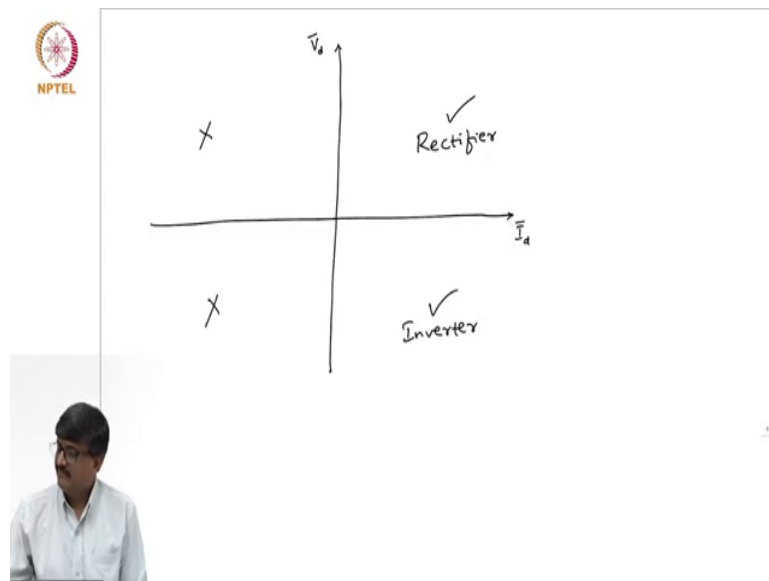


DC Power Transmission Systems
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Lecture - 35
Characteristics of 6 pulse LCC: Part 1

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So, let us try to consider operation in this plane. So, I have a plane, so on the abscissa I have the per unit current I_d bar and on the ordinate I have the per unit voltage unit. So, with the question is can I have operation in only 4 quadrants? So, by all 4 quadrants I mean; is can I have V_d positive as well as negative can I have a I_d positive as well as negative. So, if you have noticed we started with the analysis assuming that I_d , actual I_d is positive. So, I_d bar is also positive.

So, we do not get operation in the second and third quadrant. So, we have operation only in the first quadrant and the fourth quadrant of course, V_d can become negative because negative V_d corresponds to invert operation positive V_d corresponds to rectifier operation.

So, the first quadrant is nothing but, rectifier operation, in the second quadrant corresponds to no sorry, fourth quadrant corresponds to inverter operation. So, let us see; so let us see whether we can try to get some characteristics in this plane, I_d versus V_d . So, for that I need to give some numbers for these equations.

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Summary

$0 < u \leq 60^\circ$

$$V_d = \frac{V_{do}}{2} [\cos \alpha + \cos(\alpha + u)] \Rightarrow \bar{V}_d = \frac{1}{2} [\cos \alpha + \cos(\alpha + u)] \quad \text{---(1)}$$


$$I_d = I_s [\cos \alpha - \cos(\alpha + u)] \Rightarrow \bar{I}_d = \frac{1}{2} [\cos \alpha - \cos(\alpha + u)] \quad \text{---(2)}$$

$$V_d = \frac{V_{do}}{2} [-\cos \gamma - \cos(\gamma + u)] \Rightarrow \bar{V}_d = \frac{1}{2} [-\cos \gamma - \cos(\gamma + u)] \quad \text{---(3)}$$

$$I_d = I_s [\cos \gamma - \cos(\gamma + u)] \Rightarrow \bar{I}_d = \frac{1}{2} [\cos \gamma - \cos(\gamma + u)] \quad \text{---(4)}$$

$$V_d = V_{do} \cos \alpha - R_c I_d \Rightarrow \bar{V}_d = \cos \alpha - \bar{I}_d \quad \text{---(5)}$$


$$V_d = -V_{do} \cos \gamma + R_c I_d \Rightarrow \bar{V}_d = -\cos \gamma + \bar{I}_d \quad \text{---(6)}$$



So, let me call this as equation number 1. The equation in per unit modules, so this is 2, 3, 4, 5, 6. So, I will come to the case of u greater than or equal to 60 and less than equal to less

than 120 degrees later. So, let me first take off this u greater than 0 and less than or equal to 60, for u greater than 0 and less than or equal to 60 degrees.

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For $0 < u \leq 60^\circ$

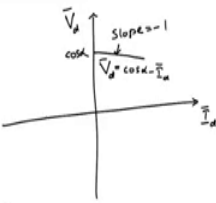

$$\bar{V}_d = \frac{1}{2} [\cos \alpha + \cos (\alpha + u)]$$

$$\bar{I}_d = \frac{1}{2} [\cos \alpha - \cos (\alpha + u)]$$

$$\bar{V}_d = \cos \alpha - \bar{I}_d$$

Constant α operation

For a given α , the equation is that of a straight line (in the graph of \bar{I}_d versus \bar{V}_d)

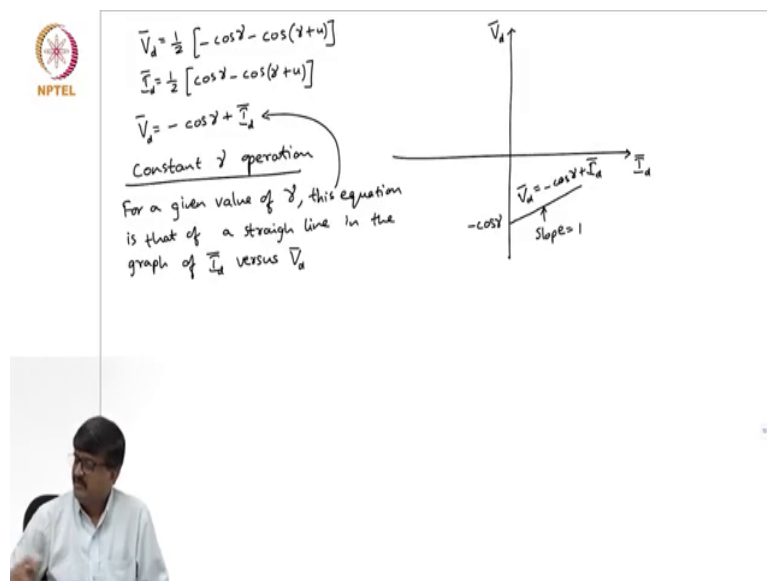
Now, let me go back to these two equations 1 and 2. So, I have equation 1, relating \bar{V}_d α and u and equation 2, relating \bar{I}_d α and u . So, using 1 and 2, so if I want a characteristic in the \bar{V}_d \bar{I}_d plane, I can get rid of say α or u . So, it means we really see that it is easy to get rid of u by adding these two equations. So, if I add these two equations then I get an equation relating \bar{V}_d \bar{I}_d and α .

So, what I am trying to say is let me just take these two equations and rewriting these equations, \bar{V}_d is half $\cos \alpha$ plus $\cos \alpha$ plus, \bar{I}_d is half $\cos \alpha$ minus $\cos \alpha$ plus. So, if I add these two equations, then α is sorry u is eliminated and I get \bar{V}_d equal to $\cos \alpha$ minus \bar{I}_d .

So, if I take the plane in which I have \bar{I}_d on the abscissa and \bar{V}_d on the ordinate. So, if I keep alpha constant; so if I keep alpha constant, then this equation \bar{V}_d equal to cos alpha minus \bar{I}_d is the equation of a straight line. So, I get a straight line and the straight line intersects the \bar{V}_d axis at cos alpha and the slope of the straight line is minus 1.

So, if I keep alpha constant what I get is constant alpha operation. So, it is if I have constant alpha operation then for a given alpha then the equation, I mean I am referring to this equation; the equation is that of a straight line. So, it is a straight line in the graph of \bar{I}_d versus \bar{V}_d instead of equations 1 and 2, suppose I take equations 3 and 4 ok.

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So, let me rewrite those equations 3 and 4. So, I have \bar{V}_d equal to half into minus cos gamma minus cos gamma plus. \bar{I}_d is equal to half cos gamma minus cos gamma plus u.


So, as I did in the previous case, I can actually eliminate u using these two equations. So, if I subtract the second equation from the first equation then I get $V d \bar{=} -\cos \gamma + I d \bar{}$. So, if I take this plane where I have $I d \bar{}$ here and $V d \bar{}$ here. So, if I keep γ constant, then this equation that I have got is the equation of a straight line. So, I have a equation of a straight line. So, the slope of this straight line is plus 1.


So, this slope is equal to 1. So, what I am trying to do is draw the straight line which is represented by this equation $V d \bar{=} -\cos \gamma + I d \bar{}$. So, when $I d \bar{}$ is 0, $V d \bar{}$ is equal to $-\cos \gamma$. So, this straight line intersects the $V d \bar{}$ axis at $-\cos \gamma$. So, what I get by keeping γ constant is constant γ operation.

So that means; if I have a constant γ then the equation that I have is that of a straight line. So, for a given value of γ , this equation is so, I mean this equation is that of a straight line in the graph of $I d \bar{}$ versus $V d \bar{}$. So, this was obtained by eliminating u from two equations.

Now, instead of eliminating u , so in the previous case also we eliminated u and got the relation between $V d \bar{}$ $I d \bar{}$ α . So, here we have eliminated even got a relationship between $V d \bar{}$ $I d \bar{}$ γ . I can take equations 1 and 2 and eliminate α . So, let us see how that can.

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$$\begin{aligned}
 \bar{V}_d &= \frac{1}{2} [\cos \alpha + \cos(\alpha + u)] \\
 &= \frac{1}{2} \left[\cos \left\{ \left(\alpha + \frac{u}{2} \right) - \frac{u}{2} \right\} + \cos \left\{ \left(\alpha + \frac{u}{2} \right) + \frac{u}{2} \right\} \right] \\
 &= \cos \left(\alpha + \frac{u}{2} \right) \cos \frac{u}{2} \\
 \Rightarrow \frac{\bar{V}_d}{\cos \frac{u}{2}} &= \cos \left(\alpha + \frac{u}{2} \right) \checkmark \\
 \bar{I}_d &= \frac{1}{2} [\cos \alpha - \cos(\alpha + u)] \\
 &= \frac{1}{2} \left[\cos \left\{ \left(\alpha + \frac{u}{2} \right) - \frac{u}{2} \right\} - \cos \left\{ \left(\alpha + \frac{u}{2} \right) + \frac{u}{2} \right\} \right] \\
 &= \sin \left(\alpha + \frac{u}{2} \right) \sin \frac{u}{2} \\
 \Rightarrow \frac{\bar{I}_d}{\sin \frac{u}{2}} &= \sin \left(\alpha + \frac{u}{2} \right) \checkmark \\
 \frac{\bar{V}_d^2}{\cos^2 \frac{u}{2}} + \frac{\bar{I}_d^2}{\sin^2 \frac{u}{2}} &= 1
 \end{aligned}$$


So, let me take the equations 1 and 2, \bar{V}_d is half cos alpha plus cos alpha plus. And I have \bar{I}_d equal to half cos alpha minus cos alpha plus u. So, I will write this as half cos alpha as alpha plus u by 2 minus u by 2. Similarly, I will write the second term as cos alpha plus u by 2 plus u by 2. So, this can be should be equal to cos alpha plus u by 2 into cos u by 2.

So, I do similarly ah some manipulations and get an equation relating \bar{I}_d alpha and u. So, I write again this as half cos alpha plus u by 2 minus u by 2, then the second term is minus cos alpha plus u by 2 plus u by 2. So, this can be shown to be equal to sin alpha plus u by 2 sin u by 2. What I do here in the this equations? So, I take a cos u by 2 which is on the right hand side to the left hand side. So, I write this as \bar{V}_d by cos u by two is equal to cos alpha plus u by 2. Similarly, if I take this equation I shift sin u by 2 to the left hand side. So, I get \bar{I}_d by sin u by 2 equal to sin alpha plus u by 2.

Now, I take the square of this equation on both sides and I take the square of this equation on both sides. So, after squaring add the left hand sides of the two equations and the right hand side of the two equations. So, if you take the square of $\cos \alpha + u$ by 2 and add it to the square of $\sin \alpha + u$ by 2, you get one on the right hand side. So, on the left hand side, we get V_d square divided by $\cos u$ by 2 plus I_d square divided by $\sin^2 u$ by two on the right hand side you get. So, if I look at this equation, I have I_d , V_d and u in this equation.

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The slide contains the following content:

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$$\begin{aligned} \bar{I}_d &= \frac{1}{2} [\cos \alpha - \cos(\alpha+u)] \\ &= \frac{1}{2} [\cos \left\{ \left(\alpha + \frac{u}{2} \right) - \frac{u}{2} \right\} - \cos \left\{ \left(\alpha + \frac{u}{2} \right) + \frac{u}{2} \right\}] \\ &= \sin \left(\alpha + \frac{u}{2} \right) \sin \frac{u}{2} \\ \Rightarrow \frac{\bar{I}_d}{\sin \frac{u}{2}} &= \sin \left(\alpha + \frac{u}{2} \right) \end{aligned}$$

$$\frac{\bar{V}_d^2}{\cos^2 \frac{u}{2}} + \frac{\bar{I}_d^2}{\sin^2 \frac{u}{2}} = 1$$

Constant u operation

For a given u , this equation is that of an ellipse in the graph of \bar{I}_d versus \bar{V}_d

The graph shows an ellipse in the \bar{V}_d vs \bar{I}_d plane. The vertical axis is \bar{V}_d and the horizontal axis is \bar{I}_d . The ellipse is centered at the origin. The vertical semi-axis is labeled $\cos \frac{u}{2}$ and the horizontal semi-axis is labeled $\sin \frac{u}{2}$. The axes are marked with 'X' at the ends.

So, if I keep u constant then what I_d gets is constant u operation. So, if u is constant then, this equation is the equation of the curve. So, what is this curve? So, this curve is that of an ellipse. So, for a given u this equation; this equation is that of an ellipse. So, in the graph of V_d in the graph of I_d versus V_d . So, if I take the graph I may have I_d here and V_d here then of


course, if I draw the complete ellipse it is here, it is like this of course, operation is not possible in the second and third quadrant.

So, I am just showing it in the second and third quadrant, but its operation is possible only in the first and fourth quadrants. So, operation is possible here, but not here. So, if I complete the ellipse I get it in all the four quadrants. So, here this distance is $\cos u$ by 2. So, in earlier if I take a line parallel to a $V d$ axis than the distance from the $V d$ axis of this line is \sin equal to. So, what we get is an ellipse, if I have constant operation.


So, point to notice if I have constant α operation then in the graph of $I d$ versus $V d$ I get a straight line with slope minus 1, for constant γ operation I get a straight line with slope plus 1 in the graph of $I d$ versus $V d$ and for constant u operation; I get an ellipse in the graph of $I d$ versus $V d$.

So, these were obtained for u greater than 0 and less than equal to 60 degrees. Suppose, u is greater than or equal to 60 degrees and less than 120 degree, then what happens?

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$60^\circ \leq u < 120^\circ$
Constant α operation
 $\bar{V}_d = \frac{\sqrt{3}}{2} [\cos(\alpha-30^\circ) + \cos(\alpha+u+30^\circ)]$
 $\bar{I}_d = \frac{1}{2\sqrt{3}} [\cos(\alpha-30^\circ) - \cos(\alpha+u+30^\circ)]$
 $\bar{V}_d + 3\bar{I}_d = \frac{\sqrt{3}}{2} \cos(\alpha-30^\circ) + \frac{\sqrt{3}}{2} \cos(\alpha-30^\circ)$
 $\bar{V}_d = \sqrt{3} \cos(\alpha-30^\circ) - 3\bar{I}_d$
For a given α , this is the equation of a straight line
in the graph of \bar{I}_d versus \bar{V}_d



Let us see. So, you can get constant alpha operation constant gamma operation and constant u operation even for this range of. So, how do I get constant alpha operation? So, essentially I am asking what will be the equation of the curve if alpha is kept constant in the graph of I d versus V d.

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$60^\circ \leq u < 120^\circ$

$$V_d = \frac{\sqrt{3} V_{d0}}{2} [\cos(\alpha - 30^\circ) + \cos(\alpha + u + 30^\circ)] \Rightarrow \bar{V}_d = \frac{\sqrt{3}}{2} [\cos(\alpha - 30^\circ) + \cos(\alpha + u + 30^\circ)] \quad (1)$$

$$\bar{I}_d = \frac{I_d}{\sqrt{3}} [\cos(\alpha - 30^\circ) - \cos(\alpha + u + 30^\circ)] \Rightarrow \bar{I}_d = \frac{1}{2\sqrt{3}} [\cos(\alpha - 30^\circ) - \cos(\alpha + u + 30^\circ)] \quad (2)$$


$$V_d = \frac{\sqrt{3} V_{d0}}{2} [-\cos(\beta - 30^\circ) - \cos(\beta + u + 30^\circ)] \Rightarrow \bar{V}_d = \frac{\sqrt{3}}{2} [-\cos(\beta - 30^\circ) - \cos(\beta + u + 30^\circ)] \quad (3)$$

$$\bar{I}_d = \frac{I_d}{\sqrt{3}} [\cos(\beta - 30^\circ) - \cos(\beta + u + 30^\circ)] \Rightarrow \bar{I}_d = \frac{1}{2\sqrt{3}} [\cos(\beta - 30^\circ) - \cos(\beta + u + 30^\circ)] \quad (4)$$

$$V_d = \sqrt{3} V_{d0} \cos(\alpha - 30^\circ) - 3 R_o \bar{I}_d \Rightarrow \bar{V}_d = \sqrt{3} \cos(\alpha - 30^\circ) - 3 \bar{I}_d$$

$$V_d = -\sqrt{3} V_{d0} \cos(\beta - 30^\circ) + 3 R_o \bar{I}_d \Rightarrow \bar{V}_d = -\sqrt{3} \cos(\beta - 30^\circ) + 3 \bar{I}_d$$

Normalization




So, for that I have to consider the equations corresponding to the appropriate range of u that is greater than or equal to 60 degrees and less than 120 degree. So, let me call this as equation number 1 for this relation this is 2, 3, 4. So, if I take equations 1 and 2, I can eliminate u and get an equation relating V_d and I_d then I can see what will be the curve that I get for a given value of α .

So, if I take 1 and 2 add them then what do I get. So, I straight away write the result. So, take equations 1 and 2 if I add equations 1 and 2, then what do I get let me just do the few steps. So, I have constant α operation is might achieve. So, V_d is $\sqrt{3} \cos(\alpha - 30^\circ) - 3 I_d$ when I_d is $\frac{1}{2\sqrt{3}} [\cos(\alpha - 30^\circ) - \cos(\alpha + u + 30^\circ)]$.

So, if I add the two equations of course, I have to multiply one of the equations by an appropriate factor and then add so, that you get (Refer Time: 19:49). So, if I take the left hand side of the first equation and add it to 3 times the left hand side of the second equation. So, I get root 3 by 2 cos alpha minus 30 degrees. And if I multiply the second equation by three and the left hand side I have to of course, multiply the second equation right hand side also by 3. So, the second terms on the right hand side gets canceled get canceled. So, what remains is plus root 3 by 2 cos alpha minus 30 degrees.

So, the equation that I get is V_d equal to root 3 cos alpha minus 30 degree minus 3 I_d . So, for a given alpha this is the equation of a straight line with slope minus 3 in the graph of I_d versus V_d . So, for a given alpha this is the equation of a straight line in the graph of I_d versus V_d . So, similarly, we can get what is known as constant δ operation.

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$$\bar{I}_d = \frac{1}{2\sqrt{3}} [\cos(\alpha-30^\circ) - \cos(\alpha+30^\circ)]$$

$$\bar{V}_d + 3\bar{I}_d = \frac{\sqrt{3}}{2} \cos(\alpha-30^\circ) + \frac{\sqrt{3}}{2} \cos(\alpha-30^\circ)$$


$$\bar{V}_d = \sqrt{3} \cos(\alpha-30^\circ) - 3\bar{I}_d$$

For a given α , this is the equation of a straight line in the graph of \bar{I}_d versus \bar{V}_d

Constant δ operation

$$\bar{V}_d = -\sqrt{3} \cos(\delta-30^\circ) + 3\bar{I}_d$$

For a given δ , this is the equation of a straight line in the graph of \bar{I}_d versus \bar{V}_d




So, if I want constant gamma operation, then I have to go that to do set of equations. So, the equations corresponding to u greater than or equal to 60 degrees and less than 120 degrees, I have to take equations 3 and 4 and then eliminate u from the right hand side of these two equations.

So, you can do some manipulations and try to show that we can get a relation between V_d , I_d and γ ok. So, write straight away the relation I leave it to you to see how can derive this. So, the equation that we get is $V_d \bar{I}_d = \sqrt{3} \cos(\gamma - 30^\circ) + 3 I_d$.

So, if I take a constant gamma, then this is the equation of again a straight line in the graph of I_d versus V_d . So, for a given value of gamma this is the equation of a straight line in the graph of I_d versus V_d . So, as in the case of u greater than 0 and less than or equal to 60 degrees, here also, we can get the constant u operation. So, let us see complicate constant u operations.

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Constant u operation

$$\bar{V}_d = \frac{\sqrt{3}}{2} [\cos(\alpha - 30^\circ) + \cos(\alpha + u + 30^\circ)]$$


$$= \frac{\sqrt{3}}{2} \cos\left(\alpha + \frac{u}{2}\right) \cos\left(\frac{u}{2} + 30^\circ\right)$$

$$\Rightarrow \frac{\bar{V}_d}{\frac{\sqrt{3}}{2} \cos\left(\frac{u}{2} + 30^\circ\right)} = \cos\left(\alpha + \frac{u}{2}\right) \quad \checkmark$$

$$\bar{I}_d = \frac{1}{2\sqrt{3}} [\cos(\alpha - 30^\circ) - \cos(\alpha + u + 30^\circ)]$$

$$= \frac{1}{\sqrt{3}} \sin\left(\alpha + \frac{u}{2}\right) \sin\left(\frac{u}{2} + 30^\circ\right)$$

$$\Rightarrow \frac{\bar{I}_d}{\frac{1}{\sqrt{3}} \sin\left(\frac{u}{2} + 30^\circ\right)} = \sin\left(\alpha + \frac{u}{2}\right) \quad \checkmark$$



So, if I want constant u operations; so let me go back to this equations. So, I have to again consider equations 1 and 2 and eliminate alpha. So, let me see, how we can do that. So, I have equations 1 and 2; equation 1 is V_d equal to $\frac{\sqrt{3}}{2} \cos(\alpha - 30^\circ) + \cos(\alpha + u + 30^\circ)$. And the second equation is I_d equal to $\frac{1}{2\sqrt{3}} [\cos(\alpha - 30^\circ) - \cos(\alpha + u + 30^\circ)]$. Now, I leave it to you to show that this is equal to $\frac{\sqrt{3}}{2} \cos(\alpha + \frac{u}{2}) \cos(\frac{u}{2} + 30^\circ)$.


Similarly, this can be shown to be equal to $\frac{1}{\sqrt{3}} \sin(\alpha + \frac{u}{2}) \sin(\frac{u}{2} + 30^\circ)$. So, if I want to eliminate alpha then I will retain the factor involving alpha on the right hand side and push the other term other factor to the left hand side and similarly, in the second equation also I will retain $\sin(\alpha + \frac{u}{2})$ on the right hand side and push the

other factors on to the left hand side. So, from this I get \bar{V}_d divided by $\sqrt{3} \cos u$ by 2 plus 30 degrees. So, this is equal to $\cos(\alpha + u)$ by 2.

So, in this equation also I push the terms on the right hand side, I push the factors on the right hand side other than $\sin(\alpha + u)$ by 2 to the left hand side. So, I will get \bar{I}_d divided by 1 by $\sqrt{3} \sin u$ by 2 plus 30 degrees. So, this is equal to $\sin(\alpha + u)$ by 2. So, if you look at either this equation or this equation on the left hand side I want the α and on the right hand side I have in the first equation $\cos(\alpha + u)$ by 2, the second equation I have $\sin(\alpha + u)$ by 2.

So, if you square both sides of these two equations and add them. So, the right hand side you get 1 and the left hand side you get an expression involving \bar{V}_d , \bar{I}_d and u .

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$$\Rightarrow \frac{\bar{V}_d}{\sqrt{3} \cos(\frac{u}{2} + 30^\circ)} = \cos(\alpha + \frac{u}{2}) \quad \checkmark$$


$$\bar{I}_d = \frac{1}{2\sqrt{3}} [\cos(\alpha - 30^\circ) - \cos(\alpha + u + 30^\circ)]$$

$$= \frac{1}{\sqrt{3}} \sin(\alpha + \frac{u}{2}) \sin(\frac{u}{2} + 30^\circ)$$

$$\Rightarrow \frac{\bar{I}_d}{\frac{1}{\sqrt{3}} \sin(\frac{u}{2} + 30^\circ)} = \sin(\alpha + \frac{u}{2}) \quad \checkmark$$

$$\frac{\bar{V}_d^2}{3 \cos^2(\frac{u}{2} + 30^\circ)} + \frac{\bar{I}_d^2}{\frac{1}{3} \sin^2(\frac{u}{2} + 30^\circ)} = 1$$

For a given u , this is the equation of an ellipse in the graph of \bar{I}_d versus \bar{V}_d .



So, if I do that I get $V d^2$ square divided by $3 \cos^2 u$ by 2 plus 30 degrees plus $I d^2$ square divided by 1 by $3 \sin^2 u$ by 2 plus 30 degrees equal to. So, if u is constant then this is the equation of an ellipse in the graph of $I d$ versus $V d$.

So, for a given u , this is equation of an ellipse in the graph of $I d$ versus $V d$. So, we can get the different characteristics such as constant α , constant γ , constant u , either for the case of u greater than 0 and less than or equal to 60 degrees or for the case of u greater than or equal to 60 degrees and less than 120 degrees. So, I stop here and we look at the other aspects of the characteristics in this graph of $I d$ versus $V d$ in relations.