DC Power Transmission Systems
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Lecture – 13
Fourier series: Part 2

I will continue our discussion on Fourier series.

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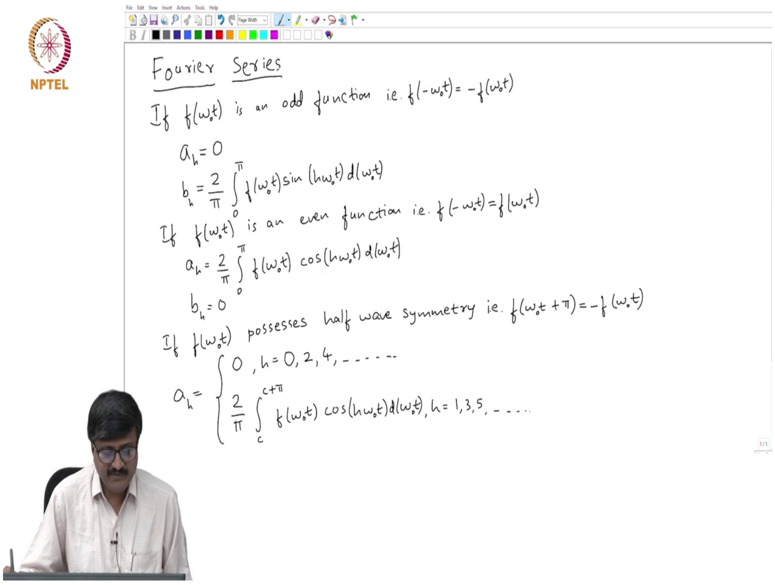

So, in the last class, we left at the point where we just defined what is known as an odd function and what is an even function. So, there are some simplifications if a function is odd; there are some simplifications if the function is even. The simplification is in the computation of the coefficients the Fourier coefficients a h or b h. Of course, we can do the other way where we compute the coefficients c h also.

So, the simplification is mainly in terms of finding the integral over a certain duration. So, it can be in the form of reduced duration for which we integrate or we already know that for some cases either a h or b h is 0; so, thereby there is a simplification ok. So, our intension is to compute either the fundamental component or the harmonic component using Fourier series. Of course though average is something which is given by Fourier series, I mean one need not get into the I mean get into Fourier series to get the average because average is something which is I mean more basic than Fourier series ok.

So, let us consider some special cases where there is some symmetry. If f of omega o t is an odd function, so, a function is said to be odd if f of minus omega o t is equal to minus f of omega o t. So, in this case, I will not get into the derivations. See if you are not familiar with these results, I would suggest that you verify this ok.

So, I am not sure whether you are heard of Fourier I mean odd function even function in the context of Fourier series. So, what is simplification in the case of odd function? When you if at all you recall, a h is?

Student: 0.

0; a h is 0 in the case of odd function. So, one has to find only b h and there is a further simplification in the computation of b h. So, one need not integrate over the full cycle of 2 pi radian. So, the simplification is one has to integrate only for a duration of pi radian. So, its 2 by pi integral 0 to 2 pi f of omega o t sin h omega o t d omega o t.

So, this the simplification for b h. So, you will see that for all values of b h. So, h takes all values from 1, 2, 3 and so on up to infinity. So, there is only b h and that to the integration is only over half a cycle that is for 180 degrees. If on the other hand, f of omega o t is an even function, so, a function is said to be even if the function evaluated at the negative of an argument is same as the function evaluated at the value of the argument ok.

So, if f of omega minus omega o t is f of omega o t, then we say that the function f of omega o t is an even function. So, in this case b h is 0 and a h has an expression which is simpler than the original expression. So, again the integration is only for pi radians. So, it is 2 by pi integral 0 to pi f of omega o t cos h omega o t d omega o t and b h is 0.

There are other types of symmetries, for example, half wave symmetry, quarter wave symmetry. So, let me define half wave symmetry. If f of omega o t possesses half wave symmetry, so, the term half wave actually indicates that the entire information is contained in half cycle. So, please note when I when we talk about periodic functions, the entire information about the function is actually there in just one cycle. So, if I know one cycle I know the function itself.

So, in the case of half wave symmetry, there is a further reduction from 2 pi radians to pi radians. So, the entire information is contained in only 180 degrees or pi radians. So, what is the definition? The definition of a half wave symmetry is I mean when we say that f of omega o t has half wave symmetry; if f of omega o t plus pi is equal to if at all you recall.

Student: Minus.

Minus f of omega o t. So, if that is true, then we say that f of omega o t has half wave symmetry. Now in this case, there is a simplification. What is simplification in this case? Do you recall? Do you recall the simplification in this case?

Student: Only odd harmonics.

Only odd harmonics. So, for all even values of h a h as well as b h is 0. So, let us look at the expression a h; so, a h is 0 when h is equal to?

Student: 0.

What about 0? If h is 0, then a h is 0. See h is 0 means it is a average value.

Student: Average is 0.

Average is 0 for half wave symmetry; so, 0, 2, 4, 6 so on. So, for all these values of h, a h is 0 and for h equal to 1, 3, 5, so on, we have an expression, but what is the simplification? We do not integrate over again one full cycle half a cycle. So, it is 2 by pi integral from any arbitrary constants c to c plus pi f of omega o t cos h omega o t d omega o t. So, this is the expression for h equal to 1, 3, 5, so on.

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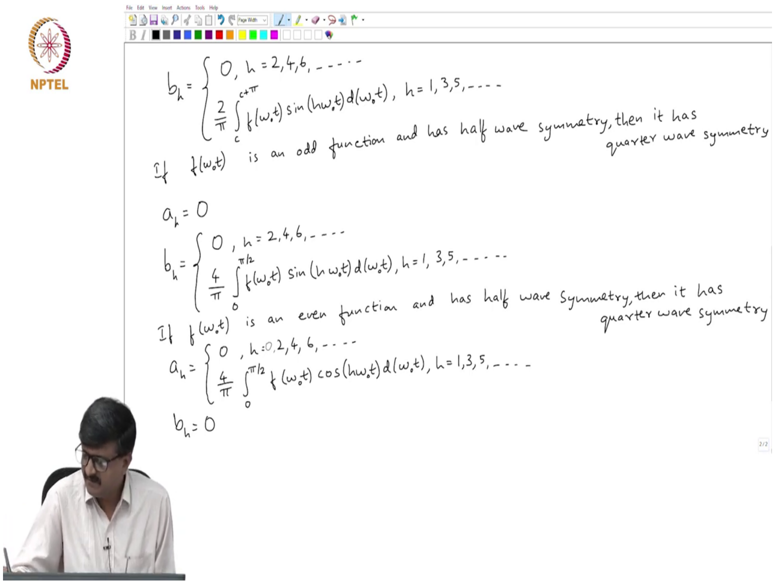

Now when it comes to b h, again b h is 0 for h equal to? b h is not defined for h equal to 0.

Student: 2, 4, 6, 8.

2, 4, 6, so on. So, again for all the even values of h, b h is 0 and for odd values of h, it is 2 by pi integral any arbitrary constant c to c plus pi f of omega o t sin of h omega o t d omega o t. So, this is applicable for h equal to 1, 3, 5, so on. So, you see that only odd harmonics are present if there is half wave symmetry. Now there is a further simplification possible if the function possesses what is known as quarter wave symmetry.

Now, before going to the general definition of quarter waves symmetry, we will take some special cases. Suppose I have combination of odd and half wave symmetry or even and half wave symmetry, then what happens ok? So, if f of omega o t is an odd function and also has half wave symmetry and has half wave symmetry, then in this case what is a h?

Student: 0.

So, we will know that for odd function a h is 0 and what about b h? So, if you look at b h again b h is 0 for?

Student: h is equal to.

Since it has half wave symmetry for even values of h, b h is 0. So, I get two expressions for b h. It is 0, if h is equal to 2, 4, 6, so on and when it comes to the odd values of h, then there is a further reduction in the duration for which we integrate. So, we do not integrate over I mean one need not integrate over pi radians, we can just integrate over quarter cycle that is pi by 2 radian. So, in this case the expression will be; please note I am not deriving any of this. If you are not familiar with these results, please verify these results. It is 4 by pi integral 0 to pi by 2 f of omega o t sin h omega o t d omega o t.

So, this expression is applicable if h is equal to 1, 3, 5, so on. So, what is to be noted is whenever there is a odd function which has half wave symmetry, there is no a h and of course, b h is 0 for even values of h and for odd values of h, b h is obtained by integration over quarter cycle.

Now let me take one more combination. If f of omega o t is an even function and has half wave symmetry, now in this case, since it is even b h is 0 and when it comes to a h again a h is also 0 for certain values of h that is even values of h.

So, a h has two expressions: one is 0 when it comes to h equal to 0 2 4 6 so on. And for odd values of h, it is 4 by pi integral 0 to pi by 2 f of omega o t cos h omega o t d omega o t, for h equal to 1, 3, 5 so on and for all values of h, b h is 0. Now instead of saying odd function and half wave symmetry, we have I mean we have just a name for a function which satisfies both the property of odd function as well as half wave symmetry.

What is that if you heard of that? Say actually the hint is in the expression. If you look at the b h expression in the case of odd function half wave symmetry, it is integration only over quarter cycle. Similarly, even in the case of even function and half wave symmetry in the expression for a h for odd values of h, the integration is only over quarter cycle. So, in fact, the information is contained in quarter cycle. So, we say that this particular function either the case of odd and half wave symmetry or even and half wave symmetry is as good as saying quarter wave symmetry.

So, we say that I will just write it here itself, then it has what is known as quarter wave symmetry. So, even in the other case if f of omega o t is an even function and has half wave symmetry, then it is said to have quarter wave symmetry. But these are only two special cases of quarter wave symmetry.

So, what is the definition of quarter wave symmetry? Say, quarter wave symmetry means the entire information is contained in quarter cycle, but these are only two special cases. There are other possibilities where I can I mean the function need not be even or odd. So, even if the function is neither odd nor even, it can still have quarter wave symmetry ok. So, that is still possible. So, we can have a very general definition of quarter wave symmetry these are just special cases.

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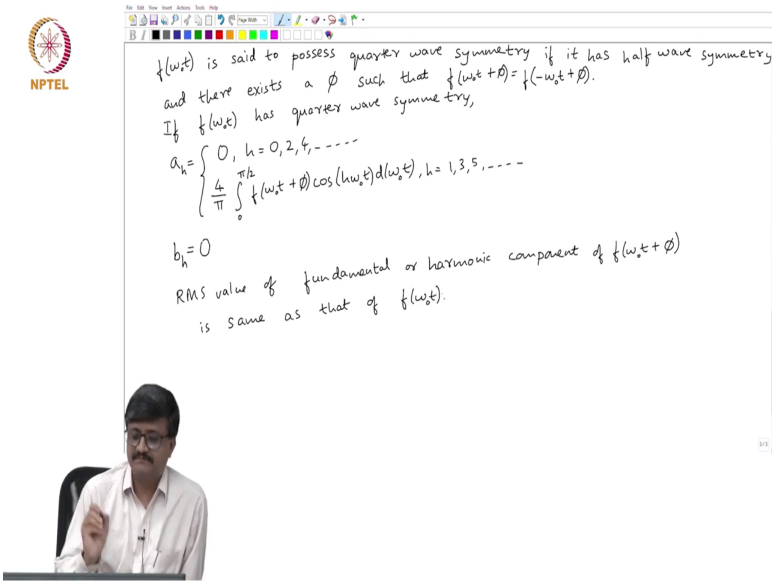

So, what I will do is I will try to give the definition. Let us see whether it makes sense f of omega o t is said to possess quarter wave symmetry. So, quarter wave symmetry is of course a special case of half wave symmetry. See, please note that in the previous two special cases as in both cases, I said half wave symmetry. But in one case, it is odd another case is even, but the point I am trying to make is a quarter wave symmetry waveform can neither be odd nor be even, but still have quarter wave symmetry. But in a I mean invariably it will have half wave symmetry ok.

So, by definition f of omega o t said to possess quarter wave symmetry if it has half wave symmetry and I put one more condition so that it has quarter wave symmetry in addition to half wave symmetry and there exists a phi, phi is some real number, there exists a phi such that.

So, what I am trying to do is I am trying to give a definition which is very general for quarter wave symmetry. So, the definition is like this; such that f of omega o t plus phi is equal to what?

Student: (Refer Time: 17:00).

See what I am trying to do is I am trying to shift the waveform of f of omega o t. See by shifting a wave form the harmonic component the RMS value of any harmonic component or fundamental or DC does not change. See by shifting to your right or left, see only by shifting vertically, you will have a change in the values, but by shifting laterally to the right or left, say when I change the argument from omega o t to omega o t plus phi, I am essentially not changing the value of any harmonic component or fundamental or average value ok.

So, what I am trying to do is I am shifting in such a way that I am shifting the axis say there is a omega o t equal to 0 axis which is a vertical axis. See our independent variable is omega o t. So, omega o t equal to 0 is the vertical axis. So, I am just shifting the position of the vertical axis such that this function becomes an even function that is all I am trying to do ok.

So, if I want to make it an even function, I should say that f of omega o t plus phi is equal to f of minus omega o t plus phi. Is that ok? So, there should exists such a phi. So, that if I shift either to the right or left depending on whether phi is positive or negative ok. So, I should be able to the I should be able to shift it to the right or left such that I mean I get an even function if I change the omega o t equal to 0 axis that is all I am trying to say. Of course it should always have half wave symmetry.

So, half wave symmetry and even function, of course we just now saw that it has quarter wave symmetry ok. So, this is the very general definition of quarter wave symmetry. So, in general f of omega o t need not be even or odd, but still it can have quarter wave symmetry ok. So, that is a definition of quarter wave symmetry ok.

So, if a function has quarter wave symmetry, then what is the expression for a h and b h? So, if f of omega o t has quarter wave symmetry, then what is a h and what is b h? Of course, one of them is easy, one of them is easy to say easy means one of them is 0. Which one?

Student: a h.

Say I.

Student: a h will be deriver if you.

Because I am trying to find a phi, so that it will shift the omega o t equal to 0 axis, in order to make it an even function. See I am not putting a minus here, f of omega o t plus phi is equal to plus f of minus omega o t plus phi. So, b h is?

Student: 0.

0. Now when it comes to a h, I need to again consider two sets of values. So, for some values it is 0, of course it has half wave symmetry. So, it is 0 for h equal to.

Student: 0, 2, 4, 6.

0, 2, 4, 6, so on ok. For h equal to 1, 3, 5; again there will be a simplification. The simplification is that I have to integrate over only quarter cycle. So, it is 4 by pi integral 0 to pi by 2, integral of what?

Student: Integral of plus pi f of omega o t plus phi.

Yeah. So, it is not integral of f of omega o t it is integral of f of omega o t plus phi. Now please note, I am trying to find an harmonic component of a waveform which is shifted to the right or left depending on whether phi is positive or negative. Now that does not alter the RMS value that is the point ok. Please note RMS value is not altered by shifting a waveform to the right or to the left; cos h omega o t d omega o t. So, this is applicable for h is equal to 1, 3, 5, so on.

So, the point to be noted here is RMS value of fundamental component or harmonic component fundamental or harmonic component of f of omega o t plus phi is same as that of f of omega o t.

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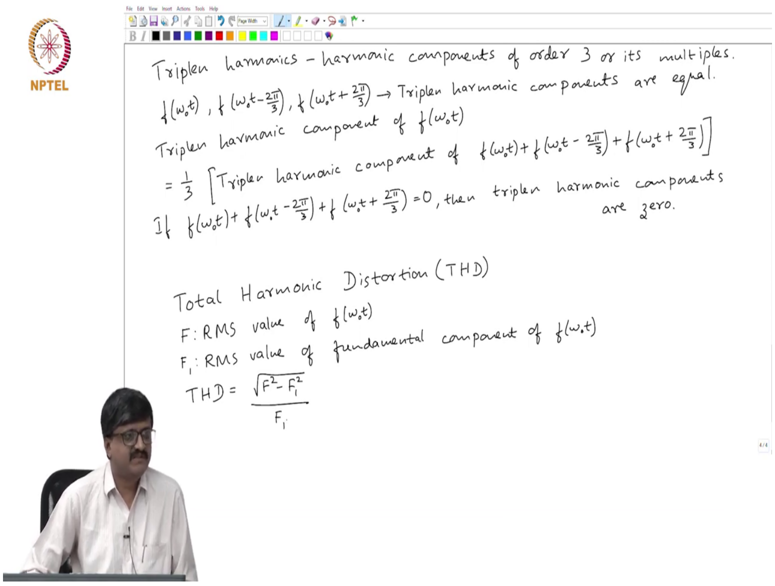

So, there is one set of harmonics which are very important in the AC system that to in the three phase AC system that is the third harmonic sixth harmonic, ninth harmonic, twelfth and so on. So, what I am talking about is the multiple of.

Student: 3

3 harmonics. So, these are known as there is a name for this, triplen harmonics. So, triplen harmonics play an important role in power systems which are three phase systems. So, triplen harmonics these are harmonic components of order 3 or its multiples. Now, if I take three phase wave forms either voltage or current, so, I will use a general notation f. So, if they are balanced.

So, please note what I am talking I mean when I say balance what I mean is not a sinusoidal and balanced. Suppose the currents or voltages in the 3 phases are identical except for a phase shift of 120 degrees. So, suppose I take a function f of omega o t as a current wave form or voltage waveform in phase a, then the current wave form or voltage wave form in phase c the phase b is just f of omega o t shifted by 2 pi. Then in phase c, it is the again f of omega o t minus 2 pi by 3 with a further phase shift.

Now, please note I am talking about quantities which are not sinusoidal. If you see it is sinusoidal then; obviously, we do not need any harmonics Fourier series at all. So, there are harmonic components in either current or voltage. So, instead of using notation i or v, I am using f ok. So, if I look at the three phases suppose the three phases I have an identical waveform for current or voltage except for a phase shift of 2 pi by 3 then what we can say about the triplen harmonic components of these? They are?

Student: No phase shift.

No phase shift. So, if I take that triplen harmonic component of f of omega o t and triplen harmonic component of f of omega o t minus 2 pi 3, they are identical; there is no phase shift as well, and that is also identical to the triplen harmonic component of f of omega o t plus 2 pi by 3; any triplen harmonic component third, sixth, ninth so on ok. So, for these three waveforms, triplen harmonic components; any triplen harmonic component are equal.

So, what I will do is I will take triplen harmonic component of the first wave form f of omega o t. So, can I write this as since they are equal if I take some of these three waveforms, the triplen harmonic component will be 3 times that found in the individual waveform. So, can I say is equal to 1 by 3 into triplen harmonic component of f of omega o t plus f of omega o t minus 2 pi by 3 plus f of omega o t plus 2 pi by 3? How, I mean how does this hold? This is because say, if I take any harmonic component, we are just doing integration ok.

So, if I add two waveforms, then the harmonic component is sum of the individual harmonics because it is just integration. We know the I mean the rules of integration. So, and another fact is the triplen harmonic components are equal for all these. So, this relation holds. Now what is the inference, what we can infer from this equation? Something which they are very familiar with in case of three phase. What do you infer from this? Can we say something about triplen harmonic component from this equation? See what is the usual value of f of omega o t plus f of omega o t minus 2 pi by 3 plus f of omega o t plus 2 pi by 3?

Student: 0.

It is 0. Normally it is 0 under very rare conditions of fault unbalanced fault we have non zero ok. So, if f of omega o t plus f of omega o t minus 2 pi by 3 plus f of omega o t plus 2 pi by 3 is equal to 0 which is true normally ok, then triplen harmonic components are 0. So, just now we saw that there are there is a possibility of one set of harmonics being 0 that is even harmonics being 0 whenever there is half wave symmetry.

Now we saw that I mean in addition to half wave symmetry; if we have the 3 waveforms, please note you are considering only on the AC side 3 phase waveforms ok. On the DC side off course, there is only 1 voltage or 1 current; on the AC side we have 3 phase voltages 3 phase currents. So, if the current or voltage on the AC side satisfies this equation; that means, f of omega o t plus f of omega o t minus 2 pi by 3 plus f of omega o t plus 2 pi by 3 is equal to

0 then the triplen harmonic components are 0. In addition to the even harmonic being 0 if at all there is a half wave symmetry ok.

So, this way we note that there are many possible harmonic waves which are just 0, we need not even try to apply Fourier series to note this ok. Now before going back to the topic, I will just define one more quantity; Total Harmonic Distortion is usually abbreviated as THD. So, suppose I use this notation F. So, see instead of using I or V for voltage and current we just using a general notation F so that it applicable for voltage or current. Suppose F is the RMS value of or periodic waveform f of omega o t and F 1 is the F subscript 1 is the RMS value of fundamental component of f of omega o t, then THD is just defined as a ratio. So, what is the definition of THD, are you familiar with THD, total harmonic distortion?

Student: It is root of.

It is?

Student: Root of.

Root of.

Student: F square minus F 1 square.

Yeah, F square minus F 1 square divided by F 1. So, I will not try to get into other possible definitions say, there other equivalent definition for this. I mean this is the one which we are using. If at all we I mean this is the easiest one to use instead of there is another definition which starts with a summation. So, that is a series in fact, that of course we say rarely useful if I want to use the expression for actually computing the value of THD, I mean if I have F and F 1 I can easily find out. Sometimes I mean you multiply it by 100 and say the THD is in percentage ok.

So, this is about the Fourier series which will be useful for finding the harmonic components. So, if any of the results that were mentioned in this classes not known to you, I would suggest you derive it ok.