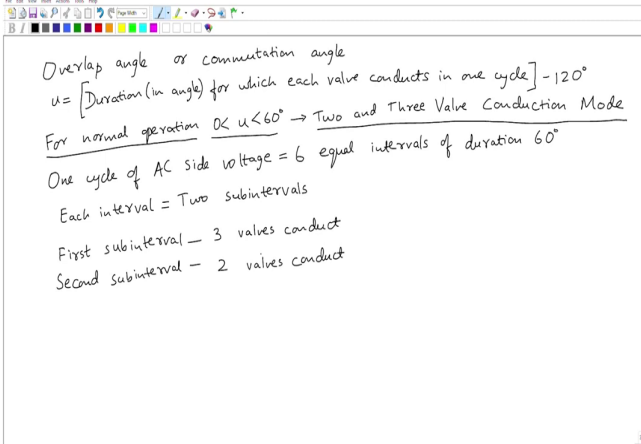




DC Power Transmission Systems
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Lecture – 22
2 and 3 valve conduction mode of 6 pulse LCC

(Refer Slide Time: 00:16)



Overlap angle or commutation angle
 $u = [\text{Duration (in angle) for which each valve conducts in one cycle}] - 120^\circ$
For normal operation $0 < u < 60^\circ \rightarrow$ Two and Three Valve Conduction Mode
One cycle of AC side voltage = 6 equal intervals of duration 60°
Each interval = Two subintervals
First subinterval - 3 valves conduct
Second subinterval - 2 valves conduct


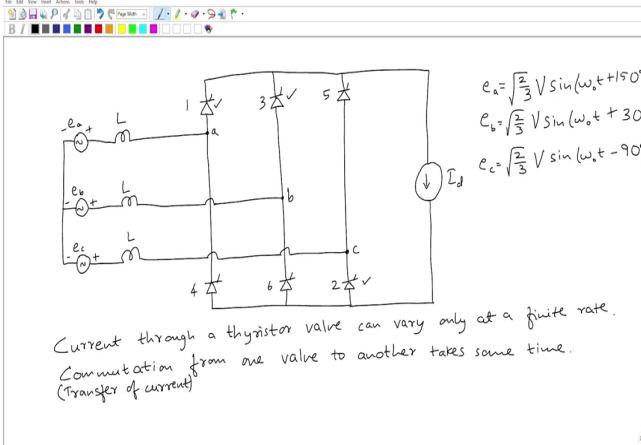


Now, this u can take in fact a any value, but we will consider some special cases. So, if I take normal operation for normal operation u is between 0 and 60 degrees. Now, what is so special about 60 degrees. See 60 degree is actually the duration of one interval. See we have defined what is known as interval each cycle of the AC side is divided into 6 equal intervals. So, one interval is 60 degrees. So, a pair of thyristor valves conduct for each of these intervals. So, u is less than 60 for normal operation we will also consider abnormal operation where u can go beyond 60.

So, let us first to concentrate on the normal operation where u is between 0 and 60 degrees. So, if I take one cycle of the AC side voltage one cycle of AC side voltage any voltage the 3 voltage sources 3 single phase voltage sources any voltage it is equal to 6 equal intervals ok. So, each of these are of duration 60 degrees. Now, what I do is I divide this interval into 2 sub intervals. So, each interval of duration 60 degrees is said to be equal to 2 sub intervals.

So, there are 2 sub intervals. There is a first sub interval and a second sub interval in each interval. Now, by definition the first sub interval is the one where there are 3 valves that are conducting and the second sub interval is the one where 2 valves are conducting. See my assumption is normal operation and hence u is less than 60 degrees ok. So, I up to some angle there are 3 valves that are conducting that correspond to overlap angle or a commutation angle and once that is over the current has completely shifted say.

(Refer Slide Time: 02:58)

$$e_a = \sqrt{\frac{2}{3}} V \sin(\omega_0 t + 150^\circ)$$

$$e_b = \sqrt{\frac{2}{3}} V \sin(\omega_0 t + 30^\circ)$$

$$e_c = \sqrt{\frac{2}{3}} V \sin(\omega_0 t - 90^\circ)$$


Current through a thyristor valve can vary only at a finite rate.
 Commutation from one valve to another takes some time.
 (Transfer of current)



If you look at the previous figure, there is a certain duration for which 1 and 3 both are conducting after certain time one stops conducting in the same interval only 3 is conducting. So, I am dividing the interval into 2 sub intervals. The first sub interval is the one where 3 valves conduct; 3 valves conduct and in the second sub interval 2 valves conduct. So, since you take any instant either 3 valves are conducting or 2 valves are conducting that is why this case is known as 2 and 3 valve conduction mode 2 and 3 valve conduction mode.

So, there are other possible modes corresponding to other possible range or other possible values of u . So, for u between 0 and 60 degrees we have 2 and 3 valve conduction mode. So, if I want to analyze I will take a one interval and we will see that it is sufficient to analyze one interval we can in fact, get the entire waveform of any quantity either on the DC side or the AC side. So, let me take one particular interval and analyze it in detail.

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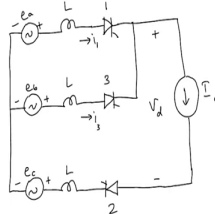


Consider the interval $\alpha < \omega t < \alpha + 60^\circ$

1st Subinterval: $\alpha < \omega t < \alpha + u$

2nd Subinterval: $\alpha + u < \omega t < \alpha + 60^\circ$

1st subinterval




$$L \frac{di_3}{dt} - L \frac{di_1}{dt} = e_b - e_a$$

$$L \frac{di_3}{dt} - L \frac{d(I_d - i_3)}{dt} = \sqrt{2} V \sin(\omega t)$$

$$\frac{di_3}{dt} = \frac{V}{\sqrt{2} L} \sin(\omega t)$$

$$i_3 = \frac{V}{\sqrt{2} \omega L} [\cos \alpha - \cos(\omega t)]$$

$$i_3(\alpha) = 0$$



So, consider the interval. Can you suggest what interval we can take? See one interval is 60 degrees what interval can be taken. Any suggestion. Suppose I take the interval the starting point of which corresponds to turning on off for valve 3 at what instant valve 3 is turned on alpha. Suppose I take the interval between alpha and alpha plus 60 degrees. This is one interval ok.

So, again this in this interval has 2 sub intervals. There is a first sub interval and a second sub interval. So, what is the first sub interval? What valve what are the values of ω or t ? What is the range of ω or t in the first sub interval?

Student: (Refer time: 05:29) alpha to alpha plus minus.

Alpha to alpha plus minus. If I take the second sub interval it is ω or t taking values between alpha plus u and alpha plus.

Student: 60 degrees.

60 degrees ok.

Now, I want to analyze the circuit for the 2 sub intervals. Let me take the first sub interval. So, what do I will try to do is I take the first sub interval that is alpha 2 alpha plus u and try to draw the circuit which is only relevant for the first sub interval. See the circuit is. In fact, drawn here already ok. Now, when I say a first sub interval there are 3 valves that are conducting 1, 2 and 3. Now there is a valve 5. There is a valve 4. There is a valve 6 which do not conduct. So, there is no current flow through valves 4, 5 and 6. In the second sub interval when only in the second sub interval which valves conduct.

Student: 3 (Refer time: 06:43).

2 and 3. So, when only 2 and 3 conduct ea does not carry current only eb and ec. So, what I will do is I will just draw a simplified diagram where I only show those components which

carry current. So, when something is not carrying a current I can just remove it is a I say open so, open circuit. So, I will just draw a circuit diagram showing only those components of the circuit which carry a current of course, all the 3 voltage sources carry current. So, I will show all the three.

So; that means, all the 3 inductances carry current. See when all these inductances are connected in series with the voltage source. So, this is e_a , e_b , e_c . Now, I draw the remaining part of the circuit in a slightly different way. If u look at the original circuit thyristor valve 1 for this first sub interval α 2 $\alpha + u$ is as good as being connected in series with e_a and L. Similarly thyristor valve 3 you please refer to the original circuit.

So, the in the if you look at the original circuit 1 is connected in series with e_a , 3 is connected in series with e_b and there is one more thyristor is conducting 2 that is connected in series with e_c . But only thing is now the direction as far as 2 is concerned the direction is in the opposite direction. So, the current flow is in entering the current is entering the.

Student: Voltage source.

Voltage source ok. So, this is 2. Now from the circuit we know that the cathode of 1 and 3 are at the same potential. They are shorted cathode of 1 and 3 are shorted. So, I will short this and the cathode of 1 and 3 is nothing, but the positive terminal of the DC side voltage and the anode of 2 is the negative side of the DC side voltage. So, on the DC side I have a current source I_d . So, this is positive terminal of V_d this is negative terminal of V_d fine.

Now, let me take the current through valve 1 say i_1 , i_1 is the current through valve 1. So, I will always show the current through a valve as the one which is flowing from anode to cathode. Similarly I will also show i_3 . What about i_2 ? I_2 is I_d . Please note i_2 the current flowing through I valve 2 is I_d ok. So, that I need not give a separate notation for that ok.

Now for the analyzing this I will apply Kirchhoff's voltage law this to this loop consisting of voltage source e_a voltage source, e_b the 2 inductances, 1 valve one and valve 3 just apply Kirchhoff's voltage law nothing more ok. So, if I do that $L \frac{di_1}{dt} - L \frac{di_3}{dt} - L \frac{di_2}{dt} + e_a - e_b = 0$ I will do one

thing. I will try to go in a different direction I will say $L \frac{di_3}{dt} - L \frac{di_1}{dt}$. So, by Kirchhoff's voltage law this is equal to $e_b - e_a$. Is that ok?

So, what is $e_b - e_a$? See we have the expression for e_b and e_a , e_b and e_a the expression is given here ok. It is $\sqrt{2} v \sin \omega t$. So, I will make one further assumption I mean no one further I use of further relation. So, $\frac{di_3}{dt} = L^{-1} (e_b - e_a)$. Of course, i_1 is i_1 and i_3 are not independent. See if I_d is the current on the DC side i_1 and i_3 are related to I_d . So, I can write i_1 as $I_d - i_3$, i_1 is $I_d - i_3$ by Kirchhoff's current law this node.

So, this is $\frac{d}{dt} (I_d - i_3) = \sqrt{2} v \sin \omega t$. So, I can or try to solve this these are differential equation in i_3 . So, solve this differential equation for i_3 that is all ok. So, this can be simplified I_d is a constant please note our representation of the DC side is a constant current source. So, I_d is a constant. So, derivative is 0. So, what we get here is $\frac{di_3}{dt} = \frac{v}{L} \sqrt{2} \sin \omega t$, is that ok?

So, left hand side there is a $2 L \frac{di_3}{dt}$. So, the $2 L$ I have taken to the right hand side $\sqrt{2}$ gets canceled with this $\sqrt{2}$ on the right hand side. So, this is the expression. So, can I get; can I get an expression for i_3 . So, i_3 as a function of ωt . So, it is $v \sqrt{2} \omega L$. You want to integrate with respect to see then if you look at the left hand side the derivative with respect to time. See for most of the waveforms our independent variable is ωt , but the derivative here that is involved is with respect to time ok. So, that is why there is ω coming there. Then what.

Student: $1 - \cos$ of \cos .

Student: $1 - \cos$ of ωt .

$1 - \cos$. How did you get that 1?

Student: (Refer time: 13:52) initial condition.

Plus $\cos \alpha$. So, it is $\cos \alpha \sin \omega t$ minus $\sin \alpha \cos \omega t$. See what you can do is take the there is a constant of integration I mean how do we get that constant of integration.

Student: (Refer time: 14:08).

How do we get that?



Student: (Refer time: 14:12).

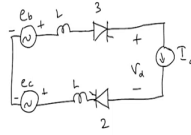
You use one condition that is i_3 at.

Student: i_3 equal to 0 at t equal to α .

So, at ωt equal to α i_3 is 0. So, using this condition you get the expression for i_3 ok. Now, let me take this expression for i_3 and see what happens to i_3 at $\alpha + u$. See α is the α is at one end of the first sub interval $\alpha + u$ is at the other end. So, what is the value of see please not this expression is applicable for any value of ωt between α and $\alpha + u$ that is all. Not any value I mean it is not applicable for less than α or greater than $\alpha + u$. So, at i_3 of α is 0, what is i_3 of $\alpha + u$?

(Refer Slide Time: 15:13)

$i_3(\alpha+u) = I_d$
 $I_d = \frac{V}{\sqrt{2}\omega L} [\cos\alpha - \cos(\alpha+u)] = I_s [\cos\alpha - \cos(\alpha+u)]$
 I_s : peak value of short circuit current, $I_s = \frac{\sqrt{2}V}{2\omega L} = \frac{V}{\sqrt{2}\omega L}$
 $V_d = e_b - L \frac{di_3}{dt} \cdot e_c = e_b - \frac{e_b - e_a}{2} \cdot e_c = \frac{e_a + e_b}{2} \cdot e_c = -\frac{3}{2} e_c$
 2nd Subinterval

 $v_d = e_b - e_c$
 Average value of V_d ,
 $V_d = \frac{3}{\pi} \left[\int_{\alpha}^{\alpha+\pi} V_d d(\omega t) + \int_{\alpha+\pi}^{\alpha+2\pi} V_d d(\omega t) \right]$
 $V_d = \frac{V_{d0}}{2} [\cos\alpha + \cos(\alpha+u)]$

I can use that expression, but what is the value of i_3 at $\alpha + u$ that is the definition of.

Student: I_d .

A first sub interval. So, it is equal to I_d ; that means, the current should have got completely transferred from valve 1 to valve 3. So, valve 3 current i_3 at $\alpha + u$ at the end of first sub interval is equal to I_d . So, if you use the expression what do you get? So, this gives I_d equal to. So, the expression is v by root 2 omega oL into $\cos \alpha$ minus \cos .

Student: $\alpha + u$.

Alpha plus u ok. Now we normally write this in terms of current which is obtained as the peak value of short circuit current. See if u just go back to the circuit the original circuit. See there are AC side terminals of the converter suppose there is a short circuit between any 2 AC side terminals, what will be the short circuit current through the source or inductance. See suppose I call this a point a, point b c. There is a short circuit between a and b rb and c rc and a, what will be the current flow through the voltage source or inductance when if there is a short circuit.

It is as good as saying I apply a line voltage. There are 2 phase voltages I apply a line voltage to an equivalent inductance of $2L$. So I can easily get the RMS value of the short circuit current by taking the RMS value of voltage divided by reactants. So, the reactants will be 2 times ωL ok. So, what I do is I define what is known as the peak value of short circuit current. We use a notation I_s for short circuit.

So, suppose I_s is the peak value of short circuit current then I_s is related to the RMS value v the frequent angular frequency ω inductance L of course, I forgot to close this bracket. So, how is I_s related to v ω and L . See if you look at the RMS value of the short circuit current it is equal to the RMS value of the voltage which is sorry. RMS value of the line voltage is just v RMS values is v line voltage divided by the total impedance or reactance which is $2\omega L$.

So, that is RMS value. If I multiply this by $\sqrt{2}$, I get the peak value. So, that is nothing, but v by $\sqrt{2} \omega L$. So, I can write I_d in terms of I_s that is what I am saying. So, I can write this previous expression as $I_s \cos \alpha - \cos \alpha + u$. So, this just for the sake of simplifying the notation instead of every time writing v by $\sqrt{2} \omega L$ I just say I_s and I_s has some meaning.

Now, let us see what happens to the instantaneous value of the voltage on the DC side. So, if I take the first sub interval what is V_d . So, if you go to the equivalent circuit see what we have here is the equivalent circuit were only the elements which do not which carry current are

shown. So, what is V_d ? I can get an expression for V_d from this circuit and the expression for i_3 which I have just derived ok.

So, can I get an expression for V_d by applying Kirchhoff's voltage law to this lower loop. See if 2 and 3 are conducting obviously, that happens in the first sub interval. They are short circuit idlth (Refer time: 19:52) they are short circuit. What about the voltage across L in which is in series with e_c . What is the voltage across the inductance which is in series with e_c or valve 2. It is 0 because the constant current is flowing. $D I_d$ by see rate of change of current is 0 because current is constant through this inductance L which is in series with e_c .

So, V_d can be related to e_b , e_c and the drop across this L which is in series with e_b ok. So, by applying Kirchhoff's voltage law I can write V_d as e_b minus $L \frac{d i_3}{dt}$ minus what. See I am referring to the circuit. I am applying Kirchhoff's voltage law to this lower loop. So, it is.

Student: Minus e_c .

Minus e_c . So, what is this $L \frac{d i_3}{dt}$. You have an expression for i_3 by dt . Just go back to the previous page $L \frac{d i_3}{dt}$ is nothing but.

Student: (Refer time: 21:13).

Can I relate that to e_b and e_a .

Student: (Refer time: 21:17).

See $L \frac{d i_3}{dt}$ is nothing, but e_b minus e_a by 2 by 2 from the first equation because $L \frac{d i_1}{dt}$ with the negative sign is nothing, but plus $L \frac{d i_3}{dt}$ ok. So, it is e_b minus e_a by 2. So, I have e_b minus $L \frac{d i_3}{dt}$ is e_b minus e_a divided by 2 minus e_c .

Student: 2 plus e_c by (Refer time: 21:47).

So, e_b minus e_b by 2 is e_b by 2. So, I have e_a plus e_b by 2 minus e_c . Now e_a , e_b and e_c are balanced so; that means, e_a plus e_b plus e_c is.

Student: 0.

0. So, e_a plus e_b is minus e_c . So, this is equal to.

Student: Minus 3.

Minus 3 by 2 e_c ok. So, this is the expression for V_d in the first sub interval. Now, if you take the second sub interval second sub interval is very straightforward of course, it is much easier. In the second sub interval I have only 2 valves conducting 2 and 3. So, if I want to draw a equivalent circuit showing only those elements which conduct current then it is the same circuit which I got earlier.

But now a few elements can be removed because i_1 will be the current through valve one will be in second sub interval i_1 is 0. So, I can just remove this e_a L and valve 1. So, the remaining elements which are shown are e_b e_c L. So, there is a valve 3 which is shown and a valve 2 which is shown I_d and the voltage across this current source I_d is V_d . Now of course, there is a showing L in this case is a redundant because the current through the inductance is.

Student: Constant.

Constant both inductance is constant as current is constant. So, there is no drop across the inductances ok. So, what is V_d in the second sub interval.

Student: E_b minus e_c .

E_b minus e_c because there is no drop across the inductance ok. So, if I want the average value of V_d can I get the average value of V_d from the expression for V_d in the first sub interval and second sub interval. See first sub interval and second sub interval constitute one interval

of 60 degrees. Now is one interval sufficient for computing the average value of V_d see on the DC side what is the minimum period.

Student: It is 60 degrees.

60 degrees on the DC side the minimum period is 60 degrees.

Student: Yes.

So, whatever happens for 60 degrees the same thing repeats after every for every I mean subsequent 60 degrees. So, the 60 degree period consisting of first and second sub interval is sufficient enough to compute the average value of V_d . So, I can say the average value of V_d . So, we will use this notation uppercase V with the subscript d . So, this can be obtained from the expression for the instantaneous value in the first interval of duration 60 degrees.

So, it is 3 by π which is nothing, but the reciprocal of π by 3 60 degrees is π by 3 radian into integral of V_d with respect to ωt from α to $\alpha + u$ plus integral of V_d with respect to ωt from $\alpha + u$ to $\alpha + 60$ degrees. So, you have to substitute 2 different expressions for V_d in the 2 integrals. In the first integral V_d is $\frac{3}{2} e_c$ in the second in integral it is $e_b - e_c$ and we have expressions for $e_i - e_b - e_c$.

So, if I substitute those expressions I get the. So, I will leave it to you to derive that this V_d can be shown to be equal to $V_{do} \frac{2}{\pi} [\cos \alpha + \cos \alpha + u]$. So, please derive this after substituting the expression for V_d and then afterwards substituting the expressions for $e_i - e_b - e_c$ ok. Of course, V_{do} is the notation which you have been very familiar with what is V_{do} .

Student: Maximum of DC voltage maximum.

Is it the maximum sorry?

Student: (Refer time: 26:55).

Maximum average value of the DC voltage for which case?

Student: (Refer time: 27:03).

That we for the case of u equal to 0 or L equal to 0. Is it true even for this case? For any non-zero value of u is it still applicable?

Student: It is only a number (Refer time: 27:18).

It is just a number. We have the I mean we have the notation V_{do} . What was the definition of V_{do} ? V_{do} is the maximum value of the.

Student: Average DC voltage.

Average DC voltage with L equal to 0.

Student: Yes.

But can we still say that it is the maximum average value with the a non-zero L . Anyway there is no need to even I mean generalize the definition of V_{do} . Once you have define the V_{do} we will just use it as V_{do} being the maximum average value with L equal to 0.

Student: Yes.

So that same notation we are using here ok. So, we will just relate V_d to V_{do} fine. Now what was the expression for V_d with L equal to 0. See L equal to 0 is equivalent to u equal to 0. So, if L is 0 or u is 0 V_d is V_{do} .

Student: $V \cos \alpha$ (Refer time: 28:14).

$V \cos \alpha$.

Student: $\cos \alpha$.

We derived that $V \cos \alpha$. So, you can get that you just put u equal to 0. If u is equal to 0 we get $V \cos \alpha$ as $V \cos \alpha$. Now due to u or L , is there a reduction in $V \cos \alpha$ or is there an increase in $V \cos \alpha$.



Student: (Refer time: 28:35).

Is there is a.

Student: Reduction.

There is a reduction ok. So, we what we can do is we can actually a quantify that reduction. So, what we will do is we will just use this relation to see how much reduction is there.

(Refer Slide Time: 28:48)


Average value of v_d , $\alpha+\pi$

$$V_d = \frac{3}{\pi} \left[\int_{\alpha}^{\alpha+\pi} v_d d(\omega t) + \int_{\alpha+\pi}^{\alpha+2\pi} v_d d(\omega t) \right]$$

$$V_d = \frac{V_{d0}}{2} [\cos \alpha + \cos(\alpha+\pi)]$$

$$V_d = V_{d0} \cos \alpha - \frac{V_{d0}}{2} [\cos \alpha - \cos(\alpha+\pi)] = V_{d0} \cos \alpha - \frac{V_{d0}}{2} \frac{I_d}{I_s} = V_{d0} \cos \alpha - R_c I_d$$

where $R_c = \frac{V_{d0}}{2I_s} = \frac{3\sqrt{2}V}{\pi} \cdot \frac{1}{2} \cdot \frac{2\omega L}{\sqrt{2}V} = \frac{3\omega L}{\pi}$

$$V_d = V_{d0} \cos \alpha - R_c I_d \quad R_c: \text{equivalent commutation resistance.}$$


So, since I am just going to the next page I will use this. So, I use this expression V_d equal to V_{d0} by $2 \cos \alpha$ plus $\cos \alpha$ plus π . So, I will write this as V_d equal to. So, without inductance that is when L is 0 or π is 0 the expression is $V_{d0} \cos \alpha$. So, now, I can write with inductance or with non-zero π V_d is $V_{d0} \cos \alpha$ minus V_{d0} by $2 \cos \alpha$ minus $\cos \alpha$ plus π . Is that ok? This is same as the previous expression

Student: Minus (Refer time: 28:48).

Student: The change.

Sorry.

Student: (Refer time: 29:54) change in the.

No I am just rewriting the previous expression. Instead of writing $V \cos \alpha$ I am saying $V \cos \alpha - V \cos \alpha$ that is all. I am just rewriting the previous expression nothing more than that. Now, why I do this way is the first term is the expression for V_d for the special case of u equal to 0 ok. So, this can be written as $V \cos \alpha - V \cos \alpha + u$. Now, just go to the previous page look at the expression for I_d , I_d is given by $I_s \cos \alpha - I_s \cos \alpha + u$.

So, $\cos \alpha - \cos \alpha + u$ is I_d by I_s . See from this equation $\cos \alpha - \cos \alpha + u$ is I_d by I_s . So, I will write $\cos \alpha - \cos \alpha + u$ is I_d by I_s . That is what I have here $\cos \alpha - \cos \alpha + u$ is I_d by I_s . So, we write this as $V \cos \alpha - R_c$ into I_d where R_c is defined as $V \cos \alpha / I_s$. So, one can put the expression for V_d . See V_d is $\frac{3}{\sqrt{2}} V_m \sin(\omega t - \pi)$ and $2 I_s$, what is $2 I_s$?

So, $\frac{1}{\sqrt{2}} I_s$ is $\frac{1}{\sqrt{2}} I_s$ into of the reciprocal of I_s . So, by definition I_s is the peak value of the short circuit current. So, it is $\frac{3}{\sqrt{2}} V_m$ in the denominator in the numerator it is $2 \omega L$. So, if you do all the cancellations what you get is $\frac{3 \omega L}{\pi}$ ok. So, the point to note is if you just look at the previous line what we have got is V_d is equal to $V \cos \alpha - R_c I_d$. So, the second term is due to L because R_c is equal to $\frac{3 \omega L}{\pi}$. If L is 0 we do not get the second term ok. So, that is why this R_c has a name. R_c is called commutation resistance. In fact, equivalent commutation resistance.

Now, it is just an equivalent resistance, but it does not result in any loss. See this resistance is not causing any loss say there is no resistance in the original circuit. So, it is just a name ok. So, R_c is just a notation it is not a physical resistor which is present there which is causing any loss ok. So, one has to just look at that as an equivalent commutation resistance ok.