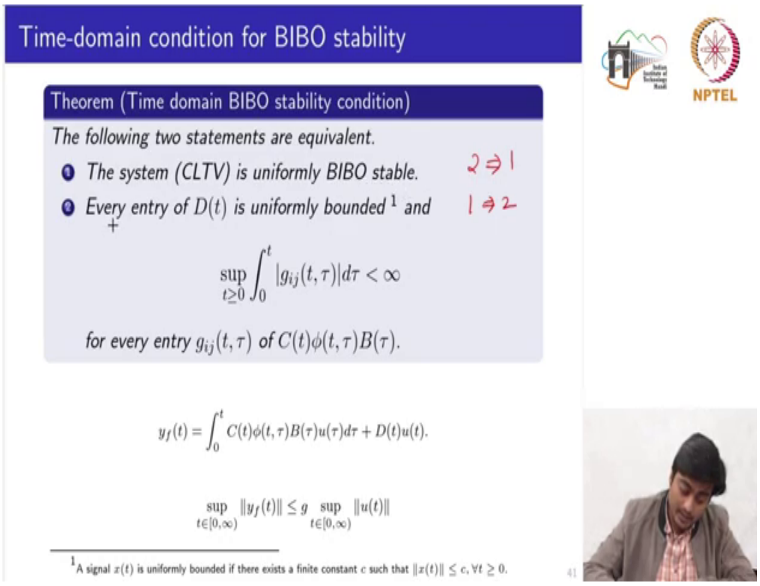

So, this proves the first implication that is 2 implies 1. Now to prove 1 implies 2, let us recall that the forced response is given by this one and this was the definition of the BIBO stability, right. So, what we need to prove that the system is, if the system is uniformly BIBO stable;

then it implies that every entry of D of t is uniformly bounded and this condition is also satisfies for every entry g i j t comma tau, ok.

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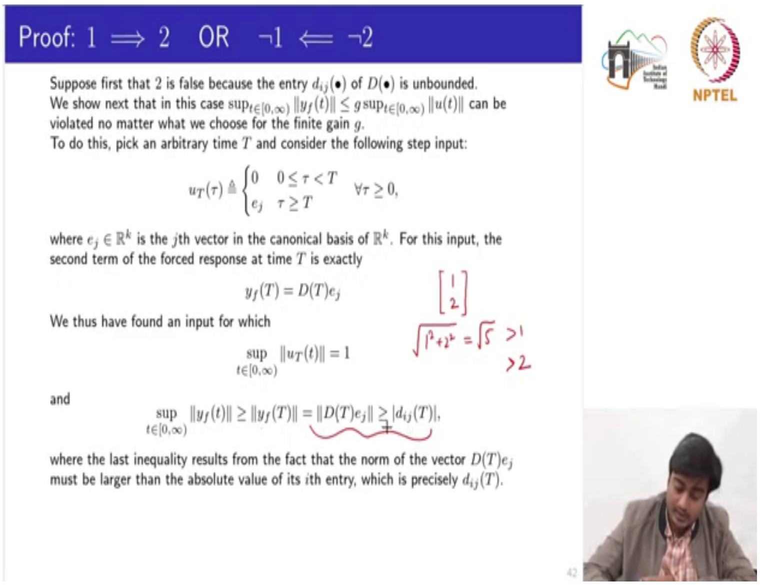

Now, we will do the proof of this implication by contradiction; meaning to say, what we want to prove that 1 implies 2. We can also prove that, if the statement 2 is incorrect or if the statement 2 is false, then it implies that the statement 1 is also false, ok. So, both these implication either I say 1 implies 2 or negation of 2 implies negation of 1. So, both of them are equivalent. So, we are going to prove this by contradiction. So, let us see. So, here we have two parts to prove; first that every entry of D of t is uniformly bounded, secondly that this condition is satisfied ok.

So, first we will see the proof of this that, the every entry of D of t or some entry of D of t is not uniformly bounded; then it implies the system is not uniformly BIBO stable. The second part of the proof says that, if this condition is not satisfies; then the system is not uniformly BIBO stable, ok. So, let us see suppose first that 2 is false, because the entry d i j of this matrix D is unbounded, ok. So we show next that in this case taking the supremum of the signal y f for all time t is less than equal to g multiplied by the supremum over all time t of this norm of the signal u of t is also violated, no matter what we choose for the finite gain g.

Meaning to say that, there does not exist any gain g. So, to do this, we pick an arbitrary time T and consider the following step input; where this signal u in subscript T of tau defines that whenever tau is lying between 0 and T the signal is 0; and when the tau is greater than or equal to T it is e j and e j is the j th vector in the canonical basis of the k dimensional vector, ok. So for this input, the second term of the forced response at time T is exactly given by y f T is equal to D of T in to e of j, all right. Because we know before time T, the input is 0. Now at time T only this part would remain, this part would go to 0, at time T only.

So, this is what it is said; that the forced response at time T is exactly equal given by this equation. So, what we have found an input which is bounded that, the supremum of this input what we are defined is basically is equal to 1. And if I take the supremum for all time T of the signal y f t is always greater than equal to y f T capital T. Because we know that y f capital T would be unbounded, right which is equal to D of T and we know some of the entry d i j is unbounded. Meaning to say that, this would be greater than or equal to the absolute value of the d i j. So, where the last inequality basically results from the fact, that the norm of the vector this must be larger than the absolute value of it is i th entry, which is precisely d i j.

This you can visualize by this let us say, if we have a vector which is given by 1 and 2. Now if I compute the norm of this vector, it would be given by 1 square plus 2 square; let us take the two norm ok, the two norm of this vector is given by under root 5. Now the norm of this vector is basically greater than the absolute value of all these entries, so root 5 is greater than 1 and root 5 is greater than 2. Basically this part comes from this that effect, that the norm of the vector would always be or it must be greater than the absolute value of it is i th entry, ok.

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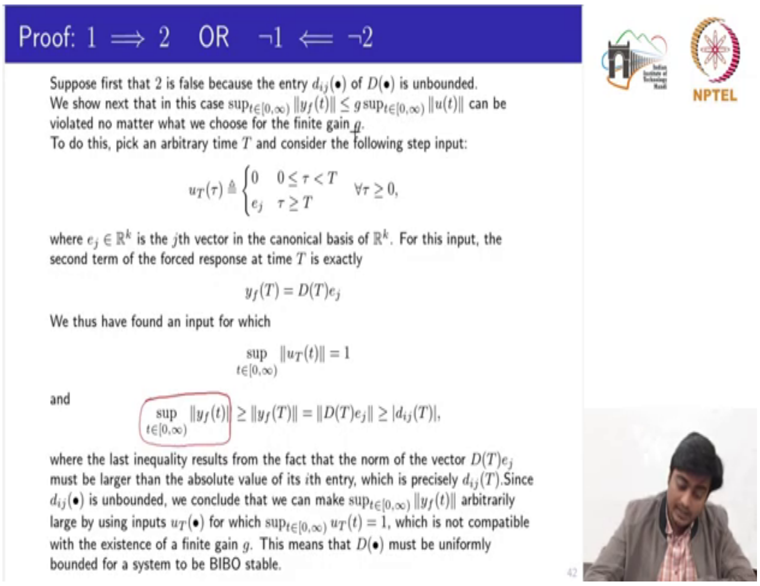

So, since d i j is unbounded, we conclude that we can make this part; arbitrarily large by using the input for which we know that this input is bounded. Meaning to say that, there does not exist a finite gain g; this means that D matrix must be uniformly bounded for a system to be BIBO stable. Because if any entry of the matrix, time varying matrix d is unbounded; then there does not exist any finite gain g, ok.

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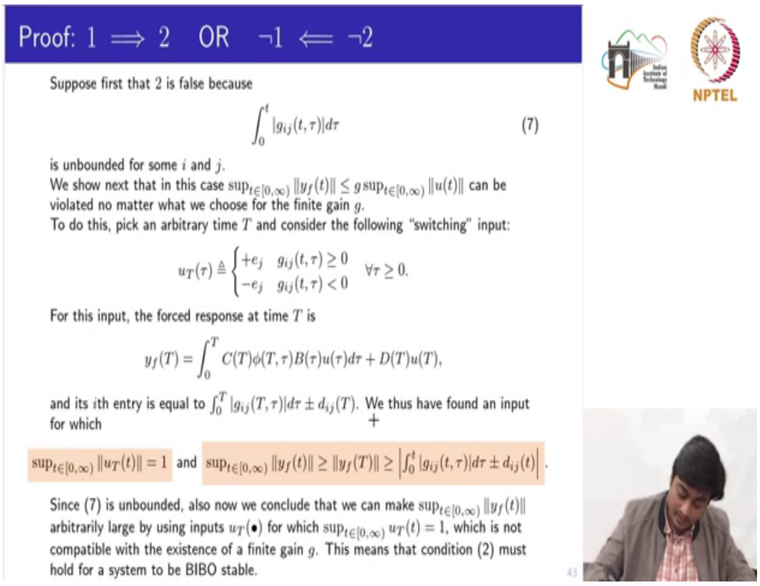

Now, the second part of the proof says, suppose the second condition in the second statement is false, right. And why it is false; because this integral is unbounded for some i and j, ok. Again similar to the previous proof, we want to show that there does not exist any g for which this equation or this inequality is satisfied. So, again we pick an arbitrary time T or we design a signal u, the control signal u which is bounded this you can see that; we defined the signal in the sense that whenever this i th, i j th entry of the of the g, t comma tau is greater than equal to 0, the input is plus e j.

And if it is less than 0, the input is minus e j and e j is the canonical basis of the n dimensional vector k dimensional vector sorry, ok. So, again if we compute the response at time T, it is given by this one; and it is i th entry would be given by this part. So, because we know that the i th entry of this part is given by this and the i th entry of D of T is basically d i j ok, only at time T.

So, now what we have found, that although the input is bounded because of the characteristic we have defined earlier. And if I take the supremum of the norm of the signal y f is always greater than equal to the norm of the signal at time T and at time T; which is basically given by this part is greater than or equal to the absolute value of it is i th entry which is given by this part, right again using the similar triangular inequalities. So this means that, this condition must be true to ensure that there exist a gain g for with the system is BIBO stable.

So, this completes the overall proof that, these two statements are equivalent and the system is BIBO stable. And the second states that the every entry of D of t is uniformly bounded in this condition or the i j th element of this matrix is satisfied.

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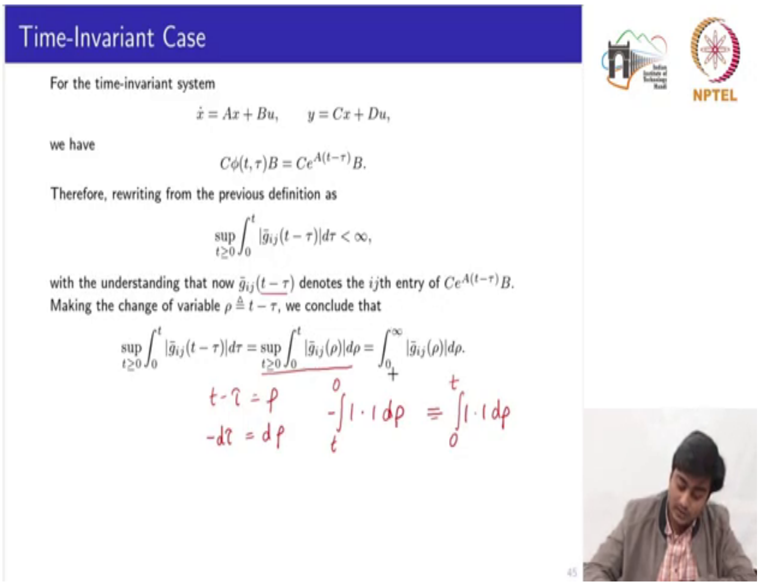

So, let us see for the time invariant case of, for the time invariant system we have A B C D matrices which do not depend on time. And we also know the state transition matrix for the LTI which is given by this exponential matrix; where if you are using t comma tau, then it turns into e to the power A in to t minus tau, the C and B matrices would remain the same.

So, therefore, rewriting from the previous definition is we can rewrite it as, that the supremum of the integral of this integrand is should be bounded; basically the it is the impulse response, that the impulse response should be bounded with the understanding that now this part denotes the i j th entry of this matrix. So, let us do a change of variable, we replaced this t minus tau part by rho; we conclude that if I put t minus tau is equal to rho, then we can replace minus, if I take the derivative of this equation it would give me d rho.

So, here d tau would be replace by d rho. So, if I make changes over here, just ignore the supremum for a time being. So, we would have the initial limit at tau is equal to 0, we would have the initial limit t; and when the final value of tau is t rho would be equal to 0 ok. And we have the which is the function of rho and d rho with a negative sign,. Now we can remove this negative sign by changing the order of this integration which is equivalent to saying that, 0 to t the function inside the absolute value and d and this is overwritten over here, ok.

Now, extent if we are taking the supremum for all time t, we can extend this time t upto infinity which eventually give us this condition. So, this must be bounded.

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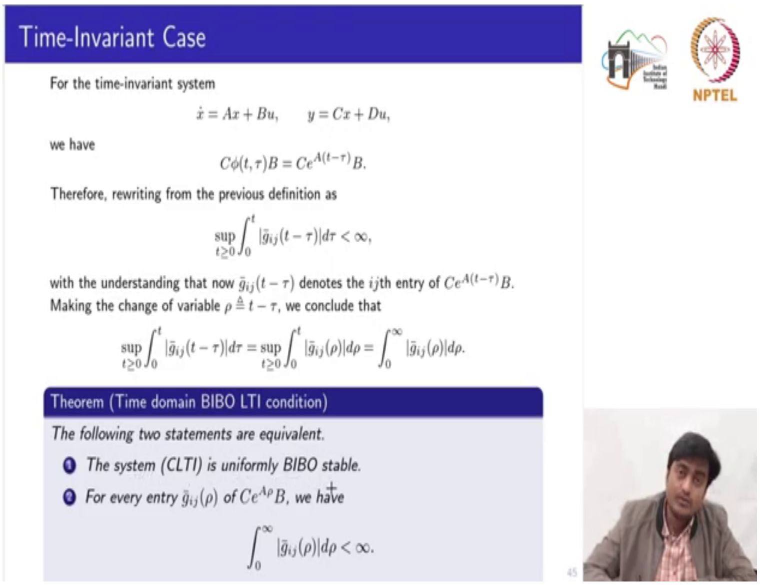

So, this gives us the next result, that the system is uniformly BIBO stable; and for every entry g bar i j of this matrix, we have the absolute value of the impulse response integrated over 0 to infinity is also finite. This also says that, the impulse response is absolutely integrable, right.

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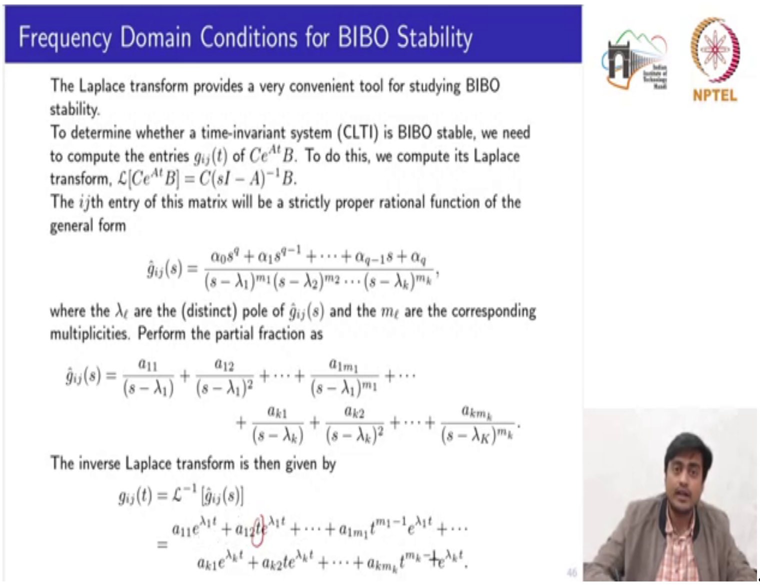

We can also see the frequency domain conditions for BIBO stability, because we know for LTI system we can use this Laplace transform tool; likewise we had seen in the first week. So, to determine whether a time invariant system CLTI is BIBO stable, we need to compute the entries g i j of this. To do this we compute it is Laplace transform and the Laplace transform of this entry is given by C into the inverse of s I minus A into B.

Now, to compute the i j th entry of this function, it could be written as the ratio of two polynomials where this is the numerator polynomial and the denominator polynomial. And all these lambdas defines the eigen values or the roots of these equations with the multiplicity m,. So, if we have m 1 is equal to 2; it means that, there are two roots located at lambda 1. So, we can do the partial friction of this equation, where we obtained these coefficients a 1 1, a 1

2 which we can compute given the transfer function. And in the denominator we would have all the powers of this first order polynomial.

So, we have s minus lambda 1 square upto to the power m 1; which denotes this multiplicity plus similarly for the other elements and again for the k th one. So, now, if I take the Laplace inverse to compute the time domain signal, I would have a 1 1 e to the power lambda 1 t plus; because we have the square by using the property of the Laplace transform I would have a 1 2 multiplied by t. So, this element is added new plus for the m 1th multiplicity we would have a 1 m 1, t raise to the power m 1 minus 1 and e to the power lambda 1 t. Now similarly goes for the lambda k th eigen value with multiplicity m k, ok.

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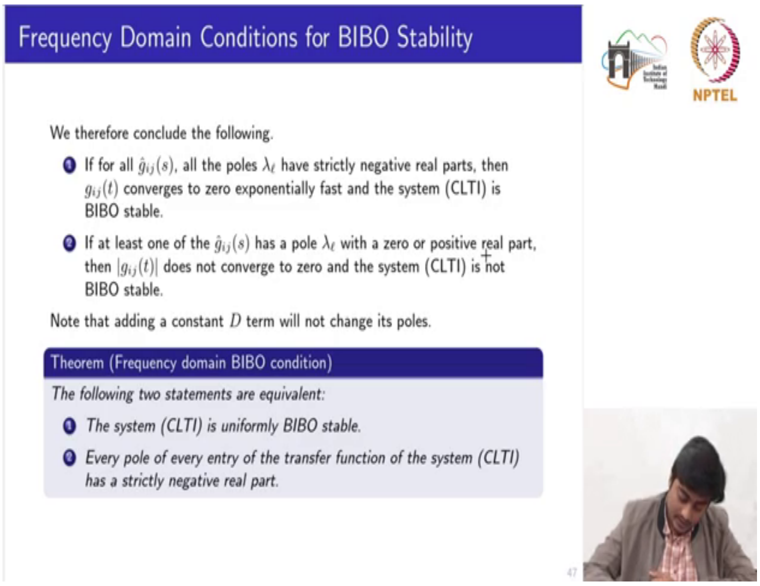

So, we therefore conclude the following; now if for all g i j of s all the poles lambda l have strictly negative real parts, then g i j of t converges to zero exponentially fast and the systems

CLTI is BIBO stable. Because first of all we have already seen that, the definition of exponential stability and asymptotic stability for the LTI system basically remains the same. And since if we look at this part that, the impulse response is basically expressed in terms of the exponentials. Now all these lambdas define the eigenvalue. Now if these lambdas on the left hand side, all these exponentials would die out to zero, exponentially.

So if at least one of the g i j of s has a pole lambda l with 0 or positive real part, then the absolute value of the g i j does not converge to zero and the system is not BIBO stable. So, if we add the D term also which is a constant term, basically it would not change the location of the poles. So, this is the key result in the frequency domain that, the system is uniformly BIBO stable; if and only if every pole of every entry of the transfer function of the system has a strictly negative real part. If the eigenvalue or if the pole of every entry of the or any entry of the transfer function is on the zero or positive then it cannot be said BIBO stable system.

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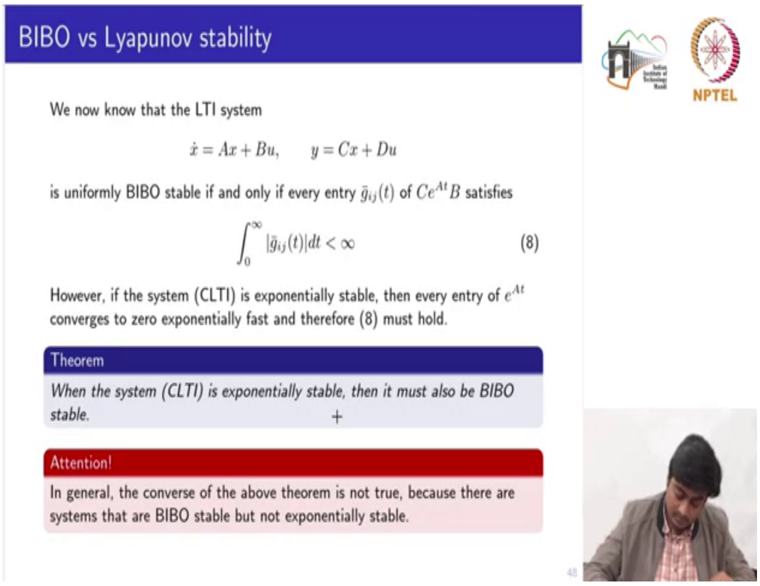

So, now there is a some relationship between BIBO stability and the Lyapunov stability. So, let us explore this for the LTI system. So, we know that the LTI system given by these two set of equations is uniformly BIBO stable; if and only if every entry g bar i j of this satisfies this condition; meaning to say that, it is absolutely integrable. However, if the system is exponentially stable then every entry of e to the power A t converges to zero exponentially fast and therefore, 8 must hold, right.

So, when the system CLTI is exponentially stable, then it must also be BIBO stable. Now this exponentially stability is basically in the sense of Lyapunov. Meaning to say that, if the system is stable or exponentially stable in the sense of Lyapunov; then it must also be a BIBO stable given a bounded input the system would yield a bounded output. But the converge of the statement is not true in general that is to say that; if the system is BIBO stable, then it we cannot guarantee that the system is exponentially stable.

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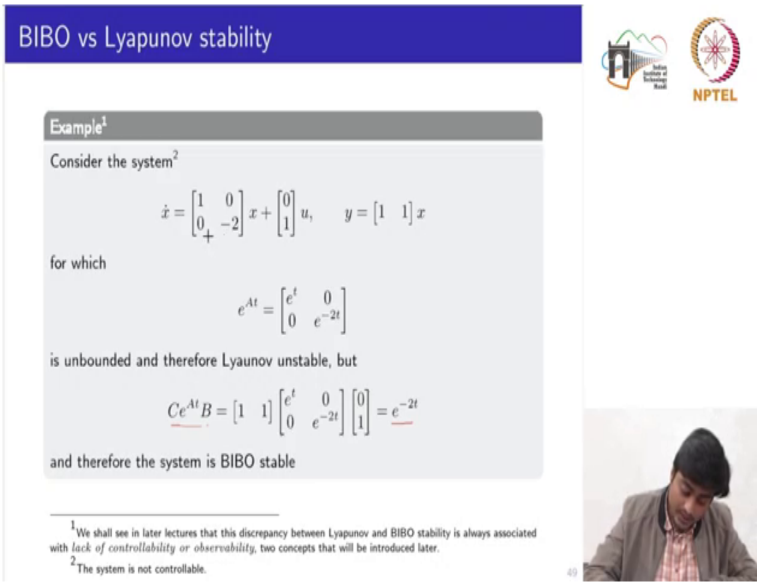

Let us consider one example to illustrate the situation. Say for example, we have the system given by this A, B, C, D matrices where A matrix is 1, 0, 0, minus 2. B vector is given by 0, 1 and C is 1, 1. So, basically it is a single input single output system of a second order, alright. So, if we compute the straight transition matrix of this system; we obtain this and we know that one of the element is e to the power t and it is not exponentially stable or unbounded and therefore Lyaunov unstable, but if we compute this part we obtain e to the power minus 2 t; meaning to say that this system is BIBO stable.

So, this is what we want to say that if the system is BIBO stable; it does not mean, or it does not implies in general that the system would also be Lyapunov stable. And we shall, so these are two footnotes written over here that we shall see in later lectures that this discrepancy between Lyapunov and BIBO stability is always associated with the lack of controllability or observability. So, these two concepts we will see in the in the later lectures. By the way that this system if by the tests or the controllability tests we were going to reduce in the next week. You would notice that this system is not controllable, right.

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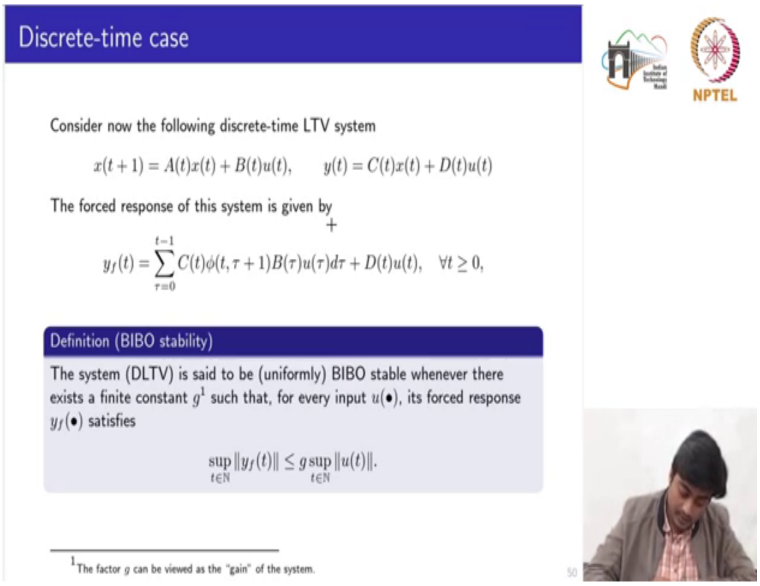

For the discrete time case; so given A, B, C, D as time varying matrix where t belongs to the set of integers. We have seen earlier that the forced response of the system is given by this equation. So, the definition of the BIBO stability in affect for the discreet time system also remains the same. That if the, we say that the system is BIBO stable whenever there exists some finite constant g; which satisfies this inequality.

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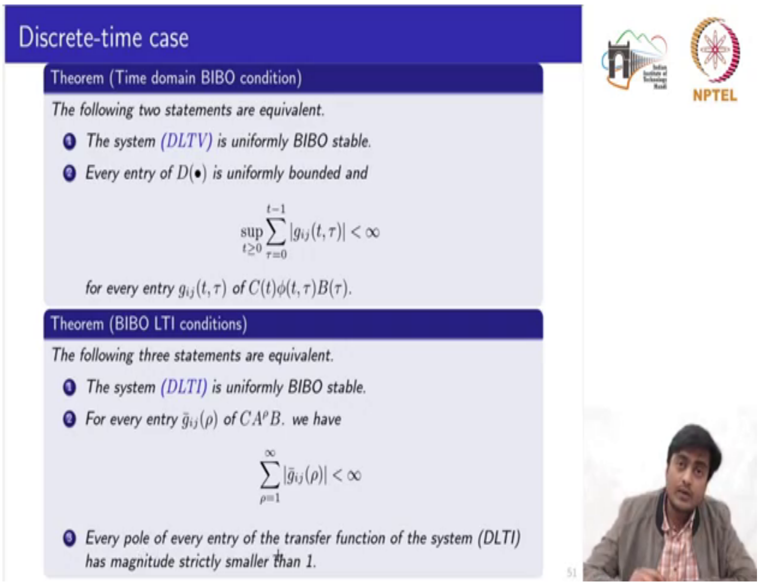

The time domain BIBO condition for the discrete time DLTV system, it is BIBO stable if and only if every entry of D is uniformly bounded and the impulse response is absolutely summable. We can also see the conditions for the LTI system.

So, we say that the system linear time invariant system is uniformly BIBO stable; if and only if for every entry g bar i of j of C A rho B; we have this condition, ok. And the third statement is basically in the in the frequency domain which say that every pole of every entry of the transfer function the system DLTI has a magnitude strictly smaller than one, basically it should be inside the unit circle.

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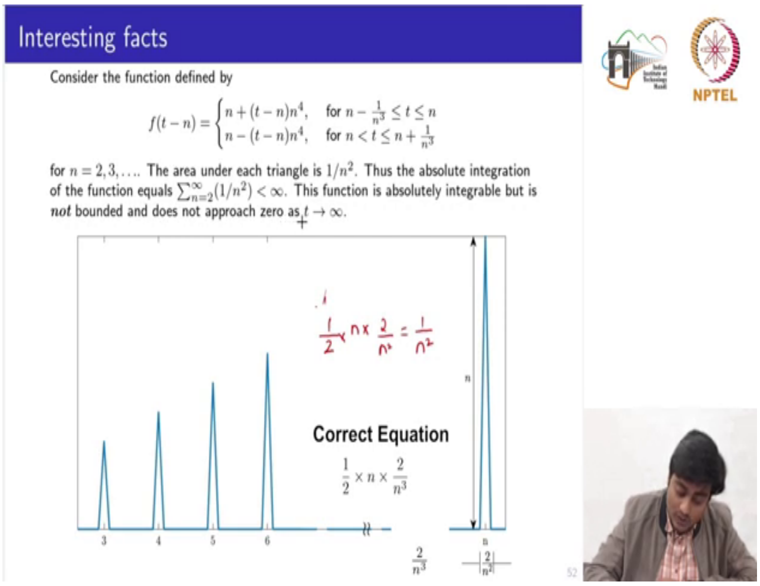

So, there are some interesting facts we which we can see now. So, consider a function which is defined by this relationship, ok. This is a continuous time function where n is a integer starting from 2, 3 and up to so onwards. So, if we plot this function; this is the plot of this function we have obtained. Now you would notice that the area under every triangle is given by 1 by n square.

So, we can see also that the for the n is equal for some nth value it is given by; a half the height n and also the base which is 2 by n square which basically comes to 1 by n square. So, so the area of all this triangle is 1 by n square. So, if we compute the integration, we see there is the absolute integration of the function is equal to or equals that; this summation would is the finite summation because as n increases from 2 to infinity, this summation would be a finite summation.

This would approach to zero. So, this function we know that is absolutely integrable, but is not bounded and does not approach to zero as t tends to infinity.

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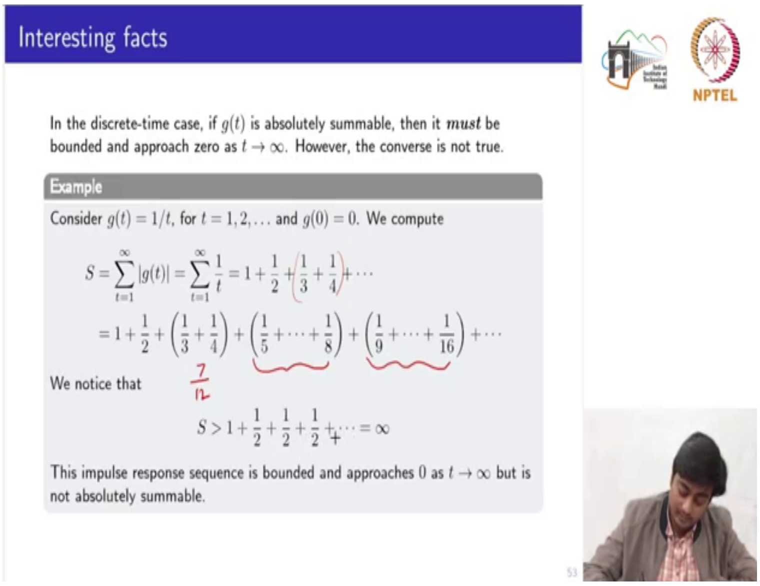

So, if we would like to see in the discrete time domain. So, in the discrete time domain; if g of t or if the signal is absolutely summable, then it must be bounded and approach 0 is t tends to infinity; which is a counter statement to this one, that this here the signal is absolutely integrable, but it is not bounded. But in the discrete time domain, if the signal is absolutely summable, then it must be bounded and approach is 0 as t tends to infinity; however, the converge is not true. Meaning to say; that if the signal is bounded and approaches 0 as t tends to infinity, then it may not implies that the signal is absolutely summable.

We can see one quick example. Consider this signal g of t given by 1 over t; where t is a belongs to a set of positive integers, ok. So, we compute this summation; the taking the sum of the absolute value of g of t starting from t is equal to 1 up to infinity. So, we see this summation is given, we can write 1 plus 1 by 2 plus 1 by 3 plus 1 by 4, ok. Now if I start clubbing the entries of the summation; let us say 1 plus 1 by 2 would remain as it is, plus if I combine these two parts. So, this part would be greater than 1 by 2.

You could see that it would give me 7 by 12. So, this part is greater than 1 by 2. Now if I club the next couple of elements, this part is again greater than 1 by 2. Similarly, if I keep combining the elements of 2 t tends to infinity; I would find that this summation is basically greater than 1 plus every entry is greater than 1 by 2. And, we know that this would approach to infinity. So, we see here that though the signal is bounded and approaches zero as t tend to infinity, because the signal it would die towards to zero. But here for this particular example that it is not absolutely summable; because the summation is basically infinity, ok.

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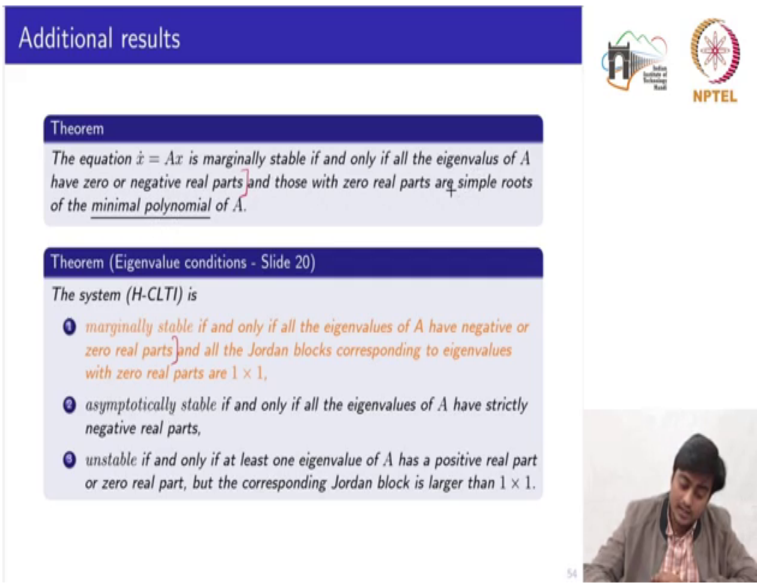

So, there are couple of additional results. So, one of the results says that this LTI system or this homogeneous LTI system is marginally stable; if and only if all the eigenvalues of the matrix A have zero or negative real parts and those with zero real parts are simple roots of the minimal polynomial of A. So, here we are introducing another polynomial which we call the minimal polynomial.

Basically, this condition is equivalent to one of the conditions we had seen earlier also; but that condition we had seen in terms of the Jordan blocks, which we have introduced on the slide 20. That the system is marginally stable if and only if all the eigenvalues have negative or zero real parts, so this part would remain same up to here.

Now, if there are some eigenvalues which lies on zero or the zero axis, then they should be simple roots of the minimal polynomial. Now in terms of the Jordan blocks we had, that the Jordan blocks corresponding to eigenvalues with zero real parts are 1 cross 1. So, either of these conditions we could use; either you compute the Jordan blocks, or you compute the minimal polynomial. Or let us say the eigenvalues of the minimal polynomial.

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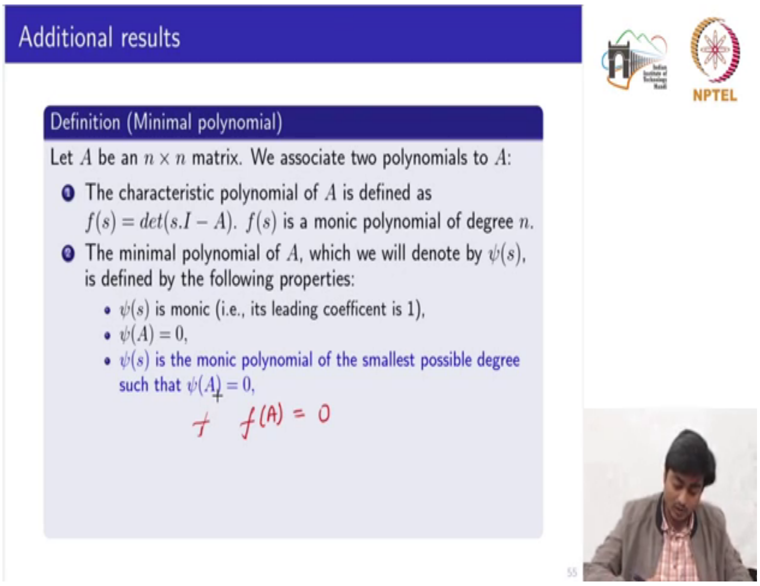

Let us see first how do we define the minimal polynomial. So, let A be an n cross n matrix. We associate two polynomials with the matrix A. So, first is the characteristic polynomial which we had already seen, which is given by this function f of s as the determinant of s I minus A; where f s is a monic polynomial of degree n, ok.

Now, another polynomial we say is the minimal polynomial of A which we will denote here by psi of s and is defined by a function having the following properties. Let us psi of s is monic; that is leading coefficient is equal to 1. Psi of A is equal to 0, which it also satisfies. So, if I put f of A, sorry if I put f of A; it is also equal to 0. So, the minimal polynomial should also satisfy this property. In addition to these two properties, it should also satisfy this psi s is the monic polynomial of the smallest possible degree such that psi of A is equal to 0, ok.

So, we will consider some couple of examples by which you would be able to get a clear picture how to compute this minimal polynomial.

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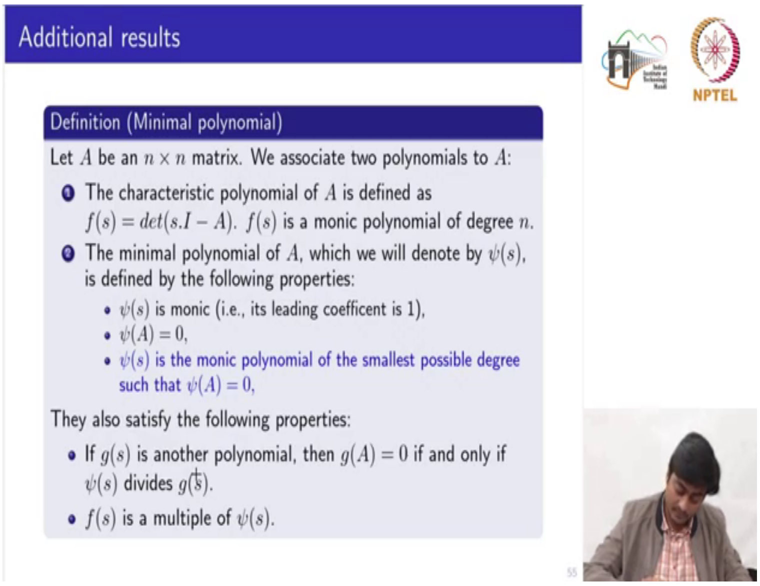

So, there are couple of other properties that if g of s is another polynomial, then g of A is equal to 0 if and only if psi s divides that polynomial g of s. And the second one f s is also a multiple of psi of s.

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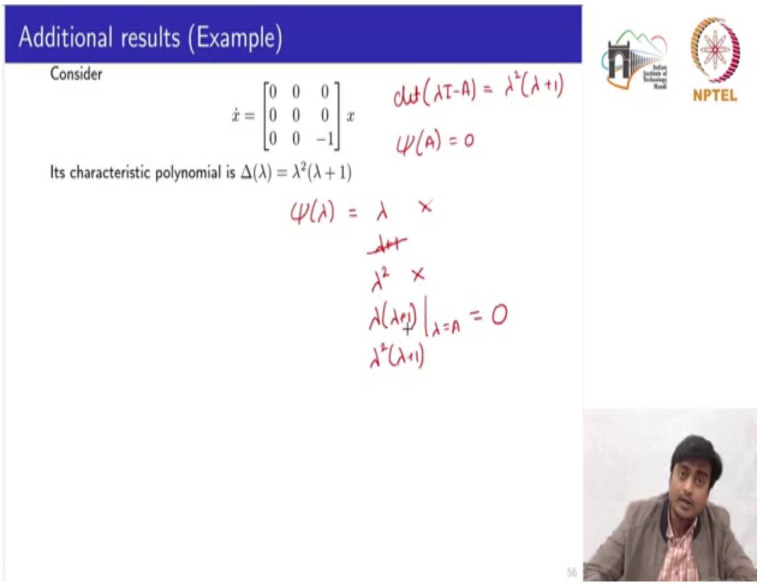

So, first we will consider an example for computing the minimal polynomial. Let us say we have this matrix, A matrix given as with this specific structure. So, if you compute the characteristic polynomial which is given by the detriment of lambda I minus A; it is given by lambda square lambda plus 1, excuse me. So, lambda square lambda plus 1.

Now I want to find a polynomial psi of s which is of lesser degree than this. It has a degree of 3 and it also satisfies psi of A is equal to 0, ok. So, a polynomial of less than degree 3 could be a; so there are a couple of options. Let us say psi of A sorry, let us say psi of lambda could be lambda which is of degree 1 or lambda plus 1; which is also a degree 1.

Now if there could be a polynomial of degree 2 which could be lambda square or lambda lambda plus 1. And also it could be equal to the characteristic polynomial lambda square lambda plus 1 which is of degree 3. Now if we try to see with all these polynomials where the psi of A is equal to 0 satisfies, then it would be the minimal polynomial.

So, if we replace lambda by A, we know that A matrix is not a zero matrix. So, this cannot be a minimal polynomial. Similarly, lambda plus 1, if I put A and plus 1. In fact, it cannot be considered as the minimal polynomial because we are considering only the minimal polynomial where we have lambda is equal to 0. So, lambda should definitely be a part of it; either it would be lambda, lambda square, lambda lambda plus 1 or lambda square lambda plus 1. So, this is in fact, excuse me that it is not an option for the minimal polynomial, right.

So, if we take this lambda square, again if I take the square of this matrix, it does not gain. If I put if I replace lambda by A, it would not be equal to 0. So, it is also not a minimal polynomial. Now if I see this part, lambda in to lambda plus 1. So, if I replace A by A; you would notice that lambda by putting lambda is equal to A, it becomes equal to 0. So, the minimal polynomial of this system is given by lambda lambda plus 1.

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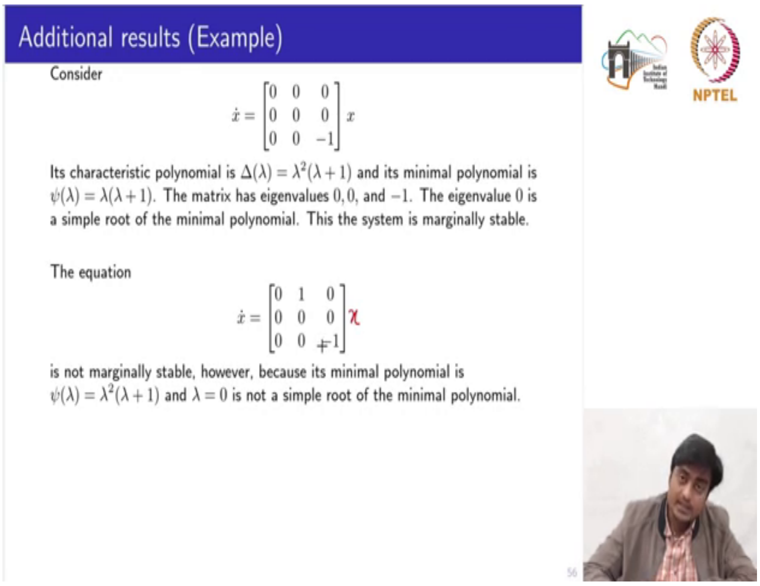

Now if I compute the eigenvalues or the roots of this polynomial it contains a simple route only at lambda is equal to 0. So, they says, that the matrix has eigenvalues. So, there are two eigenvalues at 0 and the third eigenvalue at minus 1; but if I compute the roots of the minimal polynomial it has a simple rooted 0. So, this system is a marginally stable system. Now consider another example where here instead of 0, I replace it by 1; otherwise the matrix remains the same.

The characteristic polynomial, so sorry here it should be x. So, if I compute the characteristic polynomial for this matrix, it is again equal to this one; but the minimal polynomial of this system is also equal to this one, because there does not exist any other polynomial of lesser degree than 3. And again, if I compute the eigen the roots of this characteristic polynomial;

we already know that there are two roots available at 0, which are not as simple roots. So, this system is an unstable system and this system is a marginally stable system, ok.

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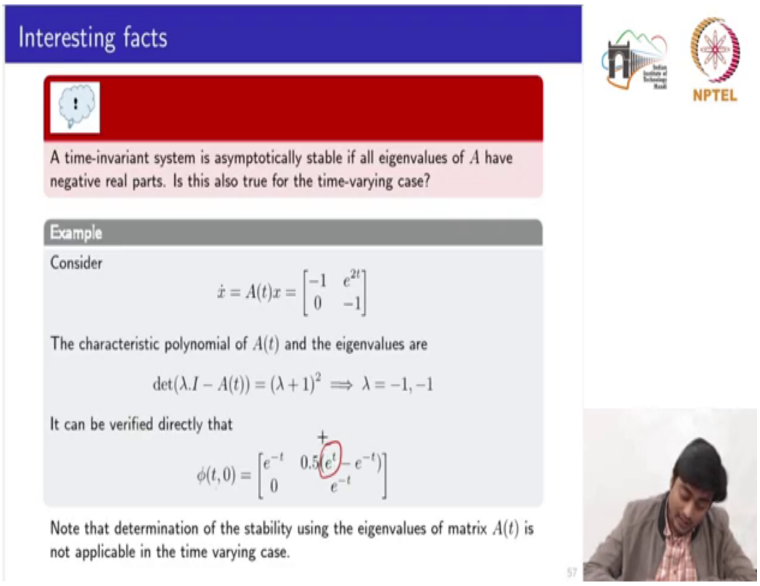

So, a time invariant system is as asymptotically a stable if all eigenvalues of A have negative real parts, so this is what we have already seen. So, now, is this also true for the time varying case. Let us try to answer this question by a by it considering a simple example.

Let us say we have this stage pace system where the, again sorry I forgot this x; so where A of t is given by this time varying matrix. Now computing the characteristic polynomial of this matrix A of t, it can be written as lambda I minus A of t. So, finally this, this becomes a characteristic polynomial and it is straightforward to see that the eigenvalues of this characteristic polynomial are at minus 1 and minus 1. So, if I apply the eigenvalue test on the

LTV system; it says that the system is a stable system because the eigenvalues are strictly at the left hand side.

Now, we can also compute or we can also verify it by computing it by computing it a straight transition matrix. So, the straight transition matrix of this system is given by this; where we could readily see that one of the component is having e to the power t which means that this system is an unstable system, which also implies that we can not apply the eigenvalue test on to the linear time varying system, ok