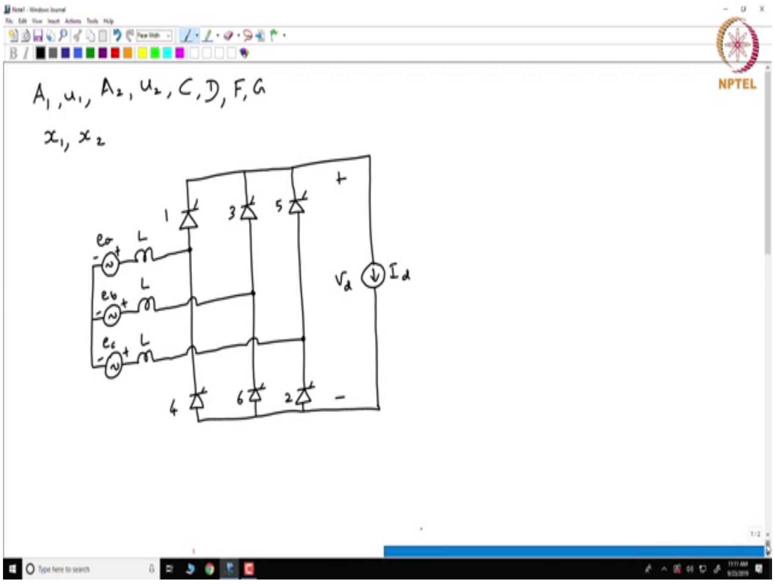
DC Power Transmission Systems

Prof. Krishna S
Department of Electrical Engineering
Indian Institute of Science, Madras

Lecture – 39
Steady state analysis of a general LCC: Application to 6 pulse LCC

(Refer Slide Time: 00:29)



So, you are trying to do what is known as the Steady state analysis of a general convertor. So, we saw that if, we can get some quantities for example, A 1 u 1 A 2 u 2 then C D, we can actually try to we can solve the equations for a given value of alpha. Of course, the other quantities like omega O is known for a system then T 1 is also known.

So, the upper case T with the subscript 1 is known. So, in general, we have to find A 1 u 1 A 2 u 2 C D F G. So, we will try to find the quantities A 1 u 1 A 2 u 2 C D F G. Of course, to find A 1 u 1 A 2 u 2 and of course, the other quantities C D F G, we need to also say what is the vector x 1 what is the vector x 2. So, what are the elements of x 1 and x 2? So, we need to also know x 1 and x 2 ok.

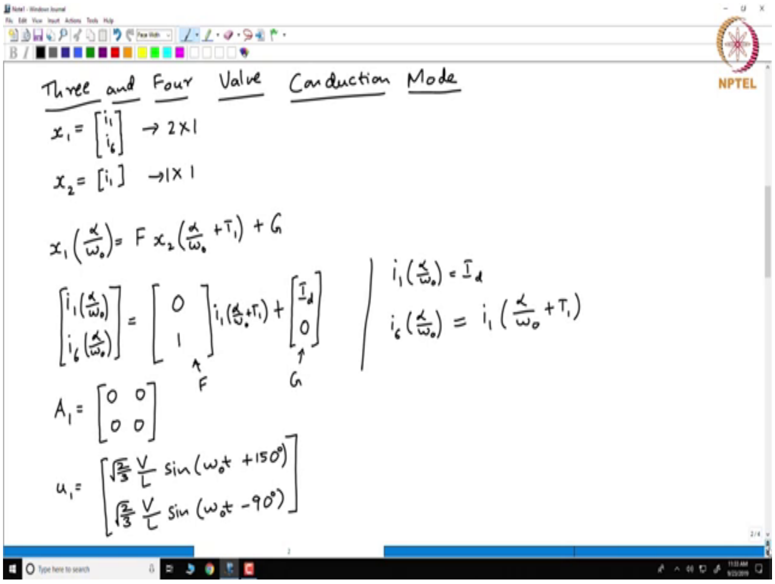
Now, let us take some special cases, I mean the first we will consider the cases that we have already studied. Then take up new cases ok. So, the cases that we have already studied are very straightforward. So, if I take the case that is already considered that is 6 pulse LCC with constant current on the DC side and 3 phase balanced voltage with inductance in each phase on the AC side. Thank you 1, 3, 5, 4, 6, 2.

So, this is our circuit now you would take one case which is already known to us that is say 3 and 4; valve conduction mode.

Student: (Refer Time: 03:35).

So, we will also consider 2 and 3 valve conduction mode.

(Refer Slide Time: 03:40)



So, 3 and 4 valve conduction mode ok. Now, what can be the state vector x 1 and what can be the state vector x 2? See there is.

Student: (Refer Time: 04:26).

One fact about x 1 and x 2 that is x 1 is just, I mean larger than x 2 by 1 ok. So, and the last element of x 1 without loss of generality is the current in the outgoing valve. I mean the current in the valve that stops conducting. So, which valves stop conducting? When I take 3 and 4 valve conduction mode in the first sub interval the 1 valve stops conducting.

Student: 6.

6. So, the second element which is first of all what is the size of x 1?

Student: 4 4.

Size of x 1 is?

Student: 4.

4 it has 4 elements how? How it has 4 elements? How does it have 4 elements? See 4 valves are conducting, but are there 4 state variables. We are familiar with circuit analysis right? I mean you do loop analysis node analysis. Here, we already considered analysis of such circuits I mean we did actually loop formation analysis. So, how many loops are there?

Student: (Refer Time: 05:39).

Student: 3 loops are there.

3 loops are there, but one of the currents is known see Id is a something which is given to us.

Student: Hm.

So, the current source is something which known. So, one of the currents is Id its a loop current which is known. So, there are only?

Student: 2.

2 loops ok. So, I can say that x 1 is having 2 elements. So, I also know that the second element or the last element of x 1 is i 6. What about the first element of x 1? So, the first element of x 1 is also the element of x 2. So, now, x 2 is only having 1 element its this just 1 element. So, how do you take the first element of x 1? See one point to note is that the state

vector is not unique, but based on the fact that the last element becomes a 0 as far as x 1 is concerned we take that as i 6. So, one of the elements of x 1 is fixed. So, the other elements of x 1 are nothing, but elements of x 2, it seems we have the freedom we have a freedom see in general we have a freedom ok.

But for this type of analysis whether do we have a freedom is the question. See there are so, many other quantities to be determined A 1, u 1, A 2, u 2, C D F G. Now there is a one equation which actually relates x 1 and x 2. What is that? In fact, there are two questions one is related through D the other one is related through F and G. So, that gives a hint about what can be the other elements of x 1 and x 2.

In fact, D is also a known matrix D is a known matrix. So, if you take the other equation which relates x 1 and x 2 through F and G that give some hint say. So, I said that I will just assume one equation like this x 1 at alpha by omega o is equal to F into x 2 at alpha by omega o plus T 1 plus G. Where F and G are constant matrices, F and G are constant matrices. Now the question is how to choose x 1 and x 2 such that I satisfy this equation.

So, if I take element wise the element wise there are two elements in x 1 the last element is known i 6 at alpha by omega o this is equal to a matrix F. Now what is the size of this matrix F, see once I know this matrix x 1 is of size 2 cross 1 matrix x 2 is of size 1 cross 1. So, what is the size of F?

Student: 2 cross 6.

Student: 2 cross 1.

2 cross 1. So, it is 2 cross 1. So, this is a 2 cross 1 matrix into x 2 ok. So, I have to find x 2. So, I will fill that x 2 here plus I will multiply this matrix F by x 2 plus G what is the size of G?

Student: 2 cross 1.

2 cross.

Student: 1.

1. Now I have not written x 2 because I have still not written x 1 completely. The first element of x 1 is nothing, but x 2. Now the question is what should be what can be chosen for x 2 which is nothing, but the other element of x 1 the first element of x 1. Say I have to if I assume that the quantity that I choose is one of the currents ok. So, as the element of x 1 the first element of x 1. The first element of x 1. So, that current is evaluated at alpha by omega o plus T 1. So, what is that current?

Student: i 1.

i 1 suppose it is i 1, suppose, it is i 1 then the left hand side is i 1 at alpha by omega o suppose this works. Let us see whether we can get F and G. So, what is the first element of F? What is i 1 at alpha by omega o 3 and 4 valve conduction mode. What is i 1? Id it Id Id. So, it is id. So, then can you tell me what should be the first element of F what should be the first element of G?

Student: (Refer Time: 11:27).

See what I am trying to say is.

Student: (Refer Time: 11:32).

i 1 at alpha by omega o is known quantity that is equal to id. Then can I relate i 6 at alpha by omega o and i 1 at alpha by omega o plus T 1 are they two related. Can I relate i 1 at alpha by omega o and i 1 at alpha by omega o plus T 1 are they two related? See this T 1 is one interval upper case T 1 is one interval that is 60 degrees.

Now, we are doing everything in time because our equations are differential equations, where the independent variable is staying. So, that is why we are changed everything to time for the time being. So, how are these two related? i 6 at alpha by omega o and i 1 at alpha omega o plus T 1.

Student: (Refer Time: 12:28).

Student: A 0 (Refer Time: 12:31).

0.

Student: Both are 0.

Both are 0 see at alpha by omega o what is i 6 I mean is it 0? Now my only my only question is whether these two are related? See one point to notice all the valve currents are identical except for phase shift of?

Student: 60 degrees.

60 degrees in time it is uppercase T 1 60 degrees is one interval upper case T with a subscript 1 is 1 interval. So, whatever happens to i 6 happens to i 1 after 60 degrees or after T 1. So, they are.

Student: Equal.

Equal they are equal.

Student: (Refer Time: 13:11).

Now, from these two equations can I fill the elements of F and G?

Student: (Refer Time: 13:15).

So, the only constraint is that F and G are constraints ok. So, what are the elements of F?

Student: (Refer Time: 13:24).

What is the second element of F?

Student: 1.

1.

What is the second element of G?

Student: 0.

0 that is easy first element of F?

Student: 0.

0 sorry 0 the second element of F.

Student: (Refer Time: 13:42).

Sorry first element of G.

Student: Id.

Id Id is a constant. So, this works so, this choice of x 1 or x 2 works so; that means, I can take this x 1 as i 1 i 6 and x 2 as i 1.

Student: (Refer Time: 14:00) what basis?

This works we do not know whether something else also works. I mean we will be happy as long as one of I mean the one which you are trying works ok. So, one can try to find out if any other choice of x 1 and x 2 will serve the same purpose. See my intention is to relate x 1 at alpha by omega o and x 2 at alpha by omega plus T 1 through F and G where F and G are constants. So, this is something which is actually satisfied always. So, after looking at many cases, I mean people have arrived at this equation.

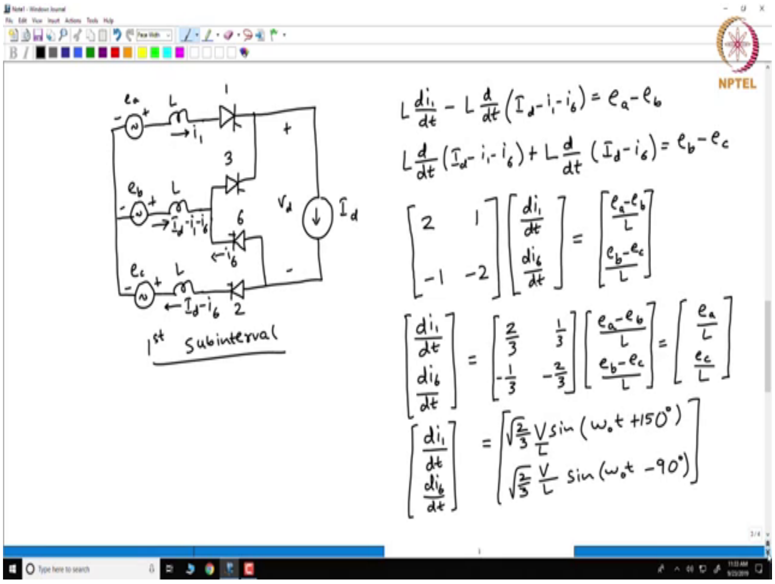
So, I just gave it as just a statement without proving. Now, we saw that it actually get satisfied for 3 and 4 valve conduction mode. So, its not only for 3 and 4, we can see that it also is satisfied for 2 and 3 valve conduction mode and also for other circuits. This is the simplest circuit. Now, let us get the other quantities A 1 u 1 A 2 u 2 and of course, see please note the C and D C and D matrices are actually known in advance if I know what is the size of x 1 what is the size of x 2.

So, C and D is something which is very easy to find out ok. So, C and D is a I will not try to find a C and D, but A 1 u 1 A 2 u 2 at F and G are to be known. So, what is A 1? What is A 1? First of all what is the size of A 1? So, it is of size 2 by 2 ok. If difficult to say what is A 1? What is u 1 that maybe easy?

Student: (Refer Time: 15:56).

What one needs to do is get the equation ok, let us do that see our circuit is like this.

(Refer Slide Time: 16:13)



If I just retain the elements through which there is a non zero current. So, this is the circuit which is applicable for the first sub interval.

So, I have ea eb ec L L L. So, I have valve 1 then below that I have valve 3, then this is valve 6, this is valve 2. So, I have taken one of the currents valve currents that is i 6 as an element of x 1. So, i 6 is one of the elements. And of course, the other element that I have taken is i 1 the other element of x 1 is i 1. So, its just a matter of applying Kirchhoff's voltage law and get the equation that is all ok.

So, I have to get two first order differential equations from that I will get A 1 u 1 A 2 u 2. Of course, A 2 and u 2 are from the second sub interval. So, this circuit is for 1st sub interval. So, this is for the 1st sub interval. So, what are the equations? If I apply Kirchhoff law L d i 1 by d t. So, what is this current? The current which is flowing through or leaving eb from the positive through the I mean for the positive terminal. Because if I want the voltage drop across the inductance in series with eb means I should know the current through eb. So, can I write this current in terms of i 1 and i 6?

Student: Id minus i (Refer Time: 19:01).

Id minus.

Student: In bracket i 1 pus i 6 Id minus Id minus.

Id minus i 1. So, Id minus i 1 gives i 3 minus i 6.

Student: Yes.

So, if I apply Kirchhoff voltage law minus L d by dt of Id minus i 1 minus i 6. So, this is equal to what; equal to what?

Student: a minus eb.

a minus eb.

So, I have to just simplify this. So, get the equation that is all I need not solve. See please note, I have to get the equation in the standard form, see the standard form is a first order differential equation where I have on the left hand side, the first derivative of the state variable with respect to time. The right hand side is actually a linear function of states and input in general ok. So, what I need on the left hand side is the derivatives, but the thing is I will have not in the required form because there are I mean there are both straight variables i 1 and i 6 in the same equation ok.

So, what I will do is I will fist write the other equation also and then try to simplify it. So, if I take the second loop, the second loop apply Kirchhoff's voltage law I get L d by dt of Id minus i 1 minus i 6 minus L what is the current if I take in this direction what is the current here?

Student: Id minus i 6.

I d minus i 6 plus L d by d t of Id minus i 6 this is equal to?

Student: eb minus ec.

eb minus ec. So, this can be written as a matrix times. So, there is both d i 1 by dt di 6 by dt. So, I will write this as d i 1 by dt d i 6 by dt. So, I will push this L on to the right hand side and the right hand side becomes ea minus eb by L and eb minus ec by L ok. So, the coefficients that remain are. So, coefficient of d i 1 by dt on the in the first equation on the left hand side is.

Student: 2 (Refer Time: 22:33).

2 and the coefficient of di 6 by dt is?

Student: 1.

1.

Student: 1.

Then in the second equation the coefficient of d i 1 by dt is?

Student: Minus 1.

Minus 1 and the coefficient of di 6 by dt is?

Student: Minus 2.

Minus 2 ok. So, this is our equation. So, to get it in the standard form pre multiply both sides of the equation by the inverse of this square matrix containing elements 2 1 minus 1 2. So, the standard form is this d i 1 by dt, d i 6 by dt. This is equal to what? What is this matrix? ea minus eb by l. So, eb minus ec by L what is the inverse of this?

Student: Minus 1 (Refer Time: 23:47) into minus 2.

Minus 1.

Student: (Refer Time: 23:50) directly 2 by 3.

2 by 3. So, it is 2 ea minus 2 eb plus eb minus ec what is this?

Student: ea.

ea by L similarly what is the second term? What is minus ea minus a plus eb minus 2 e b plus 2 ec ec by L ok. So, of course, we know what is ea and what is ec of course, if I just substitute that I get this as equal to root 2 by 3 V sin omega o t plus 150 degrees divided by l V by L sin omega o t minus 90 degrees. So, this is the equation. So, this is the right hand side of the equation, the left hand side is of course, d i 1 by dt d i 6 by dt.

Student: Normally.

What is A 1 let us go back to the previous what is 1 1?

Student: 0.

Student: 0

Its a

Student: 0.

0 matrix it is 0 0 0 0 and u 1 is what is there on the right hand side of the equation that we derived. So, it is root 2 by 3 V by L sin omega o t plus 150 degrees root 2 by 3 V by L sin omega o t minus 90 degrees. So, this is u 1. Now, if we want A 2 and u 2 we have to just consider the second sub interval. So, so far what you have done is we have found A 1 u 1, but before that we also found.

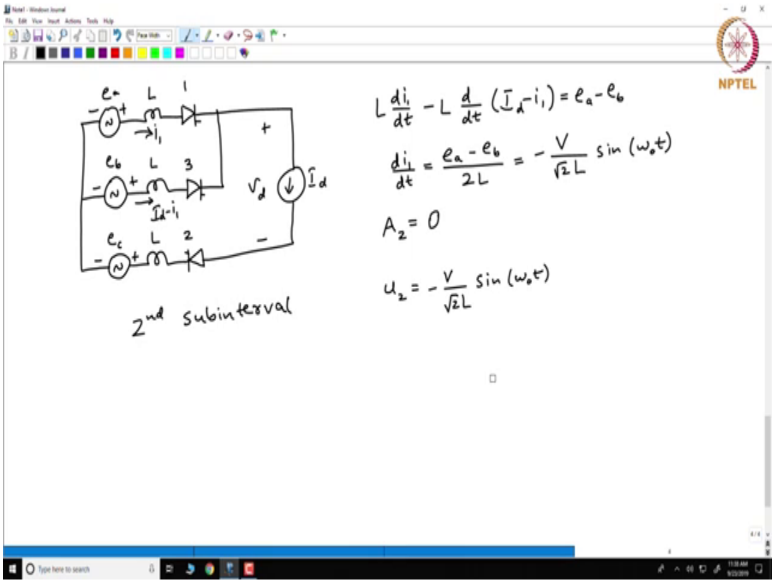
Student: FG.

FG please note this is F this matrix 0 1 is F and this matrix Id 0 is G ok. So, the only thing to be determined is a A 2 u 2 I mean C and D are known matrices once, we know the size of x 1 or x 2 the I mean we know the matrix is C and D. So, what is so, what is A 2 what is u 2 for that we have to consider the.

Student: 2nd sub interval.

2nd sub interval. So, 2nd sub interval.

(Refer Slide Time: 27:20)



So, let us consider a 2nd sub interval. So, if I just show only those elements through which there is a non zero current. So, in the 2nd sub interval, valve 6 is not conducting. So, I have ea eb ec L L L. So, we have valves 1, 3 and 2 conducting. So, in the 2nd sub interval, we have only how many state variables do you have? Only one state variable what is that state variable?

Student: i 1.

i 0. So, I will just show this state variable i 1 here. So, there are two loops one of the loops currents Id is known. So, I have to just write the equation for i 1. So, if I apply Kirchhoff's voltage law I get the equation L di 1 by dt and the current through this inductance connected in series with eb is id minus i 1. So, what I have is l di 1 by dt minus L d by of id minus i 1 is equal to ea minus ed.

So, the standard form is the derivative of the first order derivative on the left hand side, the rest of the terms on the right hand side. So, the I should have only di 1 by dt on the left hand side. So, what is there on the right hand side? ea minus eb divided by 2 L. So, ea minus eb is actually a line voltage. So, I can write this as so, what is ea minus eb?

Student: Minus root 2 V.

Student: Minus root 2 V.

Minus root 2 V so, there is 2 in the denominator. So, I can write this as v by root 2 L with the negative sign sin omega o t. Now can I say what is A 2 and what is u 2? Of course, the size of A 2 is.

Student: 1 by 1.

1 by 1 u 2 is 1 by 1 ok. So, what is u 2?

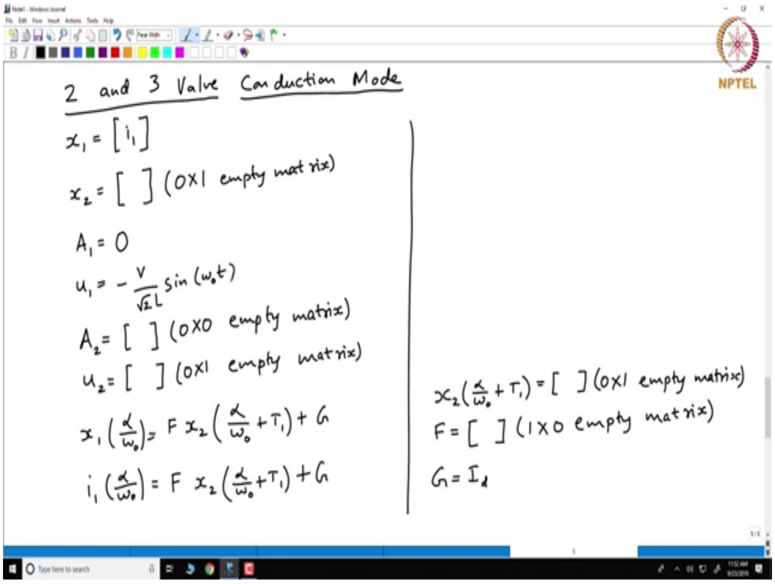
Student: Minus V.

Minus V by root 2 L sin omega o t and A 2 is?

Student: 0.

0. Now, you notice that A 1 is 0 A 2 is 0. Now because A 1 and A 2 are 0 the circuit was easy to analyze. So, we will consider some cases, where A 1 and A 2 are not 0. So, if A 1 and A 2 are not 0 though we know the solution it is a slightly complicated ok. Now before going to other cases ah, where A 1 and A 2 are non zero or let us consider the same circuit, but what we have now, is 2 and 3 valve conduction mode. See already what we see just now, is 3 and 4 valve conduction mode.

(Refer Slide Time: 31:24)



So, if I take the same circuit and consider, 2 and 3 valve conduction mode which is actually the normal mode of operation. Now can you tell me what is x 1, what is x 2? First of all what is the size of x 1? W hat is the size of x 1? Size forget x 1.

Student: (Refer Time: 32:09).

There I mean, if you look at the circuit there are 2 loops 1 loop current is known. So, there is only 1 element. So, it is just 1 element ok.

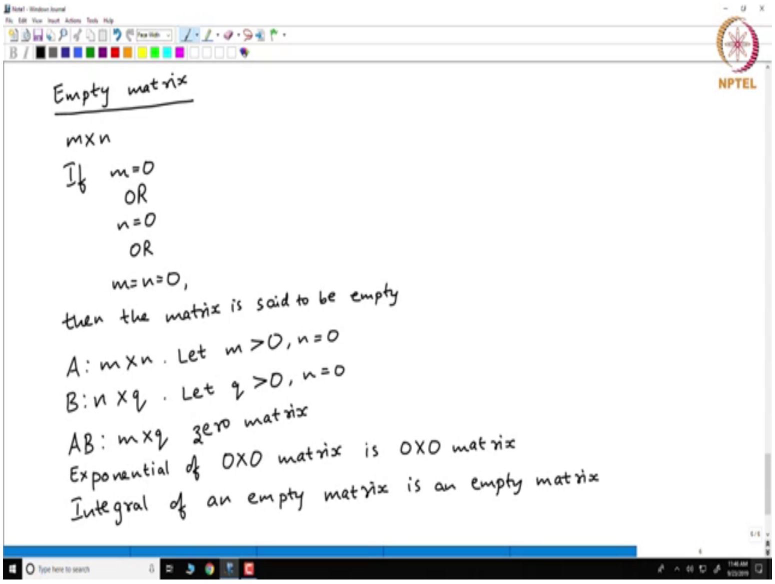
Student: (Refer Time: 32:19).

Then what about x 2?

Student: Does not exist.

Does not exist; does not exist. Now, we will introduce; what is known as empty matrix ok?

(Refer Slide Time: 32:33)



So, I will come back to 2 and 3 valve conduction mode, but I will just deviate by trying to define, I mean this is not something which is defined for this purpose I mean empty matrices are used in other areas also. Now, we know that a matrix is of size say m cross n means it has the m rows n columns. Now in I mean in I mean most of the times m and n are positive integers positive integers.

Now, one of them is 0.Now, if one of them is 0. Now, why do we get one of them 0? We will see that its easy to generalize a few things by taking either number of rows or number of columns as 0. If one of them is 0 or both 0 one of them means one of m either m or n or both are 0, then we get what is known as empty matrix. So, if m is 0 or n is 0 or m and n both are 0, then suppose the matrix is said to be empty.

Now, at this stage its not clear, what is the difference between a matrix which has 0 rows and say 1 column and 0 row and 0 column. Now, it becomes clear only when we consider some special cases ok. So, right now, we will just accept that either a row can be 0 number of rows can be 0 or number of columns can be 0 ok. And both can also be 0 everything is possible.

Now, we know how to do matrix multiplications? Suppose, I have a matrix A of size m cross n. I have a matrix B of size p cross q. Now, suppose p is equal to n. So, I mean let me take that suppose this p is equal to n or B is as good as saying B is n cross q then, I can define the product AB. So, AB is of size.

Student: m cross q m cross q.

m cross q ok. So, we will make some assumption, let m be greater than 0. Let q be greater than 0. Let n be 0 then A is an empty matrix, A is empty matrix because number of the columns is 0. B is also empty matrix because number of rows is 0, but what happens to AB?

Student: (Refer Time: 35:49) non zero.

AB is not a see, if you look at m m is not 0 q is not 0 ok. So, matrix multiplication is still defined if one of the a number of rows or number of columns is 0 as long as the number of columns in the first matrix is equal to the number of rows in the second matrix. That is the only condition required for matrix multiplication. So, here it is satisfied, but its so, happens that n is 0.

So, if n is 0 then we have A and B empty matrices, but AB the product is not empty ok. So, in that case, we define then AB is a m cross q 0 matrix. So, that is the definition. So, if I have if I have 2 empty matrices, but whose product is not empty then by definition its product is 0. So, this is just a definition; just a definition take it as a definition.

Its just definition I mean one need not be even bothered why we are trying to do like that its just a definition ok. So, doing this definition will help us in all the future analysis. Now, we will define a few more things, we see in the solution you see what is known as exponential of a matrix. So, but when it comes to exponential, what we have seen is? We have not considered I mean exponential for any arbitrary matrix. I mean exponential is always for a that is all we know at this stage I mean we are just defined exponential of a square matrix. So, when it comes to exponential of a empty matrix. So, the empty matrix should be.

Student: Square.

Should also be square. See square means number of columns is equal to number of rows. So, the exponential of a 0 by 0 matrix. So, I have an exponential of a square empty matrix is 0 by 0 matrix that is again a definition. So, if I have a square empty matrix the exponential of that is also a square empty matrix. Then in the solution see in the generalized solution, we also have what is known as an integral.

So, there is an integral of empty matrix which can come as a possibility. Say exponential comes in the integral, explanation minus A tau A 1 tau or A 2 tau into u 1 tau or u 2 tau d tau ok. So, if A 1 or A 2 is an empty matrix then exponential by definition is also empty matrix ok. So, if you take the product of that empty matrix with the u 1 or u 2 what do we get? Student: Can we take the product?

We can take product provided u 1 and u 2 should also be.

Student: Also be empty.

Empty ok. So, I mean the matrix dimensions actually match in that way so, we will see that. So, its so, happens that, if exponential of a matrix is empty, its getting multiplied by u 1 or u 2 that is also empty. So, what we get is a resultant column vector which is also empty. So, what is the integral of that see the second term in the solution it involves an integral.

So, by definition the integral of an empty matrix is also an empty matrix. See, we take integral with respect to tau right. So, integral of an empty matrix is an empty matrix ok. Now with this information, can we go back and see what is x 1 and what is x 2 what is x 2?

Student: (Refer Time: 40:16) cross (Refer Time: 40:17) 0 cross 0 cross 1.

Yeah.

Student: (Refer Time: 40:20).

Say by definition, our straight vectors are column matrices. So, it should have one column ok. So, it can be empty means the number of rows should be 0.

Student: 0.

So, x 2 is an empty matrix, I denote a empty matrix just by square brackets without writing anything within the brackets ok. So, I mean if you want you can note down this is a 0 cross 1 empty matrix. What about x 1?.

Student: (Refer Time: 40:52).

No, what can be the element of x 1.

Student: i 1.

i 1 x 1 is just i 1. Then what about A 1 say one thing to note is that in the first interval of 2 and 3 valve conduction mode, we have a circuit which is same as the circuit of the 2nd interval of.

Student: 3 and 4.

3 and 4 valve conduction mode. So, the same equation holds here. I mean for I mean the same expression holds for A 1 and u. 1 See, A 1 and u 1 for this case is same as A 2 and.

Student: u 2.

u 2 for the previous 3 and 4 valve conduction mode that we saw. So, what is A 1?

Student: 0.

0 and we u 1 we just now found.

Student: Minus V by root 2 L.

Minus V by root 2 L sin omega o t. Then what about A 2?

Student: It is empty.

Empty. So, whenever we write empty, I mean better to write the dimension because empty does not just say whether it is 0 rows or 0 columns or both are 0. So, in this case.

Student: 0 cross 0.

0 by 0; 0 cross 0 empty matrix. What about u 2? u 2 is also.

Student: 0 cross 1 empty matrix.

0 cross 1 empty matrix. C and C are obvious I mean once I know the size of x 1 and a size of x 2 C and D are obvious, but what is not obvious is F and G. So, how to find F and G? Say I have the relation x 1 at alpha by omega o is equal to F into x 2 at alpha by omega o plus T 1 plus G ok.

Now, x 1 is nothing, but i 1 is equal to F into x 2 what is x 2? x 2 is empty x 2 is empty. So, it is F into x 2 at alpha by omega o plus T 1 plus G. So, if x is x 2 is empty. So, x 2 itself is empty means, x 2 evaluated at any instant is also empty. So, these are empty matrix. So, this is of size 0 cross 1. So, this is a 0 cross 1 empty matrix. So, what is the size of F?.

Student: 1 plus 0 (Refer Time: 43:56).

So, its also an empty matrix. So, its an empty matrix of size.

Student: 1 cross 0.

1 cross 0. So, now, you see that the first term on the right hand side F is empty x 2 is empty, but product F of F and x 2 is.

Student: 0.

Non empty.

Student: 1 cross 1 (Refer Time: 44:19).

So, it is non empty its 1 cross 1 and by definition.

Student: Its a 0 matrix.

Its a 0. So, here it is just 1 cross 1 its 0. So, i 1 at alpha by omega is just G. So, what is G?.

Student: x 1 at alpha by omega G.

Now, what is that?

Student: Id.

Id.

Student: Id.

Id ok. So, here also you notice that A 1 and A 2 of course, is empty A 1 is 0. So, the analysis was very easy so, far because A 1 and A 2 are actually 0 matrix if not empty. Now, when the A 1 A 2 are not yet 0 matrices do not get confused between 0 and empty 0 is different empty is different ok. The two are different. So, if I have 0 A 1 and A 2 the solution is very easy. So, I have to just integrate that is all I have to integrate something with respect to tau and get here I have to actually use the solution of linear differential equation if A 1 and A 2 are non zero.

So, in the next class, we will consider more complicated circuits for which we will not do any I mean analysis separately, we will just use a general analysis that is applicable for any circuit and try to get x 1 x 2 A 1 A 2 u 1 u 2 F and G ok. So, we will try to consider other complicated circuits and try to do a very generalized analysis without going into specialized analysis for any of the circuits in future.

So, I will stop here we will I mean you consider as more complicated circuit in the next class. One complication comes just by adding a resistance on the AC side. There are some other complications also we will look at those things in the next class.