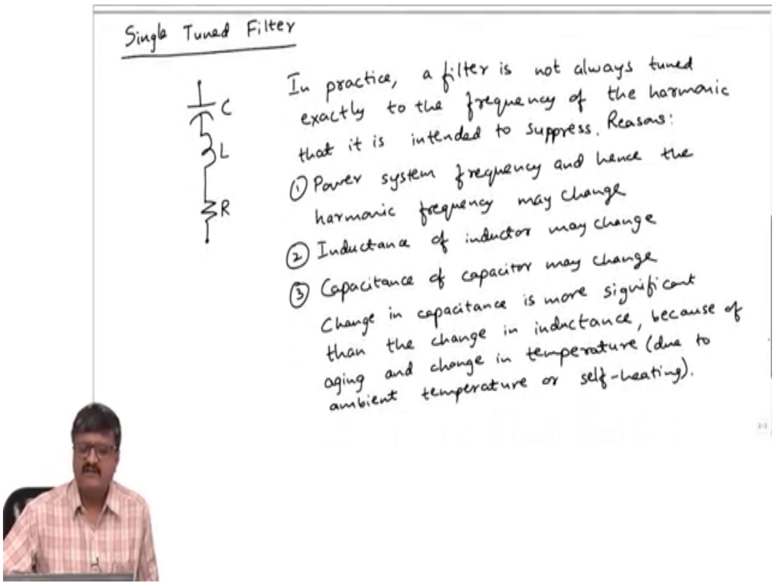
DC Power Transmission Systems
Prof. Krishna S
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 62
Single tuned filter

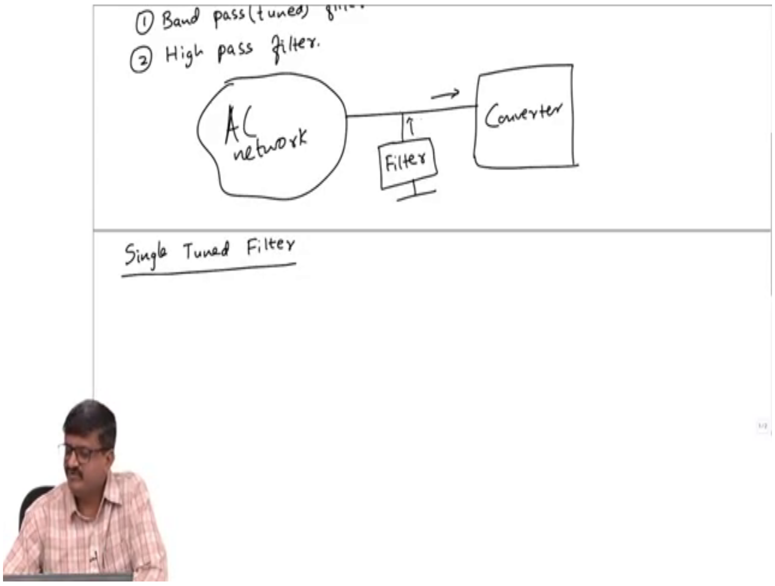
(Refer Slide Time: 00:17)



What is known as a Single Tuned Filter. So, please note we are trying to see what is happening on the AC side. So, we have seen that there are current harmonics. So, the idea is the harmonics on the AC side will not flow into the AC system; it will actually flow into the filter. Though, see we cannot actually eliminate the current harmonics in the converter. There are current harmonics in the converter that cannot be eliminated or the, that cannot be even minimized.

What one can do is try to minimize the currents in the AC system so; that means, where, where will the current harmonics flow it if it has to flow in the converter, it has to flow somewhere on the AC side. So, that that is a path is provided by the filter that is the idea. So, that it does not flow in the so, what I am trying to say is let me try to say here.

(Refer Slide Time: 01:09)



Suppose I have an AC network so, I have a converter here. So, I provide a path here let me elaborate this. So, there are harmonics in the currents on the AC side of the converter. So, if nothing is done and it will flow in the AC network also. So, if I provide a path in the form of a filter ok. So, the currents will actually be supplied I mean it is actually currents will flow through the filter. So, it will not flow through the network that is the idea ok. So, we will look at more details how this can be done.

So, let me take straight away an example a single tuned filter. So, I mean have you heard of single tuned filter by any chance heard of this name single tuned filter or at least guess what can be a single tuned filter?

Student: Single frequency.

Yeah, it is actually designed to allow current at one particular frequency I mean essentially harmonic frequency ok. So, what it what does it contain?

Student: L and C consideration.

L and C. So, it has it should have an inductance a inductor and a capacitor. So, let me draw the circuit, the circuit is like this it has a capacitor in series with an inductor in series with a resistance ok. So, we will see why we need all this. So, there is a capacitor with capacitance C, there is an inductor with inductance L and there is an effective resistance in the circuit which is R. So, this is a single tuned filter now if I want to say eliminate one particular frequency, which we can see should I consider to be most critical?

Student: The lowest order harmonic. Student: Which a lowest order harmonic.

Yeah which one I am talking about AC, AC filter.

Student: Yes.

To filter out the current harmonics.

Student: Eleventh.

Eleventh. Suppose I want to design a filter for eleventh harmonics, then I have to choose the value of L and C. So, that a, this becomes resonant circuit for the, Student: Eleventh.

Eleventh, eleventh harmonic ok. Now can I exactly design the value of L and C so, that it is tuned for the frequency of the harmonic that it is supposed to eliminate or at least supposed to suppress why?

Student: This is impedance of a megahertz that are impedance of minus j o omega C.

Ok.

Student: So, we will amplify L and C such that is to transfer and.

Now, can I always design a single tuned filter; that means, essentially select the values of L and C such that the resonant frequency is equal to say eleventh order, eleventh order harmonic frequency.

Student: Yes.

Yeah theoretically yes, but in practice what can be the problems. So, when I say I have a 50 hertz system the eleventh order harmonic is.

Student: 550.

550 hertz, but frequency is not exactly 50 frequency keeps changing. We know nominal frequency is 50. So, frequency will vary over a band. So, frequency is not exactly 50 means it varies over a small band, the eleventh order will vary over a larger band ok. So, the harmonic frequency is not a fixed number, it is a quantity which is within a certain band that is one thing.

Now, what about the value of L and C though, I select L and C no manufacturer will give an inductor and say the L will remain constant forever at this value. The manufacturer will say L is this value plus or minus by some amount even the capacitance will be this value plus or minus some amount. So, L and C are also some numbers which vary over a certain band ok. So, we cannot say that L will remain constant at this point.

Now, actually when it comes to capacitance the capacitance is more a variable I mean the band within which the capacitance value changes is much larger or much more significant compared to the band over which the inductance changes. Now that is because it is more because of aging I mean as time passes and also the change in temperature capacitance is dependent on temperature.

Now, the temperature can be due to 2 reasons. The ambient temperature itself which has nothing to do with what is going on in the capacitor or due to the heat produced in the capacitor itself ok. So, that also decides the temperature. So, temperature will have an effect on the capacitance. So, essentially C and L are not constants and the frequency itself is not constant of course, R is not affecting any of these things see R is just I mean resistance which is coming there.

We will see why we why we have R here that we will see late. But the variations of L and C will have a problems and the frequency itself is not a constant ok. So, in practice in practice, a filter is not always tuned exactly to the frequency of the harmonic that it is intended to suppress.

So, there are so many reasons. One is there can be the reasons are number 1, the power system frequency may change; power system frequency and hence see I gave an example of just a eleventh harmonic. I mean we can design also design a single tuned filter for thirteenth harmonic, then the thirteenth order, thirteenth order harmonic frequency is 13 into 50 hertz and I mean the band will appropriately increase.

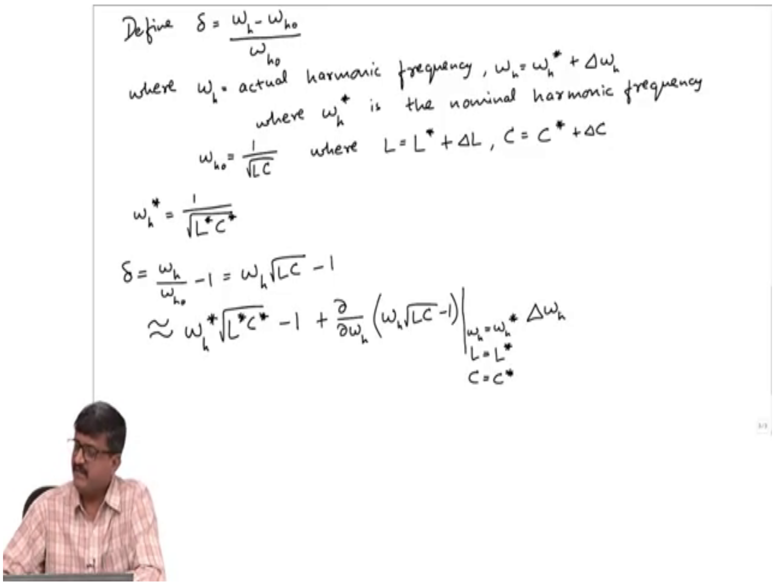
So, the band in which the harmonic frequency stays is getting larger and larger as the 50 the fundamental frequency I mean as you increase the frequency order because of the fundamental frequency itself not being a constant ok. So, power system frequency and hence the harmonic frequency may change, then inductance of the inductor may change, inductance of inductor may change, capacitance of capacitor may change.

In fact, as I said the capacitance change is more significant than inductance change. So, change in capacitance is more significant than the change in inductance. So, this is actually because of aging and because of aging. So, aging means as you keep on using capacitor for more and more time as years passed by it ages ok.

So, the capacitance itself will change and change of and change in temperature. So, change in temperature itself can be either due to ambient temperature or self-heating. So, that is why the capacitance is, the capacitance change is more significant than inductor change, but compared with inductor inductance change or capacitance change the frequency change is the in fact, more significant ok.

So, the system frequency changes are more significant compared to the inductance change or capacitance change the so, the system frequency will have a more effect in say I mean that will result in the filter not being tuned to the frequency at for which it is intended to work. So, what we will try to do is we will try to quantify this deviation in this behaviour of the filter not being exactly tuned ok.

(Refer Slide Time: 12:16)



So, we will some somehow try to quantify this. So, for that we will define a quantity. So, define a quantity called delta. This is defined as omega h minus omega h o divided by omega h o. Now let me say what is this omega h, where omega h is the actual harmonic frequency, which can be written as it is ok. Let me first say what is omega h the actual harmonic frequency. So, from the notation you should be able to make up that it is not frequency in hertz it is angular frequency please, note its angular frequency.

Now, I will write omega h as omega h star plus delta omega h where omega h star is the nominal harmonic frequency; that means, if I want to design the filter for eleventh harmonic in a 50 hertz system, 50 hertz is the nominal value. So, the nominal value of eleventh harmonic is 550 hertz ok. So, where omega h star is the nominal harmonic frequency, then omega h o is there in the definition of delta.

Omega h o by definition is 1 by square root of L C. Now are you familiar with this formula? I do not know you might have come across filters earlier in some course. So, if I want to design the filter for the frequency omega h o which is the angular frequency, then L and C should satisfy this equation omega h o is 1 by root L C, you are familiar with that right ok. Now as I said L and C themselves are not constants.

So, I write this expression for L as so, L itself is not a constant I will say L is equal to a nominal value L star plus delta L and C is equal to C star plus delta C. Now please note this omega h star L star C star are all constants, there are some fixed values. The deviation is in the form of delta omega h for frequency delta L for inductance and delta C for capacitance.

Now, this L star, C star and omega star are actually related so, can I see when everything is ideal omega h star is the nominal value it is a fixed value. So, can I establish a relationship between omega h star L star C star? So, this is exactly equal to 1 by under root L star C star. Now please note omega h o is not a constant; omega h o is not a constant because L is not a constant, C is not a constant in general whereas, omega h star is a constant.

Now, if you look at the definition delta, definition of delta omega h is not a constant omega h o is also not a constant. So, one should not assume that I am taking trying to take the difference between a variable quantity and a fixed quantity; both are variable quantities. So, we say that this delta is a quantification of the behaviour of the filter in not being tuned to the frequency for which it is intended to work ok.

Now, I mean this definition by itself may not be of much use we will try to write it in a slightly different form ok. So, I will write this definition delta as omega h by omega h o minus 1 which can be written as I will substitute the expression for omega h o. So, just now we defined omega h o omega h o is 1 by root L C.

So, I can write this as omega h under root L C minus 1. Now the next thing I am going to do is I will try to expand this right hand side in the expression for delta by using a Taylor series and the Taylor series is about the ideal values omega h star, L star, C star. Now please note delta is dependent on omega h is dependent on L it is dependent on C and all these 3 quantities have an ideal value omega h star, L star C, star ok. So, it is a function of 3 quantities. So, let me try to expand this using Taylor series, but at the same time I will not write all the infinite number of terms.

So, I will restrict it to few terms ok. So, when I do a restriction to a few terms then it becomes only approximate. So, what is the first term? See I am trying to do a Taylor series expansion about the ideal values omega h star, L star, C star. So, the first term will be nothing, but the right hand side itself with omega h replaced by omega h star L replaced by L star, C replaced by C star. So, omega h star under root L star C star minus 1, that is the first term. Then what sort of terms come next?

Student: Partial differentiation of (Refer Time: 18:34).

Yeah partial derivatives with respect to omega h with respect to L with respect to,

Student: C.

C so, then I need a partial derivative with respect to omega h of omega h under root L C minus 1 and this partial derivative is evaluated at.

Student: (Refer Time: 19:04) Omega h.

Omega h equal to omega h star, L is equal to L star C is equal to C star into what?

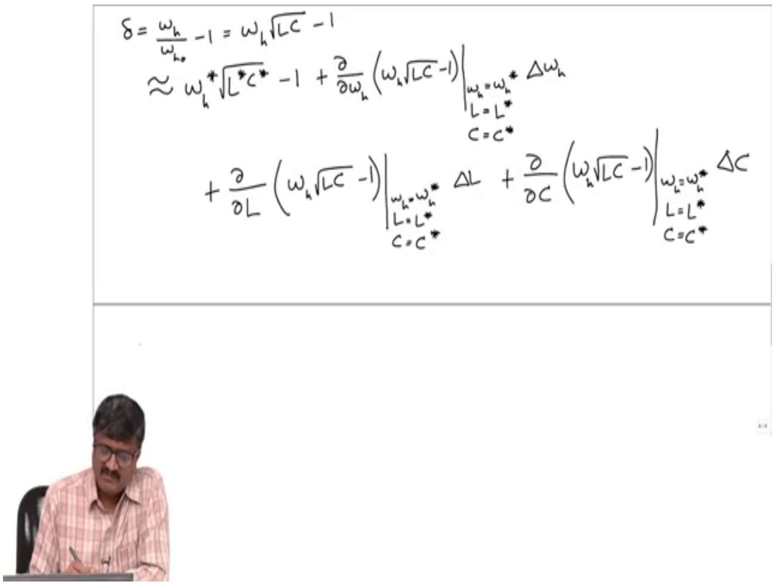
Student: Delta. Student: Delta R.

Yeah delta omega h is nothing, but omega h minus,

Student: Omega h star.

Omega h star so, in so, when I say delta omega h it actually means omega h minus omega h star ok. So, that is the partial derivative with respect to omega h. Similarly we have partial derivatives with respect to L and C.

(Refer Slide Time: 19:42)



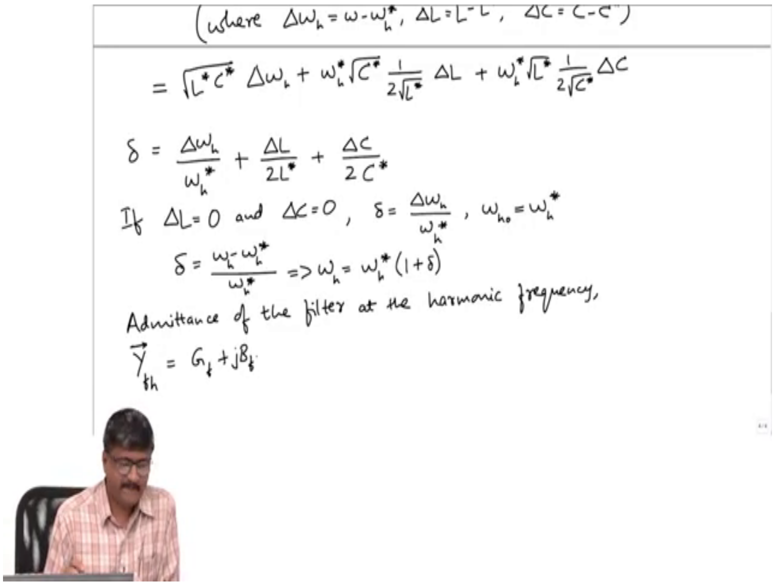
So, the next term is dou by dou L of omega h under root L C minus 1 evaluated at omega h equal to omega h star, L is equal to L star and C is equal to C star into.

Student: Delta L.

Delta L again delta L is L minus L star plus dou by dou C of omega h under root L C minus 1 evaluated at omega h equal to omega h star, L equal to L star C equal to C star into delta C. So, then subsequently, we will have the second order partial derivatives and so on. Now I will restrict myself to first order partial derivative that, that is why I have replaced the equality sign by approximately equal to ok.

So, let us see how to get these partial derivatives.

(Refer Slide Time: 20:55)



So, this is equal to what is the sum of first 2 terms omega h star L star C under root L star C star minus 1 0, it is 0 see just know we saw that omega h star is 1 by under root L star C star. So, their first 2 terms in the expression for delta now will be 0 ok. Now if I take the partial derivative that is the second, I mean the first partial derivative is with respect to omega h.

So, this is equal to so, the partial derivative of omega h under root L C minus 1 is nothing, but under root L C with L L replaced by L star and C taking value of C star into oh into delta omega h. Now please note, I have not explicitly stated here where delta omega h is omega minus omega h star delta L is L minus L star delta C is C minus C star ok.

So, this is the first partial derivative, then the partial derivative with respect to L. So, when I take partial derivative with respect to L the other quantities are assumed to be constants. So, the other quantities are omega h and root C omega h and root C and it is not just taking the partial derivative and evaluating that partial derivative at omega h equal to omega star and C equal to C star then the partial derivative of square root of L is.

Student: 1 by (Refer Time: 22:56).

1 by 2 root L that is evaluated at L star into.

Student: L minus L star.

L minus L star or delta L ok, plus the partial derivative again with respect to C so, omega h star. So, omega h root L will take a constant value that is omega h star L star. So, the partial derivative with respect to C has to be taken for root C. So, it is 1 by 2 root C and it is evaluated at C star into delta C; this is ok.

Now, let us try to simplify this. So, I will write here the first term as delta omega h square root of L star C star can be written as omega h star in the denominator ok. So, square root of L star C star is 1 by omega h star, then what is the simplification of the second term?

Student: Delta L by.

Delta L by 2.

Student: L 2 L star.

2 L star, delta L by 2 L star. So, that is because omega h star is 1 by root L star C star ok. So, similarly the third term is delta C by 2 C star. So, what we have got here is the expression for delta of course, it is an approximate, but for all practical purposes it is the expression for delta that we defined. Now as I said the variation of C that is delta C is more significant when compared with delta l, but when compared with delta omega h the variation of delta L and delta C are actually much smaller ok.

So, the effect of system frequency change will have significant effect on this delta. So, what we will try to do is, we will try to design a filter. In fact, a single tuned filter for a particular harmonic frequency. So, we will try to do a very simplified design, we will make some assumptions. So, this is the general expression for delta, but for the sake of simplifying the design. We will assume that the changes delta L and delta C are negligible ok.

So, let us do a simplified design where I will assume that delta L is equal to 0 and delta C is also equal to 0. Now please note what I have got is the general expression for delta I am just taking a simplified case for the sake of study design which will be much more simplified. So, we will see that this says this I mean in spite of this it will be much I mean it is not. So, easy design is not. So, easy it is a very complicated stuff, then delta can be written as delta omega h by omega h star.

Now, if delta L is 0 and delta C is 0 can I relate omega h o and omega h star look at the definition of omega h o, look at the definition of omega h star, look at the definition of delta L, delta C and suppose delta L is 0 delta C is 0.

Student: (Refer Time: 27:09).

Student: (Refer Time: 27:11).

Sorry.

Student: All are same.

Same.

Student: (Refer Time: 27:13).

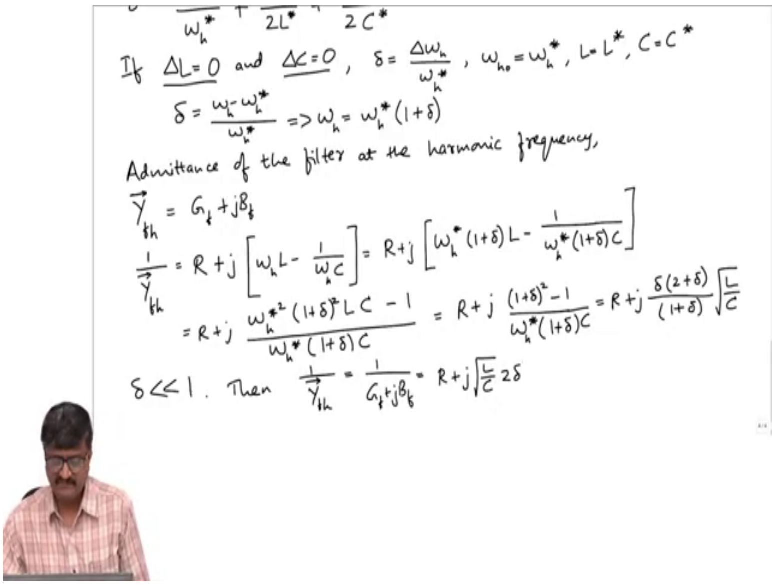
Same ok. So, omega h o is equal to omega h star ok, please note this we get only if delta L is zero delta C is 0 otherwise no otherwise no. Now if I take omega h the actual harmonic frequency then can I ok, now one point to notice that when I say let me write one more step if I take delta omega h is by definition omega minus sorry omega h minus omega h star divided by omega h star. So, that is the by, by using the definition of delta omega h.

Now, just now I said omega h star and omega h o are one and the same. So, I mean I instead of omega h star I could have used omega h o ok. So, from this, I can write omega h as omega h star into 1 plus delta ok. So, we have this filter see filter is very simple. It has an R, it has an L, it has a C all are in series R L C are in series ok.

Now, let us take the single tuned filter and try to write an expression for the admittance of the filter at a particular frequency which is the harmonic frequency. So, admittance of the filter at the harmonic frequency, now please note admittance and impedance are dependent on frequency. So, if I take admittance of the filter Y f and since I am evaluating it at the harmonic frequency I add one more subscript h and these are complex number. So, I just use an arrow to indicate that it is complex ok.

So, this can be written as conductance plus j time’s susceptance ok. So, I will not again use the subscript h its understood ok. So, G f plus G j B f G f and B f are real numbers.

(Refer Slide Time: 30:03)



So, if I take 1 by Y f h I get the impedance. So, what is the impedance, now impedance I can write see, please note G f and B f are not there in the parameter. So, in the figure of the, single tuned filter, but I have said R L and C. So, admittance if I mean the reciprocal of admittance is impedance which is equal to R plus j times the effective reactance. So, what is the effective reactance?

Student: (Refer Time: 30:36).

Yeah. So, omega h

Student: L.

L minus 1 by omega h C so, this can be written as R plus j omega h is just, now I got an expression for omega h it is omega h star into 1 plus delta into L minus 1 by omega h star into 1 plus delta into C. So, this can be written as R plus j times.

So, omega h star square 1 plus delta square L C minus 1 divided by omega h star 1 plus delta C this is. Now is there any simplification possible, yes any simplification possible?

Student: L C is equal to.

L C is equal to

Student: Minus (Refer Time: 32:11).

Yeah please note delta L is 0, delta C is zero. So, if delta L is 0 and delta C is 0 by definition of delta L it means L is equal to L star C is equal to C star that is what we mean by delta L is 0, delta C 0. So, if L is equal to L star and C is equal to C star omega h star is 1 by root L star C star. So, this gets simplified to R plus j 1 plus delta whole square minus 1 divided by omega h omega h star 1 plus delta C ok.

So, this can be written as R plus j. So, if I expand 1 plus delta whole square I get delta into 2 plus delta divided by omega h star. So, I mean I will do one more manipulation here. So, I will write this omega h star into C as what suppose I take 1 plus delta here can I write omega h star into C as.

Student: Under root of C by L.

Under root of C by l, so, can I write this as root L by C, now what is this delta? Delta is some deviation from ideal condition ok. So, if you look at the definition of delta, delta is actually close to what value ok, let us look at the definition of this original definition of delta is omega h minus omega h o by omega h o, then we got many other simplifications. So, compare to one see if you look at this definition of delta omega h by omega h o minus omega h o. So, there are there is a difference of 2 terms both terms are close to.

Student: Each other.

1, see omega h by omega h o is close to 1.

Student: Yes.

Is not 1, but close to 1 omega h o by omega h o is 1 ok. So, I am trying to take difference of 2 terms which are one is one term is 1 and other term is close to 1. So, compare to 1 delta is.

Student: Small.

Small so, that is the point. So, delta is very small compare to 1. So, if that is a case then this impedance that is reciprocal of the complex number Y f h can be written as, that is 1 by G f plus j B f can be written as R plus j. So, what is the amplification root L C is there, delta is very small.

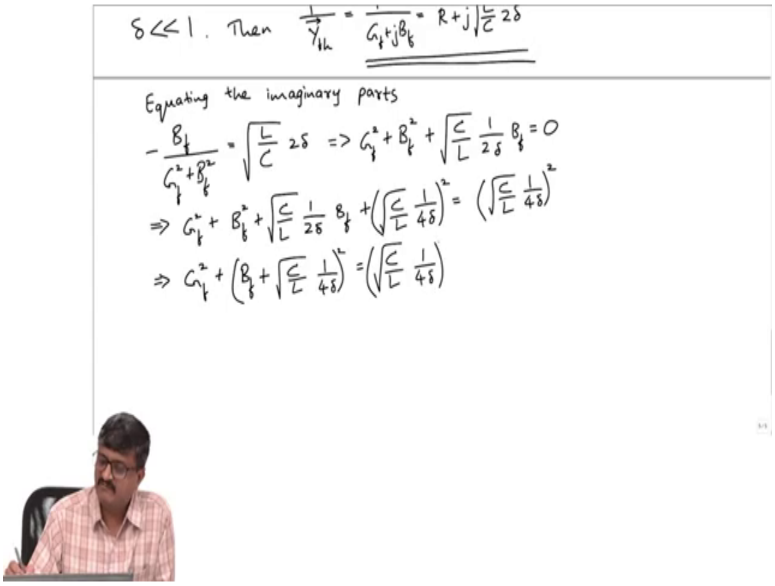
Student: (Refer Time: 35:45).

Into?

Student: 2 delta.

2 delta, 2 delta that is all. Now let me take the imaginary part of this; see the real part of the right hand side is resistance, let me take the imaginary part. So, equate the imaginary parts.

(Refer Slide Time: 36:14)



So, if I am equating the imaginary parts. So, I am taking this equation. This equation 1 by G f plus j B f is equal to R plus j under root L by C into 2 delta. So, what is the imaginary part of the left hand side 1 by G f plus j B f, G f and B f are real conductance and susceptance. So, what is the imaginary part of the left hand side?

Student: Minus B f under root of G f.

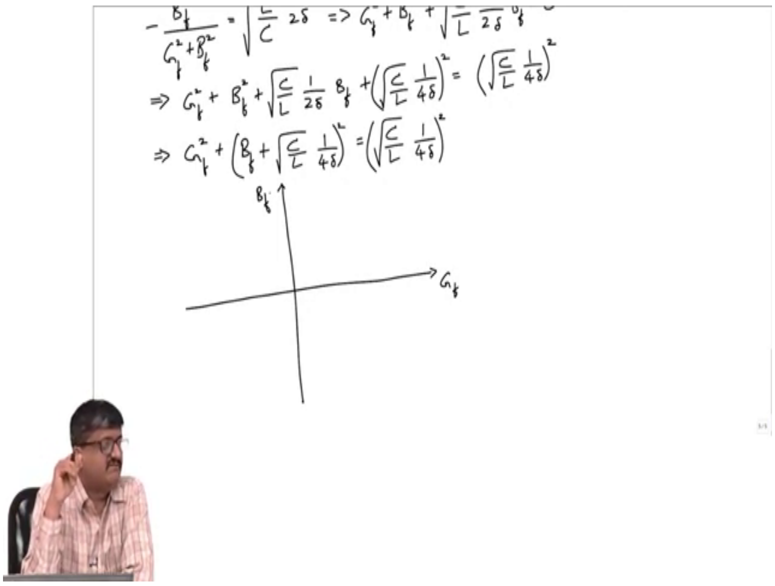
Minus B f divided by G f square plus B f square, so, this is equal to the imaginary part on the right hand side is under root L by C 2 delta. So, I will write this in a slightly different form G f square plus B f square ok, I will shift all the terms to one side. So, this becomes plus root C by L into 1 by 2 delta B f equal to 0 is this.

Now, we will do a few more manipulations I will write this as G f squared plus B f square plus root C by L 1 by 2 delta B f plus. So, I will I will add a term on both sides. So, the term is under root C by L into 1 by 4 delta whole square. So, this is added on both sides. So, what is the purpose?

Student: Getting the square.

Yeah I am actually completing the square. So, G f square plus. So, on the left hand side, what I get is G f square plus B f plus under root C by L into 1 by 4 delta whole square. So, this is equal to under root C by L 1 by four delta whole square.

(Refer Slide Time: 39:18)



Suppose I take the graph on the axis, I have G f, on the ordinate I have B f that is a complex plane with the real part equal to conductance and the imaginary parts equal to susceptance. Now the question is if I take the equation that I just got in terms of G f, B f what is the curve?

Student: (Refer Time: 39:43) its circle.

Circle, it is a circle.