Nonlinear System Analysis
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Lecture – 45
Aizermann's conjecture under passivity assumption is true

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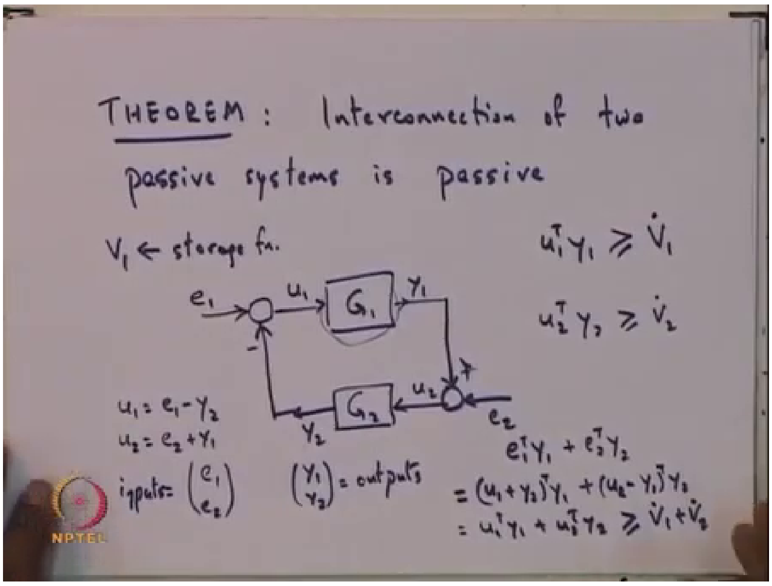

Now, how this connects up with Aizermann's idea is what I will now try to explain. And for that first let me yeah consider nonlinearities which are which are memory less ok. So, what I mean by a memory less nonlinearity is the following.

If you give a certain input to the nonlinearity you get an output, but this output is not dependent upon what happened in the system earlier in time or later in time whatever. It is an instantaneous map. So, what I am trying to say is that whatever is the input the output is completely determine by what the input is at this instant, the output at this instant is completely determine by the input at this instant. Such, maps they would call a nonlinearity.

Now, if you have a nonlinearity I mean such a map we would call memory less nonlinearity. So, if you have a memory less nonlinearity then one way.

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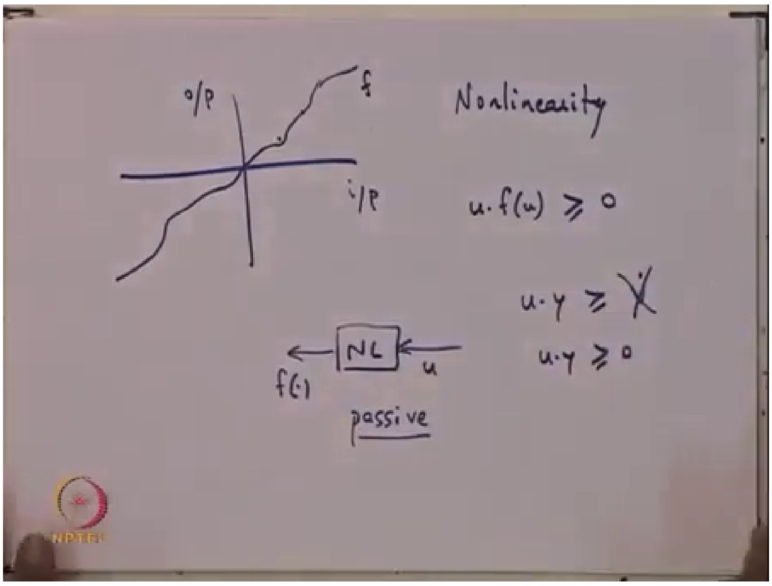

You can characterize that nonlinearity is by this map, where you have the input and here you draw the output.

So, for any given input there is a particular output for this input there is some particular output. And so, you can connect all those dots and you get a curve of course, if this curve was a straight line passing through the origin then the non-linear system is not really non-linear, but its linear, but if you have this situation it is a non-linear. So, this is a nonlinearity yeah.

So, for any given. So, this is like a lookup table if you want given any input you go through the graph and you know what exactly the output is.

Now, if you have a nonlinearity such that this curve ok. So, let me call this nonlinearity f. Then this nonlinearity is such that if the input is u then u times f of u is greater than 0 greater than equal to 0 then as far as a non-linear system is concerned. So, here is a non-linear system suppose as the input going into the nonlinearity and what you get here is f of that and what you are saying is input multiplied by output is always greater than 0.

Then, from whatever we have been discussing earlier we could call this passive. And now, if you call this passive yeah this thing is memoryless; that means, in the sense it just depends on what the instantaneous input is one could think of the storage function yeah. So, the definition of passivity is u dot y input multiplying output is greater than equal to the rate of change of the storage function.

The suppose you take the storage function to be the 0 storage function then you get this; that means, u dot y is greater than equal to 0. And so, assuming that this storage function is 0 storage function is one way that this equation is satisfied. And so, you have a passive system this non-linear system is passive and it has a storage function which is a 0 storage function. And here is the beauty of the whole thing.

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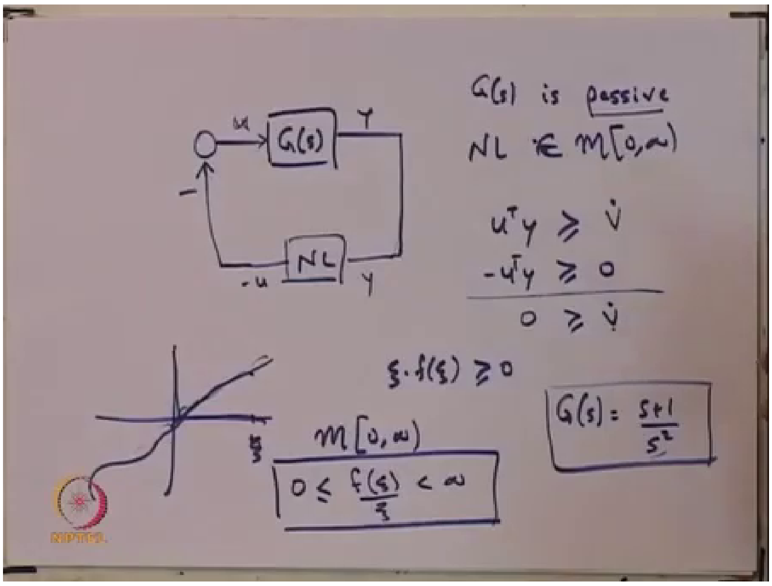

Suppose you take a matrix a linear system G of s and you take a nonlinearity. And this nonlinearity is on the first and the third quadrant incidentally nonlinearities, which are in the first and third quadrant are often denoted by this this is a nonlinearity lying in the 0 infinity sector; that means, they lie here or here the slope. Or rather if you take any u the f of u.

In fact, I had given this kind of a definition earlier if you call the input Xi f of Xi by Xi is less than infinity such nonlinearity is 0 infinity nonlinearity. And if you have any nonlinearity like this then of course, this is true Xi times f of Xi is greater than equal to 0 and from what I said in the last slide one could view this as passivity.

So, if you have a nonlinearity which belongs to this class and you have a feedback structure which looks like this, then if this G s; I mean this linear part if G s is passive. And the nonlinearity is or belongs to it is a nonlinearity like this. Then G s is passive the nonlinearity is passive. So, from what we had talked about earlier the interconnection of these two the interconnection of these two systems is also passive.

Now, this is passive what we mean by that is if this input is u and the output is y then what we are saying is u transpose y is greater than equal to v dot, where v is the storage function as far as this guy is concerned. And the nonlinearity is passive well, it is the same y here and out here what you have would be the negative of this u.

So, let me call it minus u, but this minus u and y it obeys the rules of the of the nonlinearity therefore, minus u transpose y is greater than equal to 0 yeah. This is for the nonlinearity and because it is in the 0 infinity sector this must be true this G s is passive. So, this must be true I add these two I get 0 greater than equal to v dot.

So, this is an autonomous system in which I have v provided G s is passive, I can find the storage function to be a positive definite function and I have therefore, a positive definite function whose derivative is less than 0 therefore, the resulting system is asymptotically stable. So, if you remember Aizermann's conjecture said that, if you had some trajectory which lied in the sector something like this.

And his conjecture was that if you had a G of s such that on the feedback loop if you put any gain between 0 and infinity. And it gives you a stable system then if you put a nonlinearity then the resulting closed loop system is asymptotically stable and that was proven to be false.

But what we have got is, a very similar result what we are saying is if you take any nonlinearity in the 0 infinity sector. And out here you are not going to take any transfer function such that you know you put any gain between 0 and infinity and the resulting feedback system is asymptotically stable that is not what you are going to do. What you are going to do instead is instead of that condition you are going to take a G s which is passive.

And if you do that then the resulting system is such that it is asymptotically stable yeah. So, if you recall earlier a few lectures back I had shown that in Aizermann's conjecture there is some sort of counter example I showed. And if you remember in that counter example the G s that I took was s plus 1 upon s squared.

Now, if you look at this particular transfer function. And you look at its Nyquist plot, it will be clear that this is not a passive transfer function. And as a result one cannot expect this this interconnected system to be to be passive and therefore, asymptotically stable. So, the interconnection of passive systems being passive that result, in fact, is a solution to the Aizermann's conjecture in the sense it is a positive it is a positive reply to Aizermann's conjecture in the sense that if you have a nonlinearity in the 0 infinity sector.

Then, that is like interpreting the nonlinearity as a passive nonlinearity and therefore, you interconnected with a passive linear system and the resulting system is passive and because the resulting system is passive what you have is asymptotic stability ok. So, I am out of time for this lecture and so, we would stop here today.