DC Power Transmission Systems
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Lecture – 40
6 pulse LCC with resistance included on the AC side

Let us now, consider a slightly complicated circuit and see how one can use the method of generalize steady state analysis. So, the complication is a minor one it is just adding a resistor on the AC side, but it will result in complication uh; that means, it will result in presence of one term which was not there in the previous case when there was no resistance on the AC side.

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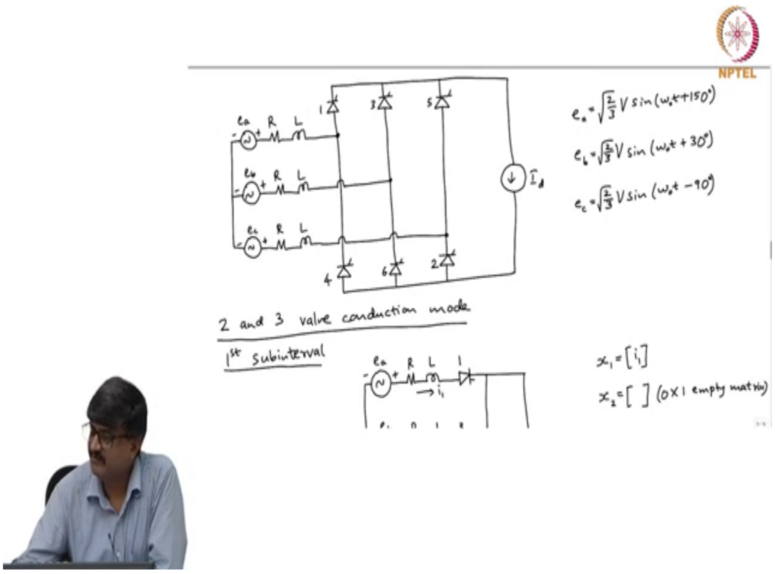

So, we will consider this circuit I have on the AC side a 3 phase balanced voltage source and this is in series with the so in each phase, there is a series resistance and an inductance; R is the resistance, L is the inductance. So, the only addition now is the resistance. So, the rest of the circuit is same as what we consider earlier, we have 3 legs and 2 thyristor valves in each leg, there is no change in the model of the DC side.

So, on the DC side we have a constant current source I d. So, these are valves 1 3 5 4 6 2. So, as in the case of the previous circuit where there was no R here also we can consider two modes of operation 2 and 3 valve conduction mode or 3 and 4 valve conduction mode, suppose I take 2 and 3 valve conduction mode. So, I can take any cycle and each cycle will have 6 intervals and in each interval of duration 60 degrees so I have 2 sub intervals ok.

So, in the 1st sub interval 3 valves conduct and in the 2nd sub interval two valves conduct. So, let me take the 1st sub interval. So, in the 1st sub interval 3 valves conduct. So, if I re-draw the circuit diagram by showing only the valves or only the elements that conduct.

So, among the 6 valves only the valves 1, 2 and 3 conduct and 4, 6 and 5 do not conduct. So, let me just re-draw the same circuit by not showing the valves 4, 5 and 6 that do not conduct.

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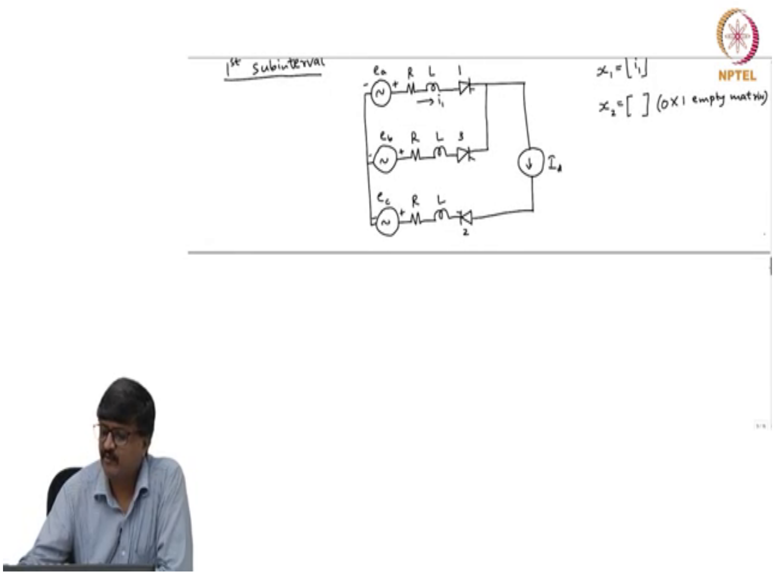

So, valve 1, is in series with the voltage source e a, valve 3, is in series with the voltage source e b, valve 2, is in series with the voltage source e c. So, this is the circuit which is obtained from the previous one you know by just removing the valves that do not conduct that is 4, 5 and 6. So, if I want to solve for the circuit oh I can apply mesh analysis or loop analysis one of currents is known I d is known.

So, if I consider the other loop. So, I have to consider the state variable or the state vector in the 1st sub interval. So, what is the state vector in the 1st sub interval? First of all what is the size of the state vector in the 1st sub interval and what is the state vector in the 2nd sub interval?

So, I mean going by what we studied in the case of a the circuit without resistance we know that in this 2nd sub interval. In fact, valves 2 and 3 conduct and the current is I d which is constant. So, there is no state variable in the 2nd sub interval.

So, x 2 is an empty matrix ;so x 2 is an empty matrix it is 0 cross 1 empty matrix. So, the size of x 1 is 1 cross 1 ok. So, it has 1 element more than that in x 2. So, if it has only one element; which is that element? So, going by our assumption it is the current through the outgoing valve. The outgoing valve is valve 1. So, it is i 1; so, the current i 1 is the element of the state vector x 1. So, how do we get the other quantities? So, let us supply Kirchhoff's voltage law and get the expression for the other quantities. So, if I apply Kirchhoff's voltage law to the loop consisting of e a, e b the 2 resistances R 2 inductances L and the valves 1 and 3, then what do I get?

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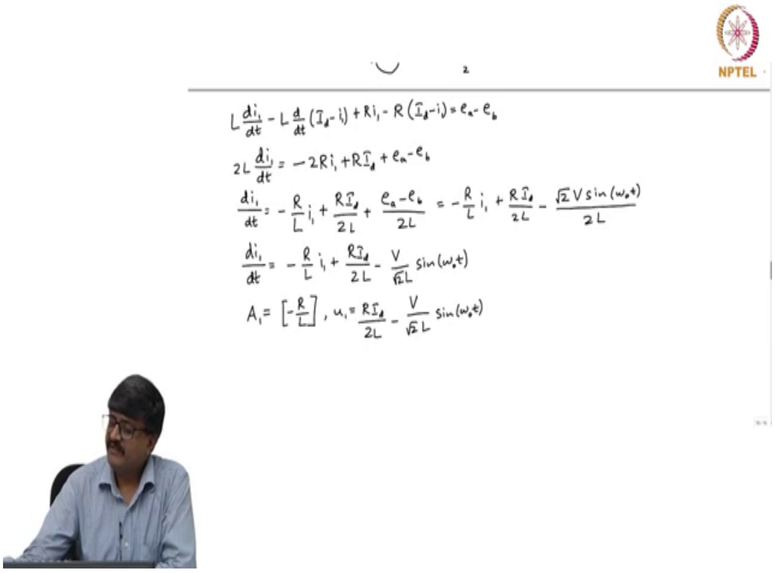

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L d i 1 by d t minus L. So, the current that is flowing here is I d minus i 1 d by d t of I d minus i 1 plus R i 1 minus R I d minus i 1. So, this is equal to e a minus e b. So, this is by applying Kirchhoff's voltage law. So, I want to bring this to the standard form. The standard form is the 1st order derivative on the left hand side; the 1st order derivative of a state variable where state variables on the left hand side. And the remaining quantities on the right hand side. So, i 1 is the state variable.

So, on the left hand side i should have d i 1 by dt. So, if I write this in the standard form d i 1 by d t is equal to. So, let me write a few more steps if I take the first 2 terms of the previous equation I get 2 L di 1 by dt. So, this is equal to so there is R i 1 appearing twice. So, if I push that to the right hand side I get minus 2 R i 1, then minus R id, if I push it that to the right hand side, I get R I d plus e a minus e b. So, in the standard form I get d i 1 by dt is equal to so, divide the entire equation by 2 L I get minus R by L i 1 plus R I d by 2 L plus e a minus e b by 2 L.

So, this is nothing but minus the right hand side is minus R by L i 1 plus R I d by 2 L e a minus e b is root 2 V sin omega o t with a negative sign. So, this is minus root 2 V sin omega o t divided by 2 L. So, I can finally, write this as d i 1 by dt is equal to minus R by L i 1 plus R I d by 2 L minus V by root 2 L sin omega o t. So, this is in the standard form. Now, from this I can say, what is A 1 and what is u 1?

So, A 1 is nothing but the co-efficient of i 1 on the right hand side. So, which is minus R by L and u 1 is nothing but the remaining terms on the right hand side. So, that is R Id by 2 L minus V by root 2 L sin omega o t. So, we have got x 1, we have got x 2, we have got A 1, we have got u 1. So, since x 2 is an empty matrix, A 2 is also an empty matrix.

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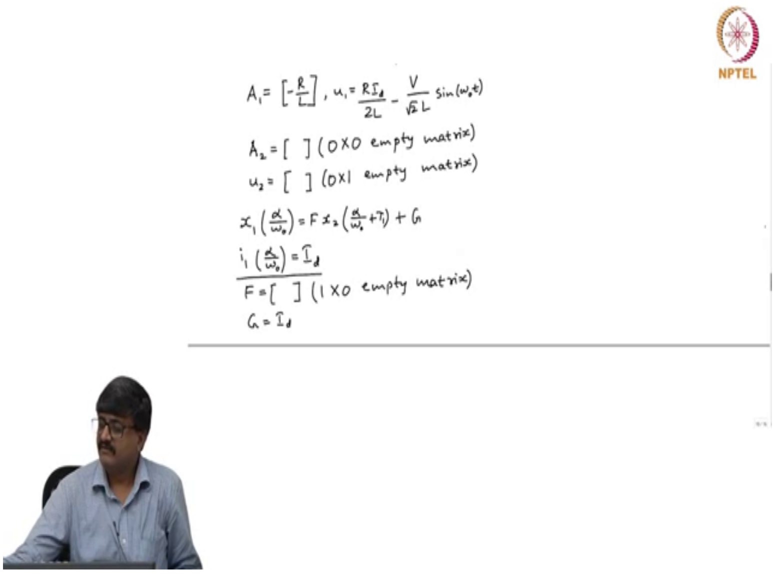

A 2 is a square matrix so, the size of a A 2 is 0 cross 0 and u 2 is also an empty matrix the size is same as that of x 2 so it is 0 cross 1 empty matrix. Now, we have to obtain F and G. So, there is an equation relating F and G, if I take x 1 at alpha by omega o this is equal to F into x 2 at alpha by omega o plus t 1 plus G. So, I have to use this equation and find F and G. So, if I take x 1 at alpha by omega it is nothing, but x 1 is i 1; x 1 is i 1. So, i 1 at alpha by omega o, so what is i 1 at alpha by omega o? So, i 1 at alpha by omega o from this circuit or from this circuit, so valve 3 is turned on at the instant alpha by omega o.

So, till then valves 1 and 2 are conducting. So, the current through valve 1 at alpha by omega o is nothing but I d. So, this is I d and of course, x 2 is empty. So, since x 2 is empty F is an empty matrix. So, the size of a F is 1 cross 0. So, from the size of x 1 and x 2, I can find the size of F.

So, since F is an empty matrix x 2 is also an empty matrix, the product F x 2 is actually a 1 cross 1 matrix which is equal to 0. So, what remains on the right hand side is G with is nothing but from the equation i 1 at alpha by omega equal to I d, I get G equal to I d. So, this is for 2 and 3 valve conduction mode. Now, if I take the same circuit and try to see what are x 1, x 2, A 1, u 1, A 2, u, 2 F and G for 3 and 4 valve conduction mode.

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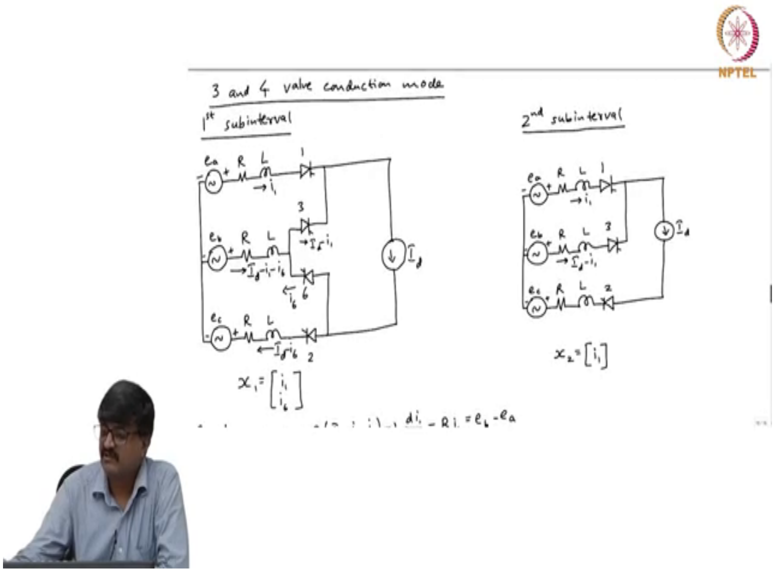

Suppose I consider 3 and 4 valve conduction mode. So, if I take the 1st sub-interval. So, this is similar to the 3 and 4 valve conduction mode of the circuit without resistance. So, the only change now is are the presence of resistance. So, we can see what are the equivalent circuits in the 1st and 2nd sub intervals. So, let us draw the equivalent circuit in the 1st and 2nd sub intervals by just showing those elements which conduct a nonzero current ok. So, in the 1st sub interval valves 6, 1, 2 are already conducting. So, 3 is turned on at the beginning of the 1st sub interval.

So, there are 4 valves conducting in the 1st sub interval that is 6, 1, 2 and 3. So, by circuit is having all the elements in the original circuit, except for the valves 4 and 5. So, valve 1 is in series with e a and valve 2 is in series with e c and both the valves 3 and 6 are conducting. So, this is the circuit for the 1st sub interval, the circuit for a 2nd sub interval is same as the circuit which was used for the 1st sub interval of the 2 and 3 valve conduction mode.

So, let me re-draw that. So, here the circuit is having all the elements except for the valves 4, 5 and 6. This is valve 1, this is valve 3 and this is valve 2. So, we have the equivalent circuit for both the sub intervals. So, can we say, what is the straight vector x 1, what is the straight vector x 2? Now, it is easy to say what is x 2 because we already consider the 2nd sub interval in the case of 2 and 3 valve conduction mode.

Because, the 2nd sub interval equivalent circuit is same as the 1st sub interval equivalent circuit for 2 and 3 valve conduction mode; so, x 2 here is nothing but what we got as x 1 in the case of 2 and 3 valve conduction mode. So, x 2 is nothing but i 1. Now, if x 2 has an element i 1 now that element should be there even in x 1.

So, the elements of x 2 will appear in x 1 so i 1 is there, but there will be one more element which is the last element which is not an element of x 2 and that element is nothing but the current through the outgoing valve. So, in this case the outgoing valve is valve 6. So, the current through the outgoing valve i 6 is the last element of x 1. So, I have i 6 as an element of x 1 and there is i 1 here. So, here I have i 1 and of course, this is I d minus i 1.

So, I got x 1 and x 2. So, the remaining quantities can be easily obtained if I apply Kirchhoff's voltage law so, if I take the equivalent circuit of the 1st sub interval and apply Kirchhoff's voltage law. So, I have to find the 2 currents i 1 and i 6. So, there are three loops one loop current is I d which is known. So, the other loop currents are to be determined. So, if I apply Kirchhoff's voltage law, what do I get?

So, let me 1st take the loop consisting of e a e b the resistance and inductance R and L in series with e a and R L n R and L in series with e b and the valves 1 and 3.

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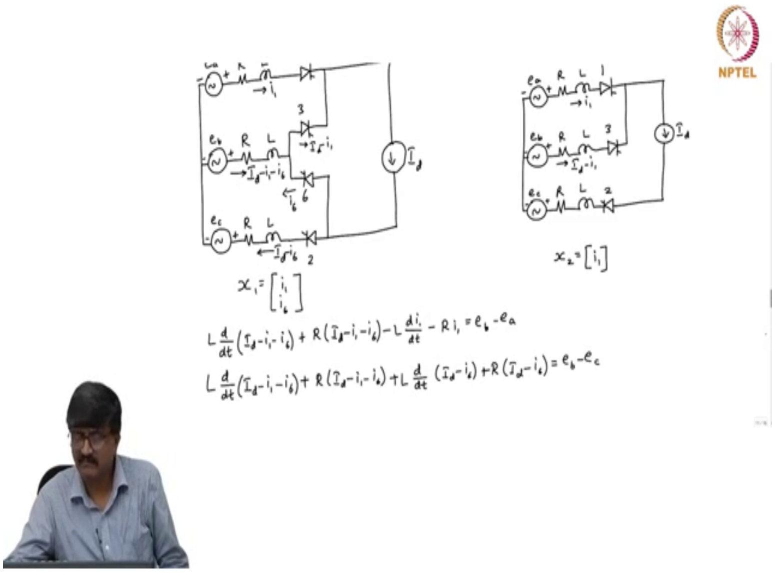

So, if I apply Kirchhoff's voltage law I get L d by dt of. So, I can also show, what is this current. Of course, your current through valve 3 is I d minus i 1. So, current valve 6 is i 6. So, from that I get the current through e b as I d minus i 1 minus i 6. So, L d by dt of id minus i 1 minus i 6 plus R into Id minus i 1 minus i 6 minus L d i 1 by dt minus R i 1 is equal to e b minus e a. So, there is one more loop consisting of a e b e c and the resistance and inductance in series with e b resistance and inductance in series with e c the valves 2 and 6 they are in another loop to which we apply Kirchhoff's voltage law.

So, I get L d by dt of I d minus i 1 minus i 6 plus R I d minus i 1 minus i 6. So, if I look at the current through valve 2. So, this is I d minus i 6. So, minus I have shown the current as entering the positive terminal of e c. So, this is plus L d by dt of I d minus i 6 plus R into I d minus i 6. So, this is equal to e b minus e c. So, I will write this in the form of a vector matrix equation.

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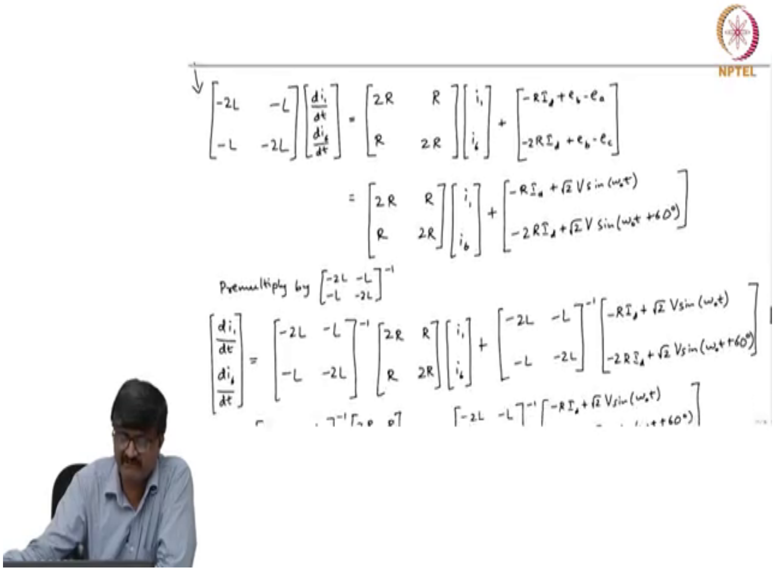

So, I will take a matrix and multiply this by a column vector whose elements are the derivatives first order derivatives of the elements of x 1 that is di 1 by d t and di 6 by dt. So, my intention is to get the equation in the standard form. So, in the standard form I have only the derivative of the state variables on the left hand side ok. So, in both the equations that we have derived so, from these 2 equations I am getting this.

So, I have to see what are the co-efficients of di 1 by dt and di 6 by dt in the 2 equations. So, if I take the 1st equation the co-efficient of di 1 by dt. So, there is minus L di 1 by dt there is one more minus L di 1 by dt. So, the co-efficient of di 1 by dt the 1st equation is minus 2 L, the co-efficient of di 6 by dt is minus L in the 1st equation. Similarly, in the 2nd equation the co-efficient of a di 1 by dt is minus L and the co-efficient of di 6 by dt.

So, there are 2 terms minus L di 6 by dt. So, this is minus 2 L. So, I push all the terms all the other terms to the right hand side. So, what do I get? So, the remaining terms I will split it into 2 categories; one, which involve the state variables i 1 and i 6 and the remain and those which do not involve i 1 and i 6. So, if I 1st take the terms involve in i 1 and i 6. So, in the 1st equation what is the coefficient of i 1?

So, if you look at the 1st equation there is a minus R i 1 and one more minus R i 1. So, if this is push to the right hand side I get 2 R. Similarly, the coefficient of i 6 in the 1st equation, there is a minus R i 6. So, since this comes to the right hand side it is R i 6. In the 2nd equation the co-efficient of i 1 is minus R. So, when taken to the right hand side it is R, the coefficient of i 6 in the 2nd equation is minus 2 R.

So, when taken to the right hand side it is 2 R and the remaining terms the remaining terms are so, there is a R I d in the 1st equation. So, if I push that to the right hand side it is minus R I d then e b minus e a. In the 2nd equation also there is a minus R I d of course, there are 2 places, where I get R I d. So, it is minus 2 R I d plus e b minus e c.

So, I can write this as. So, the right hand side is 2 R R R 2 R i 1 i 6 and if I simplify, after substituting for e a and e b I get minus R I d plus e b minus e a is root 2 V sin omega o t. Of course I have been using the same expressions for e a, e b and e c which I have been using even for the simpler case of a 0 resistance. So, see what I meant was if I look at e a e b, e c I have not change the expressions for e a, e b, e c. They are still root 2 by 3 V sin omega o t plus 150 degrees. This is root 2 by 3 V sin omega o t plus 30 degrees and this is root 2 by 3 V sin omega o t minus ninety degrees. So, with these expressions for e a e b e c e b minus e a is root 2 V sin omega o t and e b minus e c. So, this is equal to root 2 V.

So, e b is root 2 by 3 V sin omega o t plus 30 degrees and e c is root 2 V root 2 by 3 V sin omega o t minus 90 degrees. So, this is root 2 V sin omega o t plus 60 degrees. So, from this I can get A 1 and u 1. So, what I need to do is pre-multiply I am sorry I made a mistake here this is plus I have to pre-multiply both sides of the equation by the inverse of this matrix, square matrix which is appearing on the left hand side so; that means, pre-multiply by minus 2 L minus L minus L minus 2 L inverse. So, I get d i 1 by dt d i 6 by dt on the left hand side.

So, that is what we want on the left hand side which is the standard form this is equal to minus 2 L minus L minus L minus 2 L inverse 2 R R R 2 R into i 1 i 6 that is the 1st term. The 2nd term is minus 2 L minus L minus L minus 2 L inverse minus R I d plus root 2 V sin omega o t minus 2 R I d plus root 2 V sin omega o t plus 60 degrees. So, this is the equation in the standard form. So, from this we can get the required matrices. So, from this I can say, what is A 1, what is u 1? A 1 is nothing but the co-efficient of this column vector i 1 i 6 on the right side.

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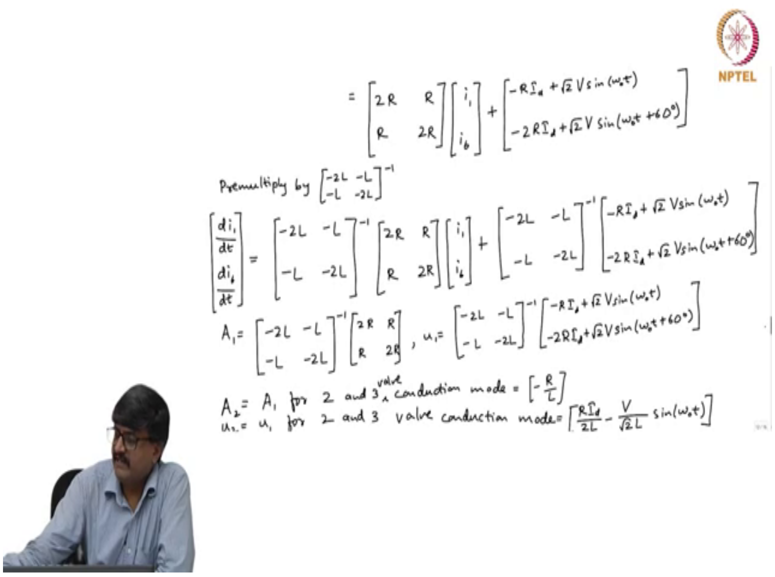

So, A 1 is minus 2 L minus L minus L minus 2 L inverse 2 R R R 2 R and the 2nd term on the right hand side is u 1. So, u 1 is minus 2 L minus L minus L minus 2 L inverse minus R I d plus root 2 V sin omega o t minus 2 R I d plus root 2 V sin omega o t plus 60 degrees. Now, this is A 1 and this is u 1. Now, can I say what is A 2 u 2 by using some previous result? So, A 2 and u 2 are obtained from the circuit diagram applicable for the 2nd sub interval.

Now, this is nothing but the equivalent circuit for the 1st sub interval of 2 and 3 valve conduction mode. So, what we got has A 1 and u 1 for the 2 and 3 valve conduction mode are nothing but A 2 and u 2 for the 3 and 4 valve conduction mode.

So, I can straight away write what is A 2 and u 2 for 3 and 4 valve conduction mode. So, A 2 is equal to A 1 for 2 and 3 valve conduction mode. So, what is that? And u 2 is nothing but u 1 for 2 and 3 valve conduction mode; 2 and 3 valve conduction mode. So, if you look at 2 and 3 valve conduction mode we got u 1 as R I d by 2 L minus V by root 2 L sin omega o t. So, the same thing is valid here R I d by 2 L minus V by root 2 L sin omega o t. So, we have got x 1 x 2 A 1 u 1 A 2 and u 2.

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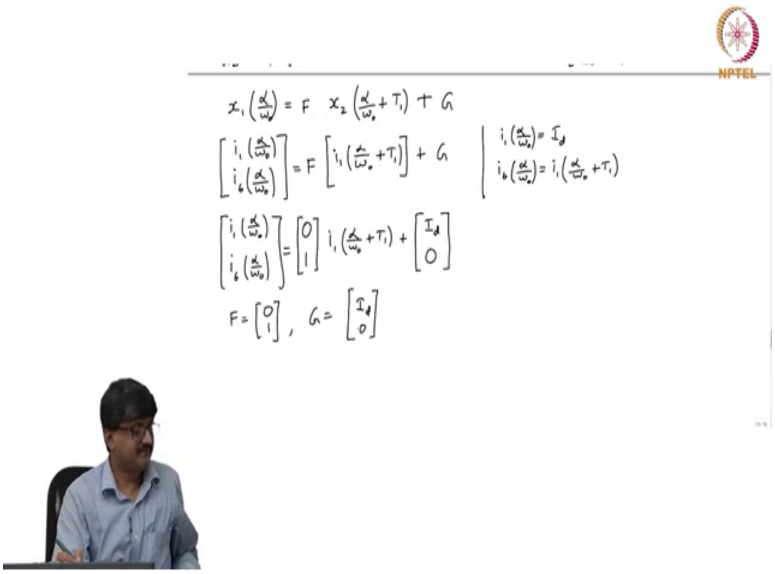

So, the only remaining quantities are F and G. So, again we have to find F and G which relates x 1 at alpha by omega o and x 2 at alpha by omega o plus t 1. So, the equation is this. So, in our case x 1 is i 1 and i 6. So, I want to relate the left hand side which is i 1 at alpha by omega o i 6 at alpha by omega o. So, this is equal to F into x 2; x 2 is nothing but so, what is x 2?

So, just now we saw what is x 2 x 2 is i 1. So, for a 2nd sub interval the only state variable is i 1. So, it is i 1 at alpha by omega o plus t 1 plus G. Now, there are a few things that we know, if I take i 1 at alpha by omega o, what is i 1 at alpha by omega o? See at alpha by omega o 3 starts conducting. So, at alpha by omega o i 1 is nothing but I d this is I d. Now, if I take i 6 at alpha by omega o. Now, I can relate actually i 6 and i 1 see whatever happens to i 6 happens to i 1 after a 60 degrees which is nothing but the upper case t with the subscript 1.

So, I can say that i 6 at alpha by omega o is nothing but i 1 at alpha by omega o plus t 1. So, from these 2 equations I can say that i 1 at alpha by omega o i 6 at alpha by omega o which is there on the left hand side is equal to i 1 at alpha by omega o plus t 1. So, 1 can easily see that the size of f is 2 cross 1. So, where 2 rows and 1 column, the size of G is also 2 cross 1. So, 1 can say that the 1st equation is i 1 at alpha by omega o is nothing but I d. So, I will put that I d in G and 0 as the co-efficient of i 1 at alpha by omega o plus 2.

So, in the 2nd equation i 6 at alpha by omega o is nothing but i 1 at alpha by omega o plus t 1. So, I put A 1 in F and 0 in G corresponding to the 2nd equation. So, we get F as 0 1 and G as I d 0. So, if I have a resistance on the AC side, then the equations are slightly complicated. Now, what is the complication? See if there is no resistance when you would have notice that there is no A 1. So, the coefficient of the state variable on the right hand side is 0.

So, if you take the 3 and 4 valve conduction mode we have a non-zero A 1 non-zero A 2. So, in if there is no resistance so, you can see that if there is no resistance this A 1 and A 2 are 0 matrices. So, that is the simplification that you get by neglecting resistance.