DC Power Transmission Systems
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Lecture – 12
Fourier series – Part 1

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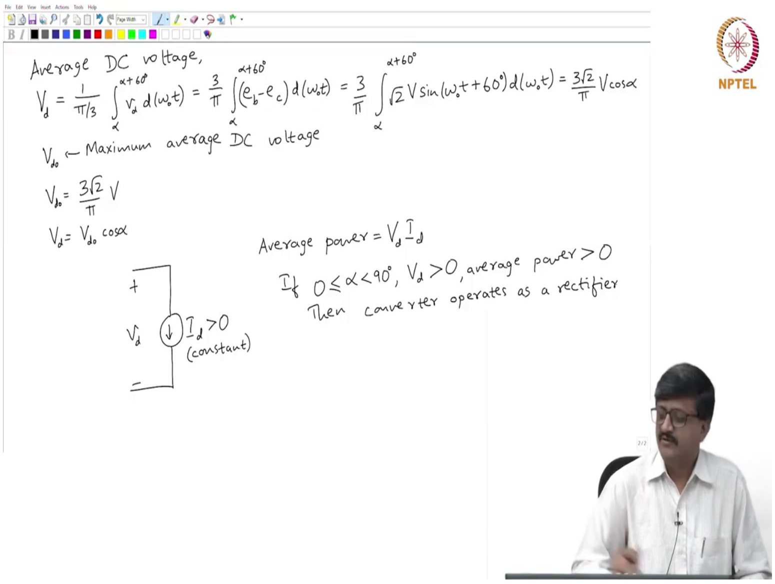

So, let me spend some time on the results of Fourier series.

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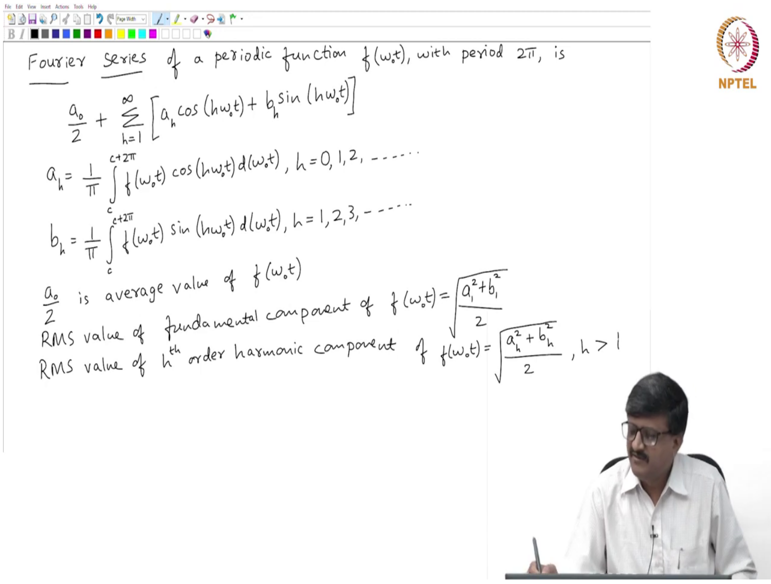

Now, the intension is to get the magnitude of the harmonic components in the voltage on the DC site and also the magnitude of the harmonic components in the current on the AC site ok. So, some results may be useful in trying to compute the harmonic components. So, what we try to do here is consider a periodic waveform; please note whether its AC site or the current or the DC site voltage, they are periodic waveforms. Now, we can say that all have a period 2 pi; it may have a period which is less than 2 pi, but 2 pi is actually a period.

Student: (Refer Time: 01:11).

So, if it repeats every I mean every 2 pi radians; so then we can say that it is a period ok. So, for our purpose we will consider all our waveforms to be periodic with period 2 pi radians. So, I will consider Fourier series of a periodic function; so I will not explicitly state; what is that quantity I am considering voltage or current; I will just use the general function f, it is a function of omega o t of a periodic function with period 2 pi in radian is. So this Fourier series is actually given by a series which has infinite number of terms.

So, the Fourier series is like this; a 0 divided by 2 plus sigma over; I use the notation h, h for harmonic; over h from 1 to infinity of a subscript h cos h omega o t plus b subscript h; sin of h omega o t; so this is the Fourier series. Now, we will not get into the theoretical aspects; whether the Fourier series converges and if it converges whether it converges to f of omega o t.

So, we will assume that it does converges and converges to f of omega; I mean we will just as I said we are just going into the results directly. So, we will use the results; we will not get into the theory behinds, that is not part this course ok. So, the intension is to get expressions for this a h and b h. So, if I take a h; so the result is like this, a h can be obtained from the given periodic function f of omega o t; it is 1 by pi integral of f of omega o t into cos h omega o t with respect to omega o t.

So, I have to integrate over period one period. So, I can choose any arbitrary lower limit, c is any arbitrary real number. So, I have to integrate over one period. So, the upper limit should be c plus 2 pi and this is applicable for h equal to 0 also then 1, 2 so on. So, for all possible values of h; including 0 and b h is say when h is equal to 0, I get a 0; a 0 is the first term ok.

So, b h is 1 by pi integral; f of omega t, sin h omega o t; d omega o t and again integration can be between any two arbitrary limits provided the difference between these two limits is 2 pi. So, it can be any arbitrary c as the lower limit and upper limit will be c plus 2 and this is applicable for all values of h; now h does not take a value 0 here; so it is 1, 2, 3 and so on ok.

So, if I take the first term a 0 divided by 2; now if you look at the equation. So, you have to look at the expression for a h with h equal to 0; so if h is equal to 0, what happens to cos h omega o t? 1. So, if you take 1 by pi integral of f omega o t with respect to omega o t between these two limits; what do you get?

Student: Two times the average.

It is two times the average because it is dividing just by pi. So, if I have divided by 2 pi, you would have got average. So, a 0 by 2 is the average value of f of omega o t. So, this is average value of f of omega o t. Now, in power systems we do not work with the peak values, we normally work with RMS values ok.

So, we are interested in what is happening to the fundamental component on the AC sites; say what is important on the AC side is fundamental component. What is important on the DC side is average component, rest are unwanted things; please note that. On the DC side, we only want the average value all others are unwanted, on the AC side we only want fundamental or all others are unwanted.

So, if I take the RMS value that is the root means square value of fundamental component of f of omega t. So, what is this RMS value of fundamental component of f of omega t?

Student: (Refer Time: 07:05).

So, can I write that say fundamental corresponds to h equal to 1; see fundamental means it is the values obtained from this a h; b h expression with h equal to 1, just as I got average with h is equal to 0. So, it is equal to a 1; are you familiar with this? Can I relate the fundamental component RMS value of f of omega o t and a 1, b 1? So, it is a 1 square plus.

Student: B 1 square.

B 1 square by?

Student: 2.

2; this is taken under square root; so this is the fundamental component. Now, similar to fundamental, I can get a even the harmonic component; that is corresponding to h equal to 2, 3, 4 so on. So, RMS value of we call what is known as h th order; harmonic component, h th order harmonic component of f of omega o t. So, this is similar to fundamental; only thing is in place of a 1, I have a h and in place of b 1; I have b h. So, it is under root a h square plus b h square divided by 2.

So, please note this is applicable for h say; for h is equal to 1, we say the fundamentals. See the name fundamental is for h equal to 1, for h greater than 1; we say harmonic that is a point ok. Fine, now sometimes it is easy to use some other Fourier series other than this trigonometric, this is known as trigonometric Fourier series and there is an another equivalent form which is exponential.

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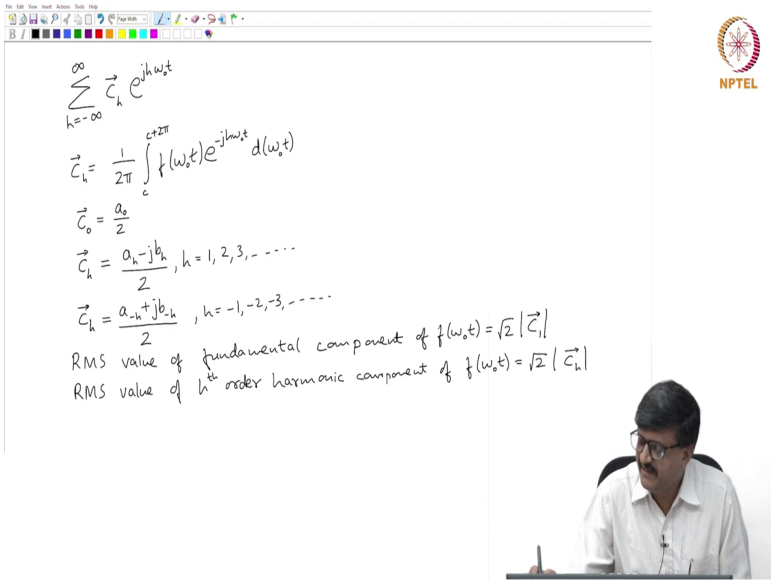

So, the exponential Fourier series is like this; sigma c h; so to explicitly indicate that c h is a complex number; I put a arrow over it ok; c h exponential of j h omega o t. So, this summation is taken over different values of h; what are the values of h? Do minus infinity to plus infinity; so it is minus infinity to plus infinity. So, here c h is the coefficient which is appearing in the Fourier series. So, c h can be obtained from the expression for the function f of omega o t.

So, it is 1 by 2 pi; I will straight away give the answer integral f of omega o t. So, you recall exponential of; I mean do you recall minus j, it is minus j h omega o t; d omega o t. So, the integration can be between any two arbitrary limits, provided the difference between the upper limit and the lower limit is 2 pi.

Now, this is just an alternate series which is equivalent to the trigonometric series; I now some times this is easy to manipulate, sometimes the trigonometric is easier to manipulate ok. So, but the many times the problem is a priori you may not know which one is easy; only after trying both, many a times after trying both will come to know one was much easier than the other or you know for the sake of completeness I am giving this.

But can we relate these coefficients in this exponential series c h to the ones in the trigonometric series a h and b h? So, if I take for a example c 0; what is c 0 in terms of the trigonometric series coefficients? A 0 by 2. You have to just look at the expression for the c h and then expression; I mean look at the expression for c 0 and a 0; then you will see easily see that c 0 is a 0 by 2.

And since a 0 is real, c 0 is real; though in general c h is complex, c 0 happens to be real ok. Then if I take any other c h; say if I take h equal to say 1, 2, 3 and so on. So, can I relate c h, a h, b h? So, can I get an expression for c h in terms of a h and b h? So, it is; it may not be straight forward one can show this. So, I will just give the result it can be shown that this is equal to a h minus j times b h divided by 2. So, this expression is applicable for positive values of h.

Student: Also we can write a h in terms of c h?

We can do that.

Student: C h minus a h?

Yes we can do that ok. Now, in c h; h can be either 0 or positive or negative. So, we have taken c 0 and positive values of h; we can also consider negative values of h in c h of course. So, when it comes to the subscript h in for c; h can be positive as well as negative ok. So, can I relate this to the coefficients a h and b h? So, here also I can relate, but it is not a h minus j b h or a I mean it is not in terms of a h and b h because h is negative. See, what I am trying to do is; I am trying to do this for negative values of h; so h equal to minus 1, minus 2, minus 3 and so on.

So, in this case if I take a h; a h is not existent, a minus h a subscript minus h. So, it is actually a subscript minus h plus j times b subscript minus h divided by 2; these things can be derived, I am just giving the results. So, that gives the relationship between the coefficients of the exponential series and the coefficients of the trigonometric series.

So, if I take the RMS value; so similarly I can take the RMS value and try to express it in terms of c h; just as I did for the trigonometric series. So, RMS value of fundamental component of f of omega o t. So, can I write it in terms of c h? In fact, when I say fundamental, it is in terms of c 1; in terms of c 1.

So, no if you recall or it may take some time to find out what is RMS value. So, if you recall I do not know; you have seen this result, it is the absolute value of c 1. Please note RMS value is always a positive real number ok; it is a positive real number; so you have to take the absolute value of c 1 multiplied by root 2.

See, the contribution to the RMS value of the fundamental which comes from both c 1 and c minus 1; it comes from both c 1 and c minus 1. So, similarly if I take any harmonic component; so RMS value of h th order; harmonic component. If you are not familiar with these results you can just verify these things; please verify if you are not familiar with this.

So, if you are never studied this earlier; you have to do it ok. So, you need to do just simple I mean, I mean I integrate functions and try to show that these are the expressions for the fundamental RMS and h th order harmonic component RMS. So, similarly for the h th order harmonic component; the RMS value is root 2 into absolute value of c h. So, again if I take any h th order harmonic component; h is of course, say when I say h th order h is positive, when I say h th order h is positive. So, the contribution is coming from both c h and c minus h ok.

So, I can use either the trigonometric Fourier series or a exponential Fourier series, but if you observe both involve taking; I mean taking integral over one full period that is 2 pi radian, whether it is a h, b h or c h. Now, many times the waveforms are so complicated longer the duration for which you have to integrate; it is more laborious. So, there are some short cuts or I mean there are some easy ways to get the expressions for a h, b h, c h.

So, if essentially we are trying to calculate the values of a h, b h or c h; I can use either one of them either one. So, if I can somehow reduce the duration for which I integrate then I mean I can quickly get the values. So, it is possible when these waveforms have some sort of symmetry ok.

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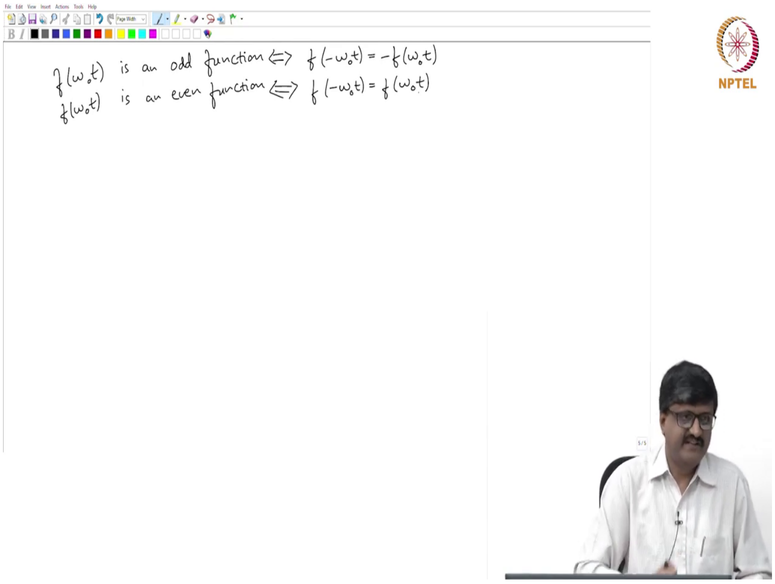

So, there are various forms of symmetry that a periodic waveform can have; so depending on that, there are simplifications ok. So, I will just start with some definitions, I will get into the details in the next class. So, all of you have heard of this odd function? So, we say that a function f of omega o t is an odd function. So, what I want to say is ok; so let me give a precise definition of this.

Student: Sir, (Refer Time: 18:06).

F of omega o t.

Student: (Refer Time: 18:08).

Is an odd function.

Student: (Refer Time: 18:14).

So, that actually means that f of minus omega o t is minus f of omega o t. Similarly, there is one more type of symmetry f of omega o t is an even function. So, the definition of this is f of minus omega o t is equal to f of omega o t. So, if a function is odd; then we get some simplification, if the function is even; we get some simplification.

There are other types of symmetries for example, half wave symmetry, quarter wave symmetry. So, due to all these symmetries; we get some simplification in the evaluation of the integral for the a h or b h or c h ok. So, we will look at all these simplifications in detail in the next class.

So, I will stop here.