


DC Power Transmission Systems
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Lecture - 32
3 valve conduction mode of 6 pulse LCC

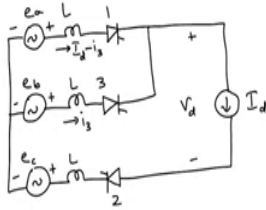
So, what we have seen so far is two and three valve conduction mode and three and four valve conduction mode.

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Three and Four Valve Conduction Mode, $u = 60^\circ$

Suppose $u = 60^\circ \rightarrow$ Three Valve Conduction Mode
 One cycle = 6 intervals each duration 60° .
 Interval $\alpha < \omega t < \alpha + 60^\circ$



$$L \frac{di_s}{dt} - L \frac{d}{dt}(I_d - i_s) = e_b - e_a$$

$$2L \frac{di_s}{dt} = \sqrt{2} V_s \sin(\omega t)$$


$$i_s(\alpha) = 0$$

$$i_s = \frac{V}{\sqrt{2} \omega L} [\cos \alpha - \cos(\omega t)]$$

$$i_s(\alpha + 60^\circ) = I_d$$

$$I_d = \frac{V}{\sqrt{2} \omega L} [\cos \alpha - \cos(\alpha + 60^\circ)]$$

$$I_d = I_s [\cos \alpha - \cos(\alpha + 60^\circ)]$$



So, for two and three valve conduction mode; the overlap angle u is greater than 0 and less than 60 degrees, for three and four valve conduction mode u is greater than 60 degrees. Now, we will see what happens if u is exactly 60 degrees. So, suppose u is 60 degrees.

Now, one can see what is the number of valves that is conducting at any instant. So, if u is less than 60 at any instant either two valves conduct or three valves conduct, if u is greater than 60 at any instant either three valves conduct or four valves conduct. When u is exactly 60, then at any instant we have only three valves conducting. So, what we get for u equal to 60 is three valve conduction.

So, we can try to do the analysis as we did for the previous few cases. So, in order to show a steady state analysis we need to just consider only one cycle in fact, then only one interval in fact. So, we have one cycle of the ac side voltage or current equal to 6 intervals. So, the 6 intervals are identical I mean; in which the each interval is of duration 60 degrees, each of duration 60 degrees. So, it is sufficient to consider one interval for the sake of steady state response.

So, let us consider one interval. So, I will always start with the interval which starts at we will always use the interval which starts at α . So, let us consider the interval; α less than ωt , less than $\alpha + 60$ degrees. So, in order to do the analysis for this interval I need to consider the equivalent circuit. So, let me show the equivalent circuit which contains only those element which carry non-zero part.

So, I have an equivalent circuit with three sources, voltage sources, e_a , e_b , e_c . So, I have valve 1 in series with e_a , valve 3 in series with e_b , in series with e_c , I have valve 2. The DC side voltage instantaneous voltages V_d and the current that is flowing to valve 3 is I_d , the current through valve 1 is i_1 which is nothing but I_d minus i_3 . So, let us try to apply Kirchhoff's voltage law to the loop containing e_a , e_b the two inductances l the valves 1 and 3.

So, by Kirchhoff's voltage law I get $\frac{d i_3}{dt} - L \frac{d}{dt} (I_d - i_3) = e_b - e_a$. Since I_d is a constant left hand side is $2 L \frac{d i_3}{dt} = e_b - e_a$. Since I_d is a constant left hand side is $2 L \frac{d i_3}{dt} = e_b - e_a$. Since I_d is a constant left hand side is $2 L \frac{d i_3}{dt} = e_b - e_a$. Since I_d is a constant left hand side is $2 L \frac{d i_3}{dt} = e_b - e_a$. So, we can solve this equation and we know the initial condition. So, first of all this equation is applicable for ωt greater than α and less than $\alpha + 60$


degrees. So, we know that i_3 at α is 0. We get the solution as i_3 equal to V by root 2 $\omega_0 L$ into $\cos \alpha$ minus $\cos \omega_0 t$.

So, this is the expression for i_3 from α to $\alpha + 60$ degrees. At $\alpha + 60$ degrees, i_3 reaches the value I_d . So, i_3 at $\alpha + 60$ degrees is equal to I_d . So, if I substitute $\omega_0 t$ equal to $\alpha + 60$ degrees on the right hand side of previous equation and i_3 is equal to I_d then I get I_d is equal to V by root 2 $\omega_0 L$ into $\cos \alpha$ minus $\cos \alpha + 60$ degrees.

So, root this V by root 2 $\omega_0 L$ as a notation which is I_s . So, we have defined a current I_s . So, I get I_d is equal to I_s into $\cos \alpha$ minus $\cos, \alpha + 60$ degrees. Now, we got a relationship between I_d I_s α and of course, u for the two and three valve conduction mode and three and four valve conduction. Now, in this case so for three valve conduction mode there is no u of course, u is exactly equal to 60. So, the relation is between I_d I_s and α .

Now, let us see whether these expressions for I_d in terms of I_s α and u for the three different operating modes are somehow related. So, let us take a equations that we have already obtained.

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For 2 and 3 valve conduction mode,

$$I_d = I_s [\cos \alpha - \cos(\alpha + u)], \quad 0 < u < 60^\circ \quad \text{--- (1)}$$

For 3 and 4 valve conduction mode,

$$I_d = \frac{I_s}{\sqrt{3}} [\cos(\alpha - 30^\circ) - \cos(\alpha + u + 30^\circ)], \quad 60^\circ < u < 120^\circ \quad \text{--- (2)}$$

For 3 valve conduction mode


$$I_d = I_s [\cos \alpha - \cos(\alpha + 60^\circ)], \quad u = 60^\circ \quad \text{--- (3)}$$

$$\lim_{u \rightarrow 60^\circ} I_s [\cos \alpha - \cos(\alpha + u)] = I_s [\cos \alpha - \cos(\alpha + 60^\circ)] \quad \text{--- (4)}$$

$$\lim_{u \rightarrow 60^\circ} \frac{I_s}{\sqrt{3}} [\cos(\alpha - 30^\circ) - \cos(\alpha + u + 30^\circ)] = I_s [\cos \alpha - \cos(\alpha + 60^\circ)] \quad \text{--- (5)}$$

$$I_d = I_s [\cos \alpha - \cos(\alpha + u)], \quad 0 < u \leq 60^\circ \quad \text{--- (6)}$$

$$I_d = \frac{I_s}{\sqrt{3}} [\cos(\alpha - 30^\circ) - \cos(\alpha + u + 30^\circ)], \quad 60^\circ \leq u < 120^\circ \quad \text{--- (7)}$$



So, for 2 and 3 valve conduction mode, we got an equation relating I_d I_s α and u . So, the equation that we got was I_d is equal to $I_s \cos \alpha$ minus $\cos \alpha$ plus u . So, since this is applicable for 2 and 3 valve conduction mode it is as good as saying these applicable for u greater than 0 and less than 60 degrees.

Similarly, we got an equation for 3 and 4 valve conduction mode. So, the equation is I_d is equal to I_s by root 3 into $\cos \alpha$ minus 30 degrees, minus $\cos \alpha$ plus u plus 30 degrees. So, this is applicable for u greater than 60 degrees and less than 120 degrees. So, the equation that we obtained just now that is I_d equal to $I_s \cos \alpha$ minus $\cos \alpha$ plus 60 degree for the 3 valve conduction mode is for u equal to 60 degrees. So, let us write that also for 3 valve conduction mode, I_d is $I_s \cos \alpha$ minus $\cos \alpha$ plus 60 degrees. This is for u equal to 60 degrees.

Now, let me see whether the equation that we obtained for 2 and 3 valve conduction mode is applicable for u equal to 60. So, to be it to be noted that its applicable for u less than 60, but is it valid for u equal to 60 it can be shown that it is true; suppose I take limit of by right hand side in the equation which gives the expression for I_d for u less than 60 degrees.

So, if I take limit of $I_s \cos \alpha - \cos \alpha + u$. So, if I take the limit as u tends to 60 degrees. No, it is easy to see that this is equal to $I_s \cos \alpha - \cos \alpha + 60$ degrees. So, the equation which is derived for u less than 60 degrees is actually applicable for u equal to 60 degrees also.


Similarly, if I take the expression for I_d in terms of $I_s \alpha$ and u for u greater than 60 degrees, so; that means, a limit of I_s by $\sqrt{3} \cos \alpha - 30$ degrees minus $\cos \alpha + u + 30$ degrees. So, if I take the limit of this expression as u tends to 60 degrees. So, this to be noted that this expression is valid for u greater than 60; so, we are trying to show that it is valid even u is equal to 60. So, it can be proved that is also equal to $I_s \cos \alpha - \cos \alpha + 60$ degrees. So, this is not as early as in the previous case, but I have bring you to show that this is true.

So, whatever we got for u less than 60 degrees and u greater than 60 degrees is applicable for u equal to 60. So, what one can say is I can have this equation which was earlier valid for u greater than 0 and less than 60 degrees to be true even for u equal to 60 degrees; that means, this is valid for u greater than 0 and less than or equal to 60 degrees. So, please note there is a difference between this equation one let me call this equation 1, this is equation 2 and say this is equation 3 and this is equation 4, 5 and 6.

So, it 1 and 6 equations 1 and 6 are almost same, but actually 6 has something more, it also has 3. So, that is because u is not less than 60 it is less than or equal to 60 degrees. Similarly, from equation 2 what I have is I_d is equal to I_s by $\sqrt{3} \cos \alpha - 30$ degrees minus $\cos \alpha + u + 30$ degrees. So, from equation 2, this is applicable if u is greater than 60 degrees and less than 120 degrees, but we just now showed that it is also applicable if u is equal to 60 degrees. So, let me call this as equation 7.

So, equation 7 is nothing but equation 2 with an additional information that its valid even for α is equal to 60 degrees. Now, let us move on to the DC side voltage.

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For $\alpha < \omega t < \alpha + 60^\circ$,

$$V_d = e_b - L \frac{di_d}{dt} - e_c = e_b - \frac{e_b - e_c}{2} - e_c = \frac{e_b}{2} + \frac{e_c}{2} - e_c = -\frac{3}{2} e_c$$

Average value of V_d ,

$$V_d = \frac{3}{\pi} \int_{\alpha}^{\alpha+60^\circ} V_d d(\omega t) = \frac{3}{\pi} \int_{\alpha}^{\alpha+60^\circ} \left(-\frac{3}{2} e_c\right) d(\omega t) = \frac{3}{\pi} \left(-\frac{3}{2}\right) \frac{\pi}{3} V \sin(\omega t - 90^\circ) d(\omega t)$$

$$V_d = \frac{V_A}{2} [\cos \alpha + \cos(\alpha + 60^\circ)]$$



So, if I take the same interval α less than ωt , less than $\alpha + 60$ degrees. So, we got an equation by applying Kirchhoff's voltage law. So, from the same circuit diagram; so, shall we look at the circuit diagram. So, I can get an expression for V_d in terms of e_b , e_c and the drop across the inductance e_c (Refer Time: 15:06) with the voltage source e_b . Of course, the valves 2 and 3 acts as short circuit and inductance in series with valve 2 is acting as a short circuit because the current through this inductance is I_d which is constant.

So, I can get an expression for V_d from the circuit. So, V_d can be written as e_b minus $L \frac{di_d}{dt}$ minus e_c . So, that should be obvious from this circuit diagram. So, V_d is e_b minus $L \frac{di_d}{dt}$ minus e_c and we also got an expression for $L \frac{di_d}{dt}$. So, $L \frac{di_d}{dt}$ is e_b


minus e_a . So, I will just substitute this expression for $L \frac{di}{dt}$ which is $e_b - e_a$ in this equation. So, it is $e_b - e_a$ is $2 L \frac{di}{dt}$.

So, it is $e_b - e_a$ by 2 minus e_c . So, this can be simplified $e_b - e_b$ by 2 is e_b by 2. So, I have e_b by 2 plus e_a by 2 minus e_c and $e_a + e_b + e_c = 0$. So, $e_a + e_b$ is minus e_c . So, we get finally, V_d equal to minus $\frac{3}{2} e_c$. So, if I look at the average value of the DC side voltage. So, let me use the same notation which we have been using. So, we will use upper case V with the subscript d .

So, I have to take the average value. So, I get the average value by taking the integration one interval. So, this is $\frac{3}{\pi} \int_{\alpha}^{\alpha + 60^\circ} V_d d\omega t$. So, if I take the interval α to $\alpha + 60$ degrees, then I know the expression for V_d . So, just now I got the expression V_d for α to $\alpha + 60$ degrees. So, it is minus $\frac{3}{2} e_c$. So, this is $\frac{3}{\pi} \int_{\alpha}^{\alpha + 60^\circ} -\frac{3}{2} e_c dt$ is $\frac{\sqrt{2}}{3} V \sin \omega t - 90$ degrees.

So, if I evaluate this integral, so I get an expression for V_d , I will leave it you to derive that V_d is equal to V_{do} by 2 into $\cos \alpha + \cos \alpha + 60$ degrees. So, V_{do} is having the same meaning as used for 2 and 3 valve conduction mode and 3 and 4 valve conduction mode. So, this is the expression for V_d . So, we have an expression for V_d in terms of V_{do} and α . So, if we recall that we had expressions for V_d in terms of V_{do} and α for the 2 and 3 valve conduction mode and 3 and 4 valve conduction mode.

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$$V_d = \frac{V_d}{2} [\cos \alpha + \cos(\alpha + 60^\circ)]$$

For 2 and 3 valve conduction mode,
 $V_d = \frac{V_d}{2} [\cos \alpha + \cos(\alpha + u)], 0 < u < 60^\circ - (1)$

For 3 and 4 valve conduction mode,
 $V_d = \frac{\sqrt{3} V_d}{2} [\cos(\alpha - 30^\circ) + \cos(\alpha + u + 30^\circ)], 60^\circ < u < 120^\circ - (2)$

For 3 valve conduction mode,
 $V_d = \frac{V_d}{2} [\cos \alpha + \cos(\alpha + 60^\circ)], u = 60^\circ - (3)$



So, let us look at what was the expression for 2 and 3 valve conduction mode and 3 and 4 valve conduction. So, for 2 and 3 valve conduction mode, the equation relating V_d , V_d o alpha and u is V_d o by 2, V_d equal to V_d o by 2 cos alpha plus cos alpha plus u.


So, this equation is valid for u greater than 0 and less than 60 degrees. We also got an equation relating V_d , V_d o alpha and u for 3 and 4 valve conduction. So, for 3 and 4 valve conduction mode, the equation that we derive is V_d is equal to root 3 V_d o by 2 cos alpha minus 30 degrees plus cos alpha plus u plus 30 degrees. So, this equation is valid for u greater than 60 degrees and less than 120 degrees. And just now we derived for the 3 valve conduction mode.

So, for 3 valve conduction mode, we got V_d equal to V_d o by 2, cos alpha plus cos alpha plus 60 degrees. So, this is applicable for u equal to 60 degrees. So, let me call this as equation 1 now, this is equation 3 and this is equation 3. Now, idea is to show that I can just use equations 1 and 2, I mean; essentially showing that equation 1 and 2 are valid for u equal to

60. So, this is similar to what we did for the equation relating V_d and V_o as a function of α and u . So, instead of having three different equations one for u less than 60 degrees, one for u greater than 60 degrees, and one for u equal to 60 degrees.

We have this equation 6 and 7, now which includes the value u equal to 60 degrees. So, we are going to do the similar thing even for V_d . So, if I look at the expression for V_d for u equal to 60 degrees of course, it is in terms of V_o and α there is because u takes specific value of 60 degrees. So, what I do is I will just try to take the limit of the expression for V_d in equation 1 as u tends to 60 degrees and show that we get the equation 3.

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$$\lim_{u \rightarrow 60^\circ} \frac{V_{d0}}{2} [\cos \alpha + \cos(\alpha + u)] = \frac{V_{d0}}{2} [\cos \alpha + \cos(\alpha + 60^\circ)] \quad - (6)$$

$$\lim_{u \rightarrow 60^\circ} \frac{\sqrt{3} V_{d0}}{2} [\cos(\alpha - 30^\circ) + \cos(\alpha + u + 30^\circ)] = \frac{V_{d0}}{2} [\cos \alpha + \cos(\alpha + 60^\circ)] \quad - (5)$$

$$V_{d1} = \frac{V_{d0}}{2} [\cos \alpha + \cos(\alpha + u)], \quad 0 < u \leq 60^\circ \quad - (6)$$

$$V_{d2} = \frac{\sqrt{3} V_{d0}}{2} [\cos(\alpha - 30^\circ) + \cos(\alpha + u + 30^\circ)], \quad 60^\circ \leq u < 120^\circ \quad - (7)$$



So, what I am trying to do is, I will take equation 1. So, at the right hand side of the equation is V_o by $2 \cos \alpha$ plus $\cos \alpha$ plus u . So, if I take the limit of this as u tends to 60

degrees. So, what I get is $V_d = \frac{V_m}{2} (\cos \alpha + \cos \alpha + 60^\circ)$. So, which is nothing, but we equate I mean the expression for V_d for u equal to 60° .

Similarly, if I take the limit of the expression for V_d which is appearing on the right hand side of equation 2 that is limit of $\sqrt{3} \frac{V_m}{2} (\cos \alpha - 30^\circ + \cos \alpha + u + 30^\circ)$. So, this can be shown to be equal to $V_d = \frac{V_m}{2} (\cos \alpha + \cos \alpha + 60^\circ)$. This not as earlier as the previous case, but it is not difficult to derive. So, I will to you to show that this equation is true. So, I get this as equation 4 and 5. So, what one can do is instead of using equation 1, 2, 3, I can just use two equations.

So, we can say that, V_d is equal to $V_d = \frac{V_m}{2} (\cos \alpha + \cos \alpha + u)$. So, instead of saying that this is valid for u greater than 0 and less than 60° , I will say this is also valid for u equal to 60° . So, let me call this equation 6. Similarly, if I take the expression for V_d in terms of $V_d = \frac{V_m}{2} (\cos \alpha - 30^\circ + \cos \alpha + u + 30^\circ)$ for 3 and 4 valve conduction mode then it is $V_d = \frac{V_m}{2} (\cos \alpha - 30^\circ + \cos \alpha + u + 30^\circ)$. So, this equation is valid for not just u greater than 60° and less than 120° , but also for u equal to 60° .

So, essentially it says that 6 and 7 can be used instead of 1, 2 and 3. This is the same as what we did for the equation relating I_d I_s α and 6 and 7 can be used instead of 1, 2 and 3. Now, as in the case of 2 and 3 valve conduction mode or 3 and 4 valve conduction let us form a table where we show all the intervals and glance that conduct the expressions for the instantaneous DC side voltage the voltage across the valve and also try to find the jumps in voltage across the valve.

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Interval	Valves that conduct	V_d	Voltage across valve 1	Jumps in voltage across valve 1
$\alpha < \omega_s t < \alpha + 60^\circ$	1, 2, 3	$-\frac{2}{3}e_c$	0	$-\frac{\sqrt{3}}{2}V \cos \alpha$
$\alpha + 60^\circ < \omega_s t < \alpha + 120^\circ$	2, 3, 4	$\frac{2}{3}e_b$	$-\frac{2}{3}e_b$	$-\frac{\sqrt{3}}{2}V \sin(\alpha + 30^\circ)$
$\alpha + 120^\circ < \omega_s t < \alpha + 180^\circ$	3, 4, 5	$-\frac{2}{3}e_a$	$\frac{2}{3}e_a$	$-\frac{\sqrt{3}}{2}V \sin(\alpha + 30^\circ)$
$\alpha + 180^\circ < \omega_s t < \alpha + 240^\circ$	4, 5, 6	$\frac{2}{3}e_c$	$-\frac{2}{3}e_c$	$-\frac{\sqrt{3}}{2}V \sin(\alpha - 30^\circ)$
$\alpha + 240^\circ < \omega_s t < \alpha + 300^\circ$	5, 6, 1	$-\frac{2}{3}e_b$	0	
$\alpha + 300^\circ < \omega_s t < \alpha + 360^\circ$	6, 1, 2	$\frac{2}{3}e_a$	0	



So, let us take all the six intervals, $\omega_s t$ greater than α and less than $\alpha + 60$ degrees, then $\alpha + 60$ degrees less than $\omega_s t$ less than $\alpha + 120$ degrees. $\alpha + 120$ degrees less than $\omega_s t$ less than $\alpha + 180$ degrees and so on. So, these are the six intervals in each cycle. So, let me see what are the valves, that conduct in each of these intervals, valves that conduct.

So, in the first interval that is from α to $\alpha + 60$ degrees. So, valve 3 is turned on at α 1 and 2 already conducting they continued. So, valves that conduct in the first interval are 1, 2 and 3. In the second interval 2, 3, 4; so, in the second interval actually, 1 has stops conducting and 4 has started conducting then, 3, 4, 5, 4, 5, 6, 5, 6, 7 sorry 5, 6, 1, 6, 1, 2 the cycle repeats.

Then, if I want the instantaneous DC side voltage V_d ; now, we already know what is the expression for V_d in the first interval, we just derived. But, we also know what is the expression for V_d , when we know what are the valves are conducting from the previous two tables, that is for the table 2 and 3 valves conduction mode or the table for 3 and 4 valve conduction modes. So, this is something which you already know. So, you can straight away write this. So, this is $\frac{3}{2} V_c \cos \alpha$ this is $\frac{3}{2} V_b \cos \alpha$, $\frac{3}{2} V_a \cos \alpha$, $\frac{3}{2} V_c \cos \alpha$, $\frac{3}{2} V_b \cos \alpha$, $\frac{3}{2} V_c \cos \alpha$.

Then voltage across one of the valves say valve 1, because we also seen the expression for voltage across valve 1 when we know what are valves that conduct. So, a few of the increase in this column are easy when valve 1 is conducting actually voltage across the valve 1 is 0. So, in the first interval valve 1 is conducting. So, it is 0, in the last two intervals valve 1 is conducting. So, voltage across valve 1 is 0. Now of course, we got the expressions for the other three intervals also, but you know point of course, noted in result.

When valve 4 is conducting then voltage across valve 1 is $-V_a$; so, in all the three remaining intervals valve 4 is conducting. So, the voltage across valve 1 is $-V_d$. So, it is $-\frac{3}{2} V_b \cos \alpha$ in the second interval, $\frac{3}{2} V_a \cos \alpha$ in the third interval and $-\frac{3}{2} V_c \cos \alpha$ in the fourth interval. So, these are the volt expressions for voltage across valve 1. We can also get the expression for the magnitude of the voltage jumps. So, jumps in voltage across valve 1. So, this is an information which is required for knowing the $\frac{dv}{dt}$ stress.

So, you see that there are four jumps, there is a jump from 0 to $-\frac{3}{2} V_b \cos \alpha$ $-\frac{3}{2} V_b \cos \alpha$ to $\frac{3}{2} V_a \cos \alpha$ $\frac{3}{2} V_a \cos \alpha$ to $-\frac{3}{2} V_c \cos \alpha$ and $-\frac{3}{2} V_c \cos \alpha$ to 0. So, if I want to know the four jumps. So, what I need to do is take the difference in these two expressions and evaluate it at the corresponding instant. So, the first jump is $-\frac{3}{2} V_b \cos \alpha$ minus 0 evaluated at $\alpha + 60$ degrees. Similarly, the second jump is $\frac{3}{2} V_a \cos \alpha$ plus $\frac{3}{2} V_b \cos \alpha$ evaluated at $\alpha + 120$ degrees and so on.

So, leave it to you to verify that these jumps are given that the first jump is $-\frac{\sqrt{3}}{2} V \cos \alpha$. The second jump is $-\frac{\sqrt{3}}{2} V \sin \alpha$ plus $\frac{\sqrt{3}}{2} V \cos \alpha$.

degrees. The next jump is minus root 3 by 2 V sin alpha plus 30 degrees and the last jump is minus root 3 by 2 V sin alpha minus 30 degrees. So, that is about the 3 valve conduction mode. So, from sake of completeness, we have also considered the 3 valve conduction mode in addition to the 2 and 3 valve conduction mode which is be the normal operation and 3 and 4 valve conduction mode.