DC Power Transmission Systems
Prof. Krishna S
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 64
Design of single tuned filter: Part 2

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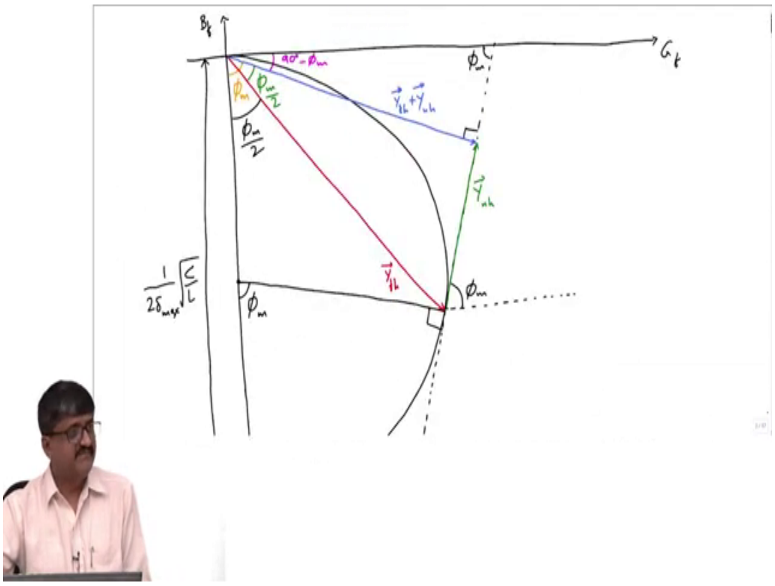

So, we were discussing the Design of single tuned filter. So, let me go to the diagram that we considered in the last class. So, it is the graph of a semicircle where it is shown as a plot in the plane where I have one axis as the real part of the admittance of the filter G f B f. So, we derived some conditions so far. So, we saw that one has to draw a tangent to the semicircle which makes an angle phi m. So, let me draw that tangent.

So, this dotted line makes an angle with respect to the boundary which is equal to phi m. So, this is Y fh and to get Y nh what I should do is first take Y fh plus Y nh which is obtained by dropping a perpendicular from the origin to the boundary ok. So, this is Y fh plus Y nh this is

90 degrees ok. So, from this I get Y nh this is Y nh. Now, if I take this angle what is this angle? It is phi m ok. So, that should be clear ok.

Now, if I take I need to use different colors otherwise it will cause confusion suppose I take this angle what is this?

Student: (Refer Time: 04:44) 90 minus.

90 minus phi m. So, 90 degrees minus phi m. So, from that I get other suppose I take this angle phi m. So, its visible not I will use a different color phi m. Now I will take yes the center of this semicircle ok. So, the center of the semicircle is on the B f axis of course, I have already said what is the diameter of this circle the diameter of the circle is 1 by 2 delta max into square root of C by L. So, that is the diameter. So, if I take the distance from this point to this point.

So, this diameter is 1 by 2 delta max square root of C by L ok. Now, if I take a straight line segment from the centre of the semicircle to this point at which I have taken the tangent ok. So, please note I have taken the tangent here. So, what is this and this this is a right angle that is a right angle ok. Now, what is this angle, see I have dropped two perpendiculars to this boundary one from the origin one from the centre.

So, both are parallel see there is one perpendicular dropped from the origin to the boundary one perpendicular dropped from the center to the boundary so, these two are parallel. So, if the other line is making an angle phi m with respect to this B f axis this is also phi m see this this is phi m means this is also phi m then if I take this angle what is this what is this angle?

Student: phi m.

Phi m by.

Student: 2.

2 please recall what you would have studied in say 8th standard or whatever school geometry nothing more than that phi m by 2. So, if that is phi m by 2 then if I take this angle the green one this is phi m by 2 the green this one phi m by 2 ok. So, this is same as the diagram that we consider in the last class I have made some additions I have shown a few angles are additional that is all.

And there is one more line a segment from the center of the semicircle to the point of tangent that is all nothing more than that ok. Now, let us continue our discussion of design of single tune filter. So, we said that we have to minimize the cost of the filter minimum cost filter.

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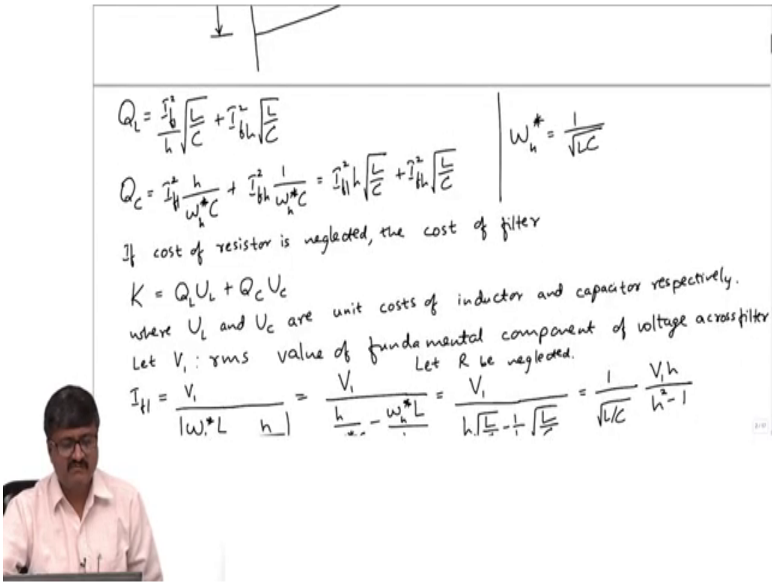

So, we got the expression for the reactive power rating of inductor. So, we got Q L the reactive power rating as. So, what was the expression can I did I get this expression if 1 squared.

Student: by h.

By h under root L by C plus I fh squared into mean towards the end of the last class I got this.

Student: (Refer Time: 10:05).

I got this ok. So, that is the reactive power rating of the inductor; similarly, if I take the reactive power rating of the capacitor. So, the if I just ignore all the harmonic components other than the harmonic component for which the filter is designed. Then the only fundamental component of the current and the ah relevant harmonic component is to be considered. So, there is a reactive power rating due to fundamental that is I f 1 squared into reactance what is the reactance of the capacitor at the fundamental frequency?

Student: (Refer Time: 10:47).

Yeah 1 by now, we do not have a notation for the fundamental angular frequency.

Student: No, sir.

So, we have a notation for the harmonic omega h star. So, omega h star by h is fundamental into C in the denominator. So, this is due to fundamental similarly, due to harmonic component it is I fh square into 1 by omega h star C. So, this can be written as, so, if I use the equation omega h star is equal to 1 by root LC I get I of 1 square into h into L by C plus I fh squared into root L by C say what I have done here is I used this see expression for omega h star 1 by square root of LC that is it ok.

Now, if I ignore the cost of resistor. So, so far I have not said why the resistor is required, but we will see that the cos I mean resistor is. In fact, required and, but thing is the resistance is very small otherwise there will be loss see here any resistive component should serve some purpose, but it at the same time I mean it should not be unnecessarily added. So, if at all it is added I mean it is very small value for serving something, but not contributing to loss.

So, if cost of resistor is neglected, then cost of filter. So, filter has three parts resistance, inductance, capacitance. So, if resistor cost is neglected then the cost of filter is due to cost of inductor and cost of capacitor. So, I will use a notation for this cost of filter as K, I cannot use C because C is already used for capacitance K, K for cost of filter. So, cost of filter is cost of inductor plus cost of capacitor. So, the cost of inductor is proportional to the rating of the inductor.

So, I write this as Q L into U L plus the cost of capacitor is Q C into U C, where U L and U C are unit costs; that means, cost per unit rating ok. So, U L and U C are unit costs of inductor and capacitor respectively. So, if I want to look at the expression for cost it is in terms of Q L and Q C, U L and U C are constants. So, Q L and Q C are dependent on I f1 and I fh say essentially we want L and C; suppose V 1 is the rms value of fundamental component of the voltage is rms value of fundamental component of voltage across filter ok.

Then can I get an expression for I f1 did I define I f1 in the last class? I f1 is the rms value of

Student: fundamental (Refer Time: 14:50)

fundamental component of the current through the filter. So, can I get an expression for I f1 in terms of V f1 sorry V 1 it is the fundamental voltage divided by the impedance magnitude of the impedance what is the magnitude of the impedance?

Student: 1 by (Refer Time: 15:09).

1 by.

Student: Mod of Y fh.

Can I write it in terms of say Y fh is something which I do not know see when I am designing a filter I want to find the value of L and C. So, as I said we will do a minimum cost filter we will try to minimize K. So, I will write K in terms of L and C. So, [vocalized- noise] K is dependent on Q L, Q L is dependent on I f1. So, I f1 should written in terms of L and C. So, what can what should be the expression in terms of L and C?

Student: (Refer Time: 15:45).

root of.

Student: (Refer Time: 15:48) square.

Ok, let me make one assumption let R be neglected. So, that it is simplified. So, it is to the impedance is only due to L and C I have also already neglected the cost of resistor I am also neglecting the value of resistor in computing the current ok. So, what is the expression for the impedance magnitude of the impedance not the complex one.

Student: (Refer Time: 16:25).

L omega h by h let me ok. So, you are saying omega h star by h into L.

Student: Minus.

Minus.

Student: h by (Refer Time: 16:40).

h by.

Student: C.

C omega h star that is what you are saying.

Student: Mod.

Mod now, that you are saying mod because you do not know which one is larger than the other, but do not we know which one is larger among the two.

Student: (Refer Time: 17:00).

First one means omega the no the reactance of the inductor is larger or reactance of capacitor is larger?

Student: Capacitor.

See there should be the reactance's of the inductor and capacitor should be same at what frequency?

Student: Resonant sir.

Resonant frequency, that is harmonic frequency for which it is designed. Now, I am computing the fundamental frequency impedance or admittance whatever. So, at fundamental frequency which one I will have a higher reactance capacitor or inductor?

Student: Capacitor.

I will repeat now the capacitor and inductor will have the same reactance at.

Student: Resonant (Refer Time: 17:42).

Resonant frequency fundamental frequency is lower than resonant frequency so.

Student: Capacitance.

Capacitive reactance will be larger. So, I do not need this modulus I know which one is larger. So, I want I f1 to be positive because I f1 is an rms value please note I f1 is a rms value that is why I want it to be positive. So, I can use the knowledge of which one is which among the two reactance's is larger and straight away write the expression. So, so I will write this as ok. So, let I need not actually erase this I mean I could have take that also omega h star L by h minus 1 by sorry h by omega h star into C modulus.

So, since we know which one is larger we could have just written this as V h by h by omega h star C minus omega h star L by h. So, if I use the expression for omega h star which is 1 by root LC I get I f1 as V 1 divided by h into 1 by omega h star C is nothing, but root L by C minus 1 by omega h again omega h star is 1 by root LC. So, it is minus 1 by h root L by C.

So, my intention is to write it in terms of L and C these are the two quantities that I have to be determined. So, this can be written as 1 by root LC is a common factor in the denominator I mean root LC root L by C is a common factor in the denominator ok.

So, this can be written as 1 by square root of LC into V 1 h divided by h square minus 1 ok. Now, if you look at either Q L or Q C there is not only I f1 I fh is also there.

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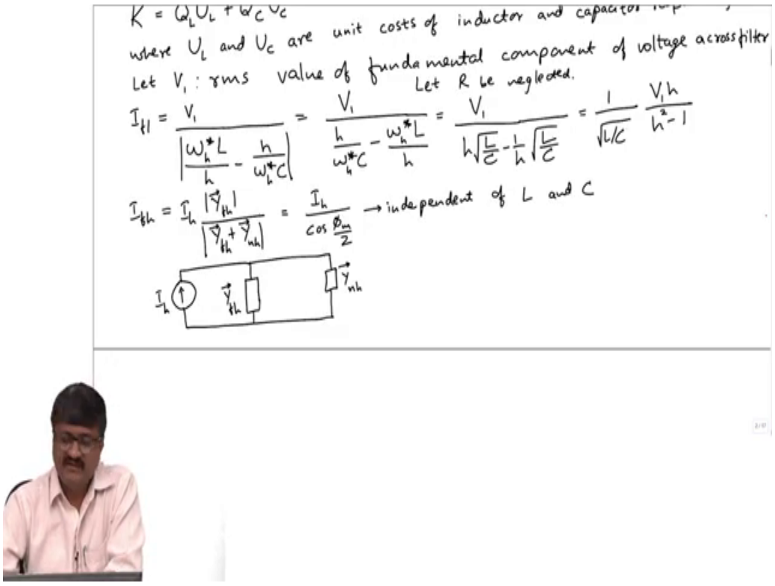

So, there is an I fh which is the rms value of the current of harmonic order h. Now, how to get I fh can I write ifh in terms of I h? Say if you we considered one particular circuit if you just recall, there was a representation of the converter in the form of a current source; I mean we started with this only then the filter is represented by an admittance the network is represented by another admittance.

So, the magnitude of the current from the converter is I h. Then the filter has an admittance Y fh at the harmonic frequency now you are considering only harmonic frequency the network has an admittance Y nh. So, can I write I fh in terms of I h and of course, and Y fh and Y nh. So, what is I fh in terms of I h it is equal to I h into what?

Student: Y nh.

Y nh by.

Student: (Refer Time: 21:27) by (Refer Time: 21:28).

That is there is something wrong there. Say you have to just take the voltages across the current source or voltage across the any of the admittances they should be same ok. So, that will be the voltage magnitudes will be same if I write this as I fh equal to I h into.

Student: (Refer Time: 21:51) Y fh.

Y fh magnitude divided by

Student: Y fh

Y fh plus Y nh absolute value just divide this equation by Y fh on both sides what do you get if you divide by Y fh magnitude? You get the magnitude of the voltage at harmonic frequency that is V h we get V h was the notation that we used in the last class. So, this can be written as I h come to this figure that we have just considered this figure. Now, look at this ratio Y fh by Y f Y fh is one quantity and ratio as one more quantity magnitude of Y fh plus Y nh.

So, these two quantities are actually the sides of a right angle triangle Y fh magnitude and magnitude of Y fh and Y nh Y fh plus Y nh. So, I can write that ratio as what in terms of phi m can I write.

Student: (Refer Time: 23:09) cos.

Cos phi m by 2. So, this is I h by cos phi m by 2. So, I h is a constant which is known phi m is a constant which is given. So, I fh is a constant please note when you look at the expression for I f1 V 1 is a constant h is a constant because V 1 is a value of the rms value of the fundamental component of the voltage. And h is also a constant similarly in the expression for I fh I h is the constant phi m is the constant, but I f1 is not a constant because I mean I f1 is dependent on the unknown quantities L and C where as I fh is independent of the unknown quantity L and C, is that clear?

So, I fh is not dependent on the unknown quantities L and C only I f1 is dependent ok. So, what I will do is I wherever there is I fh I will retain as it is just now we showed that it is a constant where as it is independent of L and C. So, this is independent of L and C where as I f1 is dependent on L and C ok.

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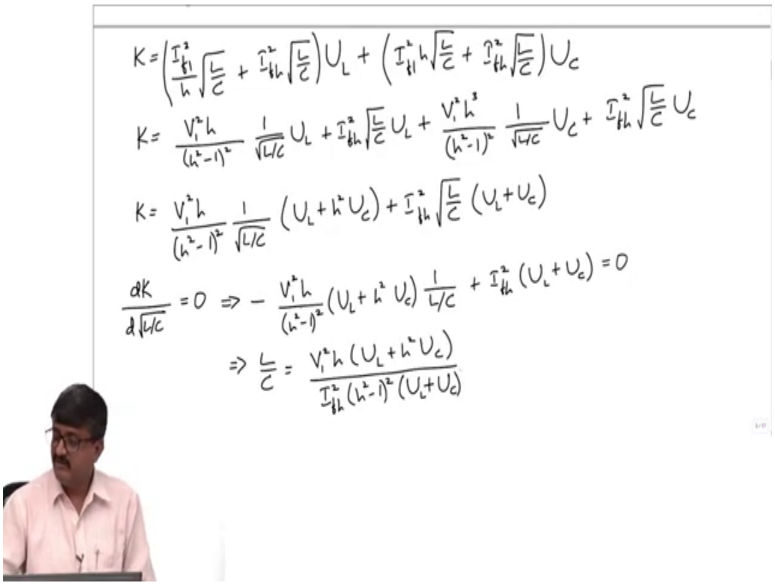

So, let me get back to the expression for cost. So, the cost K is the inductor rating into unit cost. So, if I take the inductor rating Q L. So, Q L has two terms I f 1 square. So, I substitute the expression for I f1. So, just now I derived the expression for I f1 ok.

So, yeah let me just give few more steps. So, it is I f1 squared by h into root L by C plus I fh squared root L by C this is Q L this should be multiplied by U L plus the reactive power rating of the capacitor is I f1 squared h into root L by C plus I fh squared into root L by C into U C. So, this is the cost of the capacitor. Now, what I will do is I will just substitute the expression for I f1 that we just derived in this expression for cost. So, I can write K as. So, substitute for if one I f1 is 1 by root LC root L by C into V 1 h by h square minus 1.

So, what I get is V 1 squared. So, there is an h, but h gets multiplied sorry h I mean there is a h in the denominator also so, that results in V 1 squared h divided by h square minus 1 whole squared. And there is a 1 by root L by C in the expression for I f1. So, and of course, in the cost expression also I have a root L by C. So, effectively it results in 1 by root L by C into U L. So, that is the first term say the cost has four terms two due to inductor two due to capacitor this is the first term the second term I write is write that as it is because I fh is independent of L and C ok.

So, I am just substituting the expression for I f1 similarly if I take the first term in the cost function of the capacitor I can substitute for I f1. So, I get V 1 square now there is h cube now divided by h square minus 1 whole square into 1 by root LC one by root sorry L by C into U C plus I fh square root of L by C, U C ok. So, is this just substituting the expression for I f1 ok. So, that gives an a I mean I will try to simplify this. So, I will write this as I will try to take the terms from coming from the fundamental and together.

So, they have a common factor V 1 squared h divided by h square minus 1 whole squared into 1 by square root of L by C into U L plus h squared UC. I am taking the first term and the third term in the previous expression plus the second term and the fourth term has a common factor I fh squared root L by C into U L plus U C. So, essentially there are two terms. So, if you look at this expression for cost look at all the quantities V 1 is constant h is a constant U L is a constant U C is a constant I fh is a constant. So, the only unknown quantities are.

Student: L and C.

L and C ok. Now, it up I mean I can say that the only unknown quantity right now is not just two, but only one which is?

Student: L by C.

L by C or square root of L by C I can say square root of L by C is an unknown quantity. So, if I want to minimize cost the necessary condition is the first derivative of cost with respect to this unknown quantity square root of L by C should be 0.

Student: should be.

Student: both them should be (Refer Time: 29:30).

Both of them should be equal yeah, let us see suppose I take the first order derivative. So, what do I get. So, from this i get an expression for L by C or root L by C that is all I mean I can get that. So, if I take the derivative or do I get, please note the you are taking that derivative with respect to square root of L by C square root of L by C. So, can I say that the derivative of the first term in the expression for K is minus V 1 squared h divided by h square minus 1 whole squared into U L plus h squared U C into what into what the derivative of the first term.

Student: C by L.

C by L into C by L. So, or 1 by L by C then the derivative of the second term is easy it is just I fh square into U L plus U C you are differentiating with respect to square root of L by C that is all ok. Now, what does this give this gives the value of L by C this I has actually giving the value of L by C from the necessary condition. So, from this I can say from the necessary condition for minimum cost L by C should be equal to V 1 squared h into U L plus h squared U C divided by I fh squared into h square minus 1 whole squared into U L plus U C is this ok.

Now, this is the necessary condition, but whether it gives minimum or not is yet to be determined for how do I find that?

Student: (Refer Time: 31:43).

I have to take the.

Student: (Refer Time: 31:46).

Second derivative and see whether it is it should be what second derivative evaluated at this value of L by C should be.

Student: (Refer Time: 31:55) greater than (Refer Time: 31:56).

Positive it should be positive you are minimizing.

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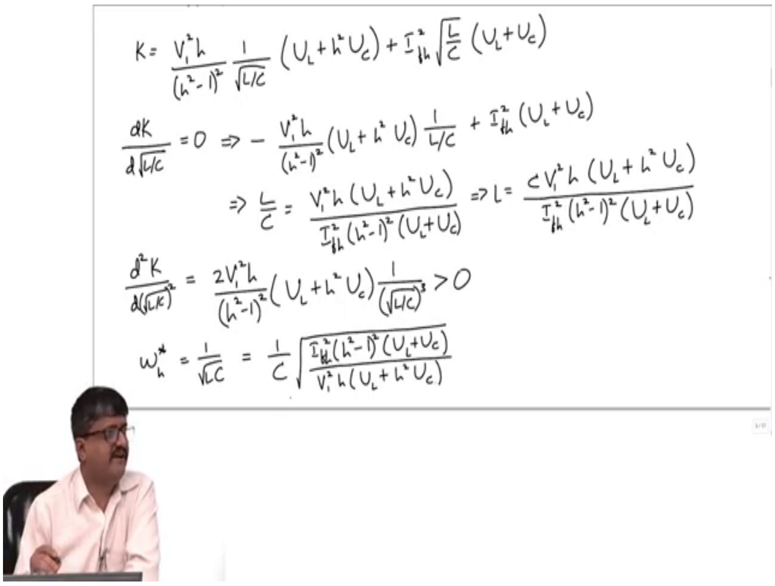

So, if I take the second derivative of K with respect to root L by C. So, and this should be evaluated at the value which is just now obtained. So, what is the expression for the second derivative of K? We have the first derivative of K. So, differentiate it again with respect to L by C. So, if you look at the first derivative expression the second term in the first derivative is independent of L by C.

So, what I get is 2 times V 1 squared h divided by h square minus 1 whole squared U L plus h squared U C into 1 by square root of L by C to the power of.

Student: 3.

3. Now, you put any non zero value I mean sorry not any non zero value any positive value for root L by C you will see that this second derivative is positive. Now please note you look

at the expression for L by C this L by C is positive and otherwise it does not make sense L is positive C is positive both should be positive so, ok. So, this is greater than 0. So, the it is satisfies the sufficient condition ok, but it still does not give value of L and C it gives the ratio it just gives the ratio. So, how to find L and C.

Student: (Refer Time: 33:31).

Y fh is even more complicated Y fh means you should know r also.

Student: Omega (Refer Time: 33:38).

So, omega h star is 1 by root LC. So, I use these two equations. So, I have the expression for L by C I also have the expression for omega h star omega h star is known please note omega h star is known. So, from this I can say that. So, what I do is. So, I write this as the previous equation as L equal to C into V 1 h V 1 squared h into U L plus h squared U C divided by I fh squared into h square minus 1 whole squared U L plus U C ok. So, I used this expression for h and substitute it in the expression for omega h star.

So, what do I get 1 by C into square root of I fh square into h square minus 1 whole squared into U L plus U C divided by V 1 square h into U L plus h square U C. So, this equation will give me what?

Student: Value of c.

Value of c I know the left hand side omega h star and I know the coefficient of 1 by C all quantities are known. So, I get C once I get C I can find.

Student: L.

L once I can find L and C can I find R.

Student: Yes.

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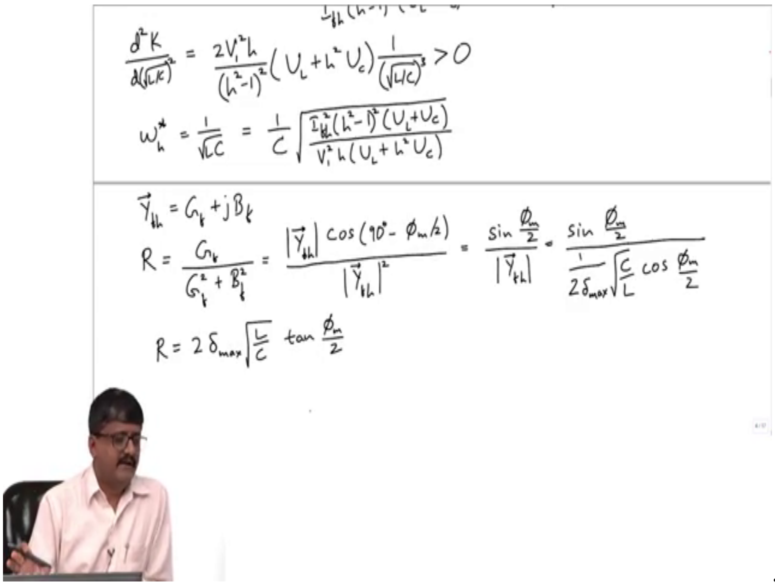

Yeah, how? So, can I relate R G f and B f what is R in terms of G f and B f, see it is the real part of the reciprocal of Y fh real part of the reciprocal of complex Y fh. So, can I write this as G f divided by G f divided by.

Student: Gf square (Refer Time: 35:52) B f square.

G f square plus B f square. Now, again come to this figure please note yes.

Student: How is yfh is (Refer Time: 36:05).

How Y fh is known come back to this now how to determine Y fh we discussed in the last class if you recall we take delta max the worst value of delta max get the semicircle. So, from this semicircle can I get the value of ok. How did we determine Y nh we know how to determine Y nh we determined the value of Y nh from the boundary and we got Y fh from the tangent yeah, I did not get what exactly you are asking how to get Y fh how we get yeah how we get Y fh how we get Y fh?

Student: (Refer Time: 37:05).

Can we get the semicircle see just now I said how to find see delta max we take the worst value. So, how do we get the semicircle see for drawing a semicircle I need this diameter know.

Student: Yeah.

So, do I know the diameter?

Student: Yes.

How just now we determined.

Student: (Refer Time: 37:27).

Root C by L we just now determined. So, I know how to find the diameter. So, I know how to find the semicircle. So, if I know how to find the semicircle then what should I do how to get? Yeah, the question is still valid see till now I have not said how to get a diameter we just now said how to get a diameter. So, once I know how to get a the value of a diameter I know how to draw the semicircle. So, how to get Y fh? I mean it is a question still I mean is not completely answered yeah, how to get Y fh?

Student: From the radius of the semicircle.

Radius of the.

Student: Semicircle from the mx and (Refer Time: 38:05) to the angle phi.

Yes, from that you get ok. So, this figure will help in I mean getting all the quantities. Now, if I take G f see please note G f is the real part of Y fh. So, can I say G f is Y fh into cosine of some angle what is that angle? See if I just project Y fh onto the G f axis I get the value of real part G f.

Student: 90 minus phi.

90 minus phi m by 2. So, let us write that. So, G f is the magnitude of Y fh into cosine of 90 degrees minus phi m by 2 ok. So, that is G f now, what is G f square plus B f squared it is nothing but the magnitude of Y fh square ok.

So, cos 90 minus phi m by 2 is sin phi m by 2 and this Y fh in the numerator the magnitude of Y fh in the numerator gets cancelled. So, we are left with Y fh magnitude in the denominator ok. So, this is sin phi m by 2 divided by what is Y fh? Yeah, we again use the figure. So, if you look at this figure ok. So, let me draw one more line segment which will be easy to answer this question. I will take this line segment what is the angle between this red line and this?

Student: 90 degrees.

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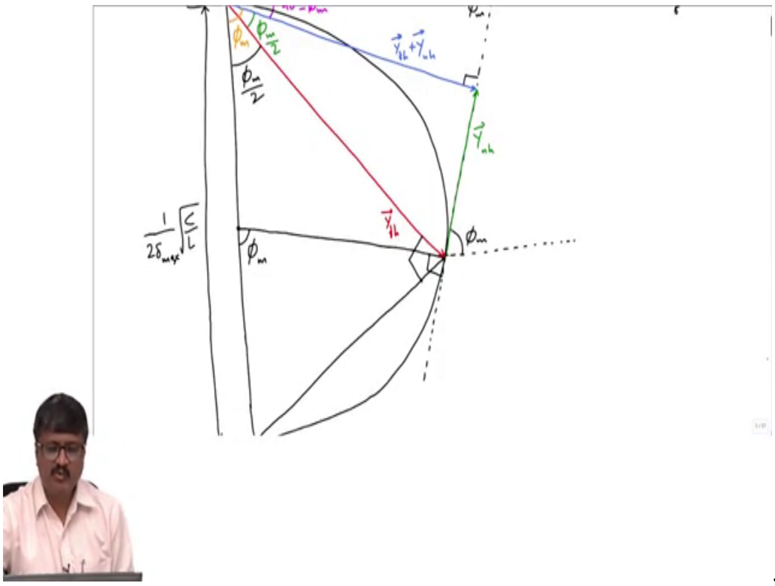

90 degrees. So, this is 90 degrees. So, from this diagram, I know two I mean I know that one of the hypotenuse is 1 by the diameter 1 for this triangle right angled triangle. See I take this red line and the just newly drawn straight line segment and another one line segment as the diameter these three form a triangle.

So, one side is 1 by 2 delta max root C by L another one has a magnitude Y fh. So, from that can I say that Y fh is the diameter into cosine of some angle that angle is phi m by 2. So, Y fh is 1 by 2 delta max root C by L into cosine of phi m by 2. So, finally, I get the expression for R as 2 delta max if I do simplification into root L by C into what?

Student: Tan.

Tan phi m by 2. So, I know all these quantities delta max is known it is the worst value of delta L and I know just now I found L and C. So, we know how to find L and C and phi m is given. So, from that I get R. So, we this is a single tune filter it is a simplest possible design by making some assumptions ok.

So, first of all we made the assumption of ignoring the variation of the values of inductance L and C itself. So, we designed this filter by just considering the variation of the system frequency that is all. So, if we consider even the variation of L and C, it becomes even more complicated ok. So, this is the simplest possible design, but we will not go to anything beyond this. So, this is what is known as single tuned filter.