DC Power Transmission Systems Prof. Krishna S Department of Electrical Engineering Indian Institute of Technology, Madras

Lecture – 29 Analysis of 3 and 4 value conduction mode of 6 pulse LCC: Part 1

(Refer Slide Time: 00:16)



So, i 6 is equal to root 2 by 3 V by omega o L into sin alpha minus sin omega o t plus i 6 of alpha. Now, this expression is applicable in the first subinterval. Now, one has to note that alpha is the instant at which valve 3 is turned on, see valve 3 is turned on at omega o t equal to alpha. Then, what about valve 2? Valve 2 is turned on at omega o t equal to?

Student: (Refer Time: 01:21).

Alpha?

Student: Minus 60 degree.

Minus 60, then valve 1 is turned on at omega o t equal to alpha minus 120. Then if I take the previous valve that is turned on that is before valve 1 the one valve that is turned on is?

Student: 6.

6. So, valve 6 is turned on at omega o t equal to alpha minus 180 degrees. Now, what is a duration for which a valve conducts? Duration for which any valve all valves conduct for the same duration. So, duration for which a valve conducts so, duration is an angle what is the duration? 120 degrees plus u ok.

Now, when does valve 6 stop conducting? So, it is turned on at omega o t equal to alpha minus 180 degrees. So, valve 6 stops conducting; that means, the current through the valve 6 goes to 0; at omega o t equal to alpha minus 180 plus see alpha minus 180 is mentioned it is turned on.

Student: (Refer Time: 03:05).

So, I have to add the duration for which it conducts. So, it is 180 degree plus u. So, this is nothing but alpha minus 60 degrees plus u. So, from alpha 2 alpha minus 60 plus u is taken as the first subinterval that is the basis for taking this alpha minus 60 plus u as the end point of the first subinterval ok. So, what is happening to i 6 at this instant alpha minus 60 plus u?

Student: 0.

0. So, because valve 6 stops conducting. So, i 6 at alpha minus 60 degree plus u is 0. So, I substitute this expression for i 6 at this particular instant which is 0, in the general expression for i 6 that I have got ok. So, one should note that this is applicable for what valve use of

omega o t. So, this is not a expression which is applicable always this applicable for omega o t greater than or equal to alpha and less than or equal to?

Student: (Refer Time: 04:29) plus u minus 60 degrees.

Alpha plus u minus 60 degrees. So, its of course, applicable at alpha plus u minus 60 degrees. So, if I substitute this here why I do this? I do this to get the expression for.

Student: (Refer Time: 04:45).

I 6 of alpha say only after knowing i 6 of alpha I have so, the solution for I 6, till then I do not have the solution for i 6 ok. So, if I do that I get. So, what I have to do is, wherever there is omega o t replace it by alpha plus u minus 60 and i 6 is equal to 0. So, root 2 by 3 V by omega o L sin alpha minus sin alpha plus u minus 60 degrees plus i 6 of alpha. So, this gives the expression for i 6 of alpha.

So, i 6 of alpha is root 2 by 3 V by omega o L into sin alpha plus u minus 60 degrees minus sin alpha. So, substitute this explanation for i 6 of alpha in the expression for i 6 that I have got earlier. So, again the same equation, substitute this here. So, from that I get i 6 equal to root 2 by 3 V by omega o L. So, if I substitute this sin alpha, which is there in the equation gets canceled with the minus sin alpha in the expression for i 6 of alpha.

So, what I am left with is, sin alpha plus u minus 60 degrees minus sin omega o t ok. So, let me call this as equation number 3. So, in the last class we got equations 1 and 2 so, this is 3. This is the expression for i 6, now as we saw there are 2 loop currents that are unknown, see one loop current is already known that is I d the other 2 loop current.

So, I am referring to the circuit that was learn in the last class. So, the other loop current is of course, we can solve for any of the 3 quantities. So, we wrote the equations see in terms of i 6 and i 1. So, will solve for i 1. So, I will not rewrite those equations 1 and 2; I will just try to use those equations and try to get an expression for i 1.

(Refer Slide Time: 07:32)



So, we had equations 1, I just multiply it by equation 2 and subtract it from equation no, I will multiply if equation 1 by 2 and subtract it from equation 2. So, this is for getting the expression for i 1. So, what do we get?

Student: (Refer Time: 07:57).

3 L d i 1 by d t is equal to e b I presume you are having the equations with you I am ok, e b minus e c minus 2 e b 2 because I am multiplying equation 1 by 2 plus 2 e a. Is this ok? Or you want me to write equations 1 and 2. I have written I mean I just want do not want to repeat ok, I am presume you have those equations. So, this can be simplified.

So, this is minus e b minus e c plus 2 a and e i e b e c are balanced sinusoidal voltages. So, minus e b minus e c is equal to e a plus 2 a, that is 3 a. So, if I substitute the expression for e a;

of course, one simplification that is directly possible is in this equation there is a 3 on both sides. So, L d i 1 by d t is equal to e a.

So, this means [inaudible] L d i 1 by d t is equal to root 2 by 3 V sin omega o t plus 150 degrees. So, this has to be solved for i 1. So, if I solve this for i 1 I get root 2 by 3 V by omega o L into what? So, it is dependent on the instant at which the subinterval starts ok. So, it starts from alpha.

So, what is the, what is the solution? Cos of say either I can integrate and just find the constant of integration or I can do a definite integral. Say how I do definite integral? See if I want to say I can do both these; I mean will give you the same result I do either I just say integrate this and take a constant of integration find the constant of integration or I can just take this.

(Refer Slide Time: 10:42)

So, if I d i 1 by d t is root 2 by 3 V by L sin omega o t plus 150 degrees. So, if I employ the definite integral, it is nothing but integral of the right hand side sorry root 2 by 3 V by L sin omega o t plus 150 integrate with respect to time. Now, integration with respect to or time will be a bit difficult. So, what I do is, I will multiply and divided by?

Student: Omega.

Omega o. So, I will just multiply by omega o and divided by omega o is that omega o is a constant. So, that I can put limits on angle instead of time.

Student: Ok.

So, what is the lower limit?

Student: (Refer Time: 11:43).

Student: I knew the I know the both are (Refer Time: 11:45).

Both are equivalent in fact. Now that this is straight forward because, you get the answer in one step otherwise you get it in terms of constant of integration and then you find the concept of integration again put back the put it back in the equation. Now, this is that gives the answer in one shot.

I think you are familiar I presume you are familiar with this. See, you have to integrate from some lower limit and the upper limit is omega o t. I presume if there is no cos of confusion; I mean the dummy variable and the upper integral upper limit are one of the same.

Student: Yes.

And I take the value of i 1 at the lower limit. So, what is the lower limit?

Student: Alpha.

It is, see this expression is applicable from what instant?

Student: Alpha to alpha plus (Refer Time: 12:39).

From alpha so, and this is alpha. So, this gives the answer in one step, otherwise you have to write it in terms of a constant of integration and then get the constant of integration again. So, this gives i 1 is equal to root 2 by 3 V by omega o L into. So, integral of sin is minus cos. So, I get cos alpha plus 150 degrees, minus cos omega o t plus 150 degrees plus what is i 1 at alpha?

Student: I d.

I d. Now is this ok? See what is the other way? Other way is you just do the integration with the constant of integration, I mean both will give the same answer I mean. Now, this is because i 1 of alpha is I d. So, let me call this equation number I have used equations up to 3. So, let me call this 4 this question 4. Now, we have the expression for i 1 and i 6 a in the first subinterval.

Let us go to the second subinterval; sorry the equation 3 is the expression for i 6 in the first subinterval equation 3 ok. Then equation 4 is the expression for i 1 in the first subinterval. Now, let us go to the second subinterval.

(Refer Slide Time: 14:41)



So, again we will draw the effective circuit diagram in the second subinterval. So, while effective I mean showing only those elements that conduct current ok, effective circuit diagram then the second subinterval. So, the second subinterval actually starts at alpha plus u minus 60 degrees to?

Student: Alpha plus 60 degrees.

Alpha plus 60 degrees. So, which are the valves that conduct in the second subinterval? 1, 1 3.

Student: 2.

2, 1 2, 1 3 are conducting. See in the first subinterval, 6 1 2 3 were conducting 6 as stop conducting at the end of first subinterval. So, the valves that conduct are 1 2, 1 3. So, if I just draw the effective circuit of course, I have to show all the 3 sources e a, e b, e c this is 1, this is 3, and this is 2. So, the cathodes of 1 and 3 are at the same potential. So, this is I d this is V d. So, the there is a current i 1, which is flowing through valve 1 and the current through valve 3 which is i 3 is nothing but for this subinterval it is I d minus i 1.

So, I am again employing Kirchhoff's voltage law or in other words mesh analysis to solve for the unknown current i 1; one of the currents is already known. So, there are 2 loops one loop current I d is known the other current is i 1. So, let us solve for i 1 by employing Kirchhoff's voltage law. So, if I apply Kirchhoff's voltage law to this loop, I get L d by d t of I d minus i 1 minus L d i 1 by d t is equal to e b minus e a I d is a constant. So, derivative with respect to time of I d is 0.

So, what I get is, 2 L d i 1 by d t is equal to e a minus e b. So, 2 L d i 1 by d t is e a minus e b the line voltage. So, what is the expression for e a minus e b? So, peak value is root 2 V. So, e a as a phase angle of 150, e b as a phase angle of 30 degrees so, e a minus e b is minus root 2 V.

Student: Sin omega.

Sin omega o t.

So, it is L d i 1 by d t root 2 L d i 1 by d t is minus V sin omega o t. So, solve for i 1. So, again if I tried to directly use the definite integral I get an expression for i 1 as V by root 2 omega o L into what? See, if I take this i 1 I have to integrate after pushing this root 2 L to the right hand side I have to integrate with respect to omega o t after dividing by omega o and the limits are.

Student: (Refer Time: 19:59).

If I let me some more steps. Minus V by root 2 L 1 by omega o integral of sin omega o t d omega o t what are the limits?

Student: Alpha plus u minus 60.

Alpha plus u minus 60 degrees upper limit is?

Student: Omega o t.

Omega o t, plus i 1 at?

Student: Alpha plus u minus 60 degrees.

Alpha plus u minus 60 degrees. So, from that we get the expression for i 1 as V by root 2 omega o L. So, integral of minus sin is cos. So, I get cos omega o t minus cos alpha plus u minus 60 degrees, plus i 1 at alpha plus u minus 6 degrees. Now, this equation will be of will be useful only if I know what is i 1 at alpha plus u minus 60. So, what how to get i 1 at alpha plus u minus 60?

Student: Previous (Refer Time: 21:33).

So, this is nothing but the value of i 1 at the instant of starting of the first sorry second subinterval, see second subinterval starts at alpha plus u minus 60. So, the value of i 1 at this instant is same as the value of i 1 at the instant of the I mean ending of this first subinterval; please note i 1 is continuous why i 1 is continuous?

Student: (Refer Time: 21:59).

i 1 is the current through the inductance. Now, please not i 1 is the valve current but is it also the current through the inductance. See in that in these effective circuit it is of course, say these things can be used only if say value or the starting of the first subinterval second subinterval is equal to value of the end of the first subinterval can be used as long as i 1 is continuous is i 1 continuous.

Student: (Refer Time: 22:35).

i 1 is continuous why?

Student: (Refer Time: 22:39).

Because, the current no current through the inductance is i a, how is i a is continuous does not mean i th i 1 has to be continuous. So, I mean how do you; how do you say it is continues? Any?

Student: (Refer Time: 23:05).

Sorry.

Student: Both are (Refer Time: 23:11).

In the so, if you go back to the original circuit so, was it connected to in series of the same inductance ok?

Student: (Refer Time: 23:41).

Can I relate 2 valve currents and the face current i a? It is i 1.

Student: (Refer Time: 23:49).

Minus.

Student: i 4.

i 4. Now, what was i 4 in the previous first subinterval?

Student: 0.

0 second subinterval? 0. So, for these 2 subintervals i 4 is 0. So, therefore, i a is i 1. So, i 1 is the current through the valve ok. So, for i a is equal to i 1 for alpha, sorry alpha less than omega o t less than alpha plus 60 degrees. So, due to which i 1 happens to be the current through inductor not always in this, interval first interval which consists of 2 subintervals ok. So, I can use the previous equation. So, what is the previous equation? Previous equation is equation 4. So, this equation 4 is applicable up to alpha plus u minus 60.

See it is applicable in the first subinterval. See please note this let me repeat this i 6 expression given by equation 3 is applicable for omega o t greater than alpha and less than alpha plus u minus 60 degrees. And one can check that it is also applicable for omega o t equal to alpha and omega o t equal to alpha plus u minus 60. Similarly, the expression for i 1 given by equation 4 is applicable for omega o t greater than or equal to alpha and less than or equal to alpha plus u minus 60 degrees.

So, I use equation 4, which is applicable even for alpha plus u minus 60 degrees and get the expression for i 1 plus alpha i 1 at alpha plus u minus 60 degrees. So, from 4 i 1 plus i 1 at alpha plus u minus 60 degrees is equal to. So, what you need to do is, wherever there is omega o t replace that by alpha plus u minus 60 degrees.

So, what we get is, root 2 by 3 V by omega o L cos alpha plus 150 degrees minus cos alpha plus u minus 60 degrees that replaces omega o t plus 150 plus I d. So, that gives i 1 at alpha plus u minus 60 degrees as root 2 by 3 V by omega o L cos alpha plus 150; cos alpha plus u plus 90 degrees can be written as sin alpha plus u that is all, sin alpha plus u plus I d. So, this

gives the expression for i 1 at alpha plus u minus 60. So, I substitute this in the expression for i 1 here. So, that gives me the expression for i 1 in the second subinterval.

So, i 1 is V by root 2 omega o L cos omega o t minus cos alpha plus u minus 60 degrees plus root 2 by 3 V by omega o L cos alpha plus 150 degrees plus sin alpha plus u plus I d ok. So, let me use some number for this equation, equation number 5. So, this is equation 5. So, equation 5 is applicable from alpha plus u minus 60 degrees to alpha plus 60 degrees.

(Refer Slide Time: 29:11)

 $i_1(x + 60^{\circ}) = i_6(x^{\circ})$ Expression for i_6 in the 1st subinterval (equation (5)) and replace wat by x + 60° = Expression for i_1 in the 2nd subinterval (equation (5)) and replace wat by x + 60° $= \sum_{k=1}^{\infty} \frac{V}{\sqrt{2}} \sum_{ij=1}^{N} \left[sin (a + u - 60^{i}) - sin a \right]$ = $\frac{V}{\sqrt{2}} \sum_{ij=1}^{N} \left[cos (a + 60^{i}) - cos (a + u - 60^{i}) \right] + \left(\frac{5}{3} \frac{V}{\omega_{j,k}} \right] \left[cos (a + 150^{i}) + sin (a + u) \right] + \hat{\Gamma}_{a}$ = $\frac{V}{\sqrt{2} \omega_{j,k}} \left[cos (a - 30^{i}) - cos (a + u + 30^{i}) \right]$
$$\begin{split} & \widehat{\underline{L}}_{5} = \frac{\widehat{(2 \vee V)}}{2\omega \cdot L} \\ & \widehat{\underline{L}}_{4} = \frac{\underline{L}_{5}}{\sqrt{3}} \left[\cos(\alpha - 30^{\circ}) - \cos(\alpha + \omega + 30^{\circ}) \right] \end{split}$$

Now, if I take the currents i 1 and i 6, see I have got the expression for i 1 and i 6 in the first subinterval I got the expression for i 1 in this second subinterval of course, in the second subinterval i 6 is 0 ok. So, can I relate i 1 and i 6, see all the valve currents are identical except for a phase shift. So, if you take the 2 current valve currents i 1 and i 6 where identical expect for a phase shift. So, what is the phase shift between i 1 and i 6?.

See whatever happens to i 1 would have happened to i 6, 60 degrees ago say what some instant valve 6 is turned on means, after 60 degrees valve 1 is turned on after 60 degrees valve 2 is turned on ok. So, whatever happens to any valve say i 6, valve 6 the same thing happens to one after 60 degrees same thing happens two after another 60 degrees and so on ok. So, if I take i 6 of omega o t it will be equal to i 1 of omega o t.

Student: Plus 60 degree.

Plus 60 degrees not minus plus so, I mean is it plus or minus? i 1 is lagging i 6 by 60 degrees.

Student: Plus.

So, it is plus get this correct ok. Now, what I will do is, I will try to use the 2 equations which gives expression for i 1 in the first subinterval and the second subinterval. And I will replace this in this equation omega o t by alpha. So, if I take i 6 of alpha it is equal to i 1 of alpha plus 60 degrees. See the previous the first question is applicable for any value of omega o t ok. So, I just put omega o t equal to alpha.

Now, what I will do is, I will take the expression for i 6 in the first subinterval. So, I have the expression for i 6 in the first subinterval. So, it starts from alpha to alpha plus u minus 60. So, replace omega o t by alpha in the expression for i 6 and take the expression for i 1 in the second subinterval which is applicable from alpha plus u minus 60 to alpha plus 60. See what I am trying to do is, take the expression for i 6 in the first sub interval.

So, this is equation number. So, if you go back we got the expression for i 6 as equation number 3; this is equation 3 and replace omega o t by alpha. Then what I am saying is this equal to the expression for take the expression for i 1 in the. So, I want to use this equation i 1 at alpha plus 60 is i 6 is alpha. So, for that I have to take the expression for i 1 in the?

Student: Second.

2nd subinterval. So, the expression for i 1 in the 2nd subinterval is given by equation 5 so, this equation 5. And replace omega o t by alpha plus 60 degrees ok. So, what does this give? So, let me do this or I will take the expression for i 6 in the 1st subinterval and replace omega o t by alpha ok.

So, wherever there is omega o t I will just put alpha ok. So, what I get is root 2 by 3 V by omega o L sin alpha plus u minus 60 degrees minus sin alpha. So, that is the expression for i 6 in the first subinterval after replacing omega o t by alpha.

So, this is equal to expression for i 1 just now I got the explanation for i 1. So, this is equal to V by root 2 omega o L cos alpha plus 60 degrees minus cos alpha plus u minus 60 degrees, plus root 2 by 3 V by omega o L cos alpha plus 150 degrees plus sin alpha plus u plus I d.

Now, I will leave it you to simplify this equation some trigonometric manipulations have to be done. So, it can be shown that I d, which is appearing as the last term on the right hand side is equal to V by root 6 omega o L, into cos alpha minus 30 degrees minus cos alpha plus u plus 30 degrees.

So, this I am leaving it to you to derive, its just simplification of the previous equation. Now, if you recall we defined a quantity called short circuit current at the peak value of short circuit current I s. So, I s the peak value of short circuit current is root 2 V by 2 omega o L. So, I can write the expression for I d in terms of I s as I d is equal to I s by root 3. So, I just replace V by root to omega o L by s that is all. So, its I s by root 3 cos alpha minus 30 degrees minus cos alpha plus u plus 30 degrees.

So, this is the relationship between I d, I s alpha and u in the case of 3 and 4 valve conduction mode. So, we got similarly a relationship between I d I s alpha and u for 2 and 3 valve conduction mode, this is applicable for 3 and 4 valve conduction mode.