


LDPC & Polar codes in 5G Standard
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LDPC Codes in 5G: protograph, base matrix, expansion


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5G NR base matrices

- Two base matrices
 - BG1: 46 x 68 and BG2: 42 x 52
- Block structure of base matrices

$$\begin{bmatrix} A & E & O \\ B & C & I \end{bmatrix}$$
- BG 1
 - A: 4 x 22, E: 4 x 4, O: 4 x 42 all zero
 - B: 42 x 22, C: 42 x 4, I: 42 x 42 identity
- BG 2
 - A: 4 x 10, E: 4 x 4, O: 4 x 38 all zero
 - B: 38 x 10, C: 38 x 4, I: 38 x 38 identity



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Hello and welcome to this lecture on the base matrices in the 5G new radio standard. Okay, like I mention the 5G standard specifies parity check matrixs for LDPC codes using this base matrix and expansion method, except that there are two base matrices and several possible expansions. Okay, and the bass matrix itself is quite large, it is not a very small base matrix in my example I showed you a 3 by 6 based matrix. The base matrices in the 5G standard, there are two of them, one is 46 by 68, the other is 42 by 52.

Okay, so it is quite a large base matrix. Okay, 46 by 68 and 42 by 52, okay, quite large. Okay, and the base matrices have a very useful structure. Okay, so even though they are large, so turns out you can, carve out some small matrices from them and this matrices have a very, very easy structure, so, in particular, there is a block structure. Okay, so they have a block structure like this. Okay, the first block is A, first row block is A, E and O and the second row block is B, C and I okay, so in the base matrix one, the first base matrix BG1. A is a 4 by 22 matrix, E is a 4 by 4 metrics, O is a 4 by 42 all zero matrix, so we can say all zero, should be careful here, so it is actually all minus 1, remember.

Okay, so the base matrix is all minus 1, so it is get replace by, you know, when it expand gets replaced by the all zero matrix. Okay, so B is 42 by 22, it is must much larger, right so 42 by 22, C is 42 by 4 okay, and I, part is actually an identity, 42 by 42, it will have an identity

structure. Okay, so I will show you the same examples later on. Okay, so this block pictures very important, the first row block usually is tilt with separately in the encoding part and the second part is dealt with separately. Okay, so it is important to note this block difference.

BG 2 also has a very similar structure. Okay, remember BG 1 is 46 by 68, so you can see a 4 plus 42 is 46 and then 22 plus 4 plus 42 is 68. Okay, and the base graph two also has a very similar block structure. A 4 by 10, 4 by 4 and 4 by 38 and then B is 38 by 10, 38 by 4, 38 by 38. Okay, easy enough to describe, the standard goes into details on what this, each of these things are, I will show you some, some pictures next to illustrate how this looks okay.

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NPTEL

Example: BG2
 ignored
 dot (blue)

0₁
 all-zero

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NPTEL

Example: BG2
 ignored
 dot (blue)

0₁ S2
 all-zero

function

0₂
 S2

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So, how does the block look? This is how it looks, this is my A part, okay, this is BG2 by the way, this is E, this is O. Okay, so you can see O is like the empty part, so this is, I should tell

you how I got this picture first, so this is a visualisation of the base matrix, what I have done is, minus 1 is ignored. Okay, if the base matrix entry is minus 1, I just put space there. Okay, I am not showing it because it gets replaced by all zeros. Okay zero to any other value, 0, 1 etc. right, I am showing as a dot. Okay, the blue dot. That is the visualisation, just to show you how the structure looks, that the nonzero values are.

So you see this A part is fairly dense. Okay, not of nonzero values, E is also, E also has a very peculiar special structure is called the double diagonal structure. Okay, you can see, if you come and see closely, you will see E has one main diagonal and the off diagonal, right so you can see the main diagonal and the off diagonal, this is the double diagonal structure in E, this is all, all zeros, so this nothing in this, I mean all minus once all zeros.

Okay, and then you have a B part which is the big part and then, what did I called this, C which is this a long rectangular small part and this is the identity part and you can see this is just diagonal, in fact, all this entries are zeros. Okay, as an they are replace by an identity, so overall, even after expansion this will be identity. Okay, this will be all zero after expansion. Okay, and the top O part will be all zeros, so this is how the matrix will blow up.


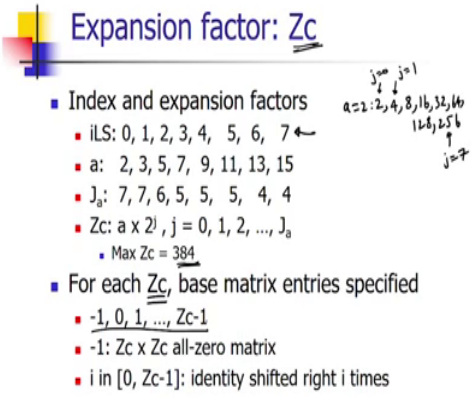
Okay, so even though like I said that this is very big matrix, is 42 by 52, you can sort of stop here if you like, you know for an instance, you can do this. Okay, so this small part is another base matrix, you can ignore all this other things, you can sort of puncture, all this things out it, you do not have to evaluate those parities, only use this part of the base matrix to transmit your code word. Okay, you can do that, that sort of thing is called rate matching, so you can choose to do that.

In fact the first two message blocks are not transmitted at all in 5G. Okay, they are always puncture, this to guys are always puncture. Okay, so I do not want to spend too much time on the rate matching and all these thing that happen in 5G, I want to focus on the LDPC code first and then maybe in a later lecturer point out, how the rate matching actually works, so even though this whole parity check matrix is very big, you can see this large identity part allows you to just cut off wherever you like. In case so you can get smaller base matrices and different rates quite easily from this matrix. Okay.

So, like I mention I am not going to spend too much time on thinking about rate matching in this lectures, but it is good to know where it came from. Okay, so this is how the matrix looks, the entries themselves are very different. I will show you some of the entries in few

examples in the later slide. Okay, so hopefully this is clear. If you go to the standard document, there will be a clear description of how to construct this base matrices also.


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Expansion factor: Z_c

- Index and expansion factors
 - ILS: 0, 1, 2, 3, 4, 5, 6, 7 ←
 - a: 2, 3, 5, 7, 9, 11, 13, 15
 - J_a : 7, 7, 6, 5, 5, 5, 4, 4
 - Z_c : $a \times 2^j$, $j = 0, 1, 2, \dots, J_a$
 - Max $Z_c = 384$
- For each Z_c , base matrix entries specified
 - 1, 0, 1, ..., $Z_c - 1$
 - 1: $Z_c \times Z_c$ all-zero matrix
 - i in $[0, Z_c - 1]$: identity shifted right i times

Handwritten notes on slide:
 $a = 2, 4, 8, 16, 32, 64$
 $j = 0, 1, 2, 3, 4, 5, 6, 7$



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Okay, so now what about the expansion. Okay, now the expansion, the base matrix I told you has size 42 by 52, 46 by 68. Okay, that is the base matrix size. Okay, now, there is something called index, which is ILS in the standard, it is just the way to number the, order the expansions and then there are two factors here, which control the expansion, there is a factor called A and then there is a factor called J_a and the expansion is A into two part J, for J going from 0 to J_a , so there are a lot of expansions possible, so if you look at A equals 2, it corresponds to expansion of 2, 4, 8, 16 right, all the way up to 56 right, so 32, 64, 128, 256 right, so this is, this corresponds to J equals 0, this is J equals 1, all the way up to J equals 7.


Okay, so those are all the expansions possible and you can figure out where the largest welcome from it turns out the largest expansionist 384 okay, so this is just given to sort out all the expansions, now for every base matrix and for every expansion there has to be, you have to fix the values. Okay, you have to specify what the values are. Okay, so for each Z_c the base matrix value are specified, they take values from minus 1 to Z_c minus 1.

Okay, alright, so let me repeat this once again, for each expansion factor, defined then this fashion, there are a lot of expansion factors like I mention that for every A, you have to multiply with two part J and J going from a certain range, and so you have a lot of expansion factors and for every expansion factor you have to specify the standard, clearly specifies,

what the base matrix entries are. Okay, so you have to look at the standard document and figure out what the base matrix entries are for each expansion factor.

So, in short given a base matrix and given an expansion factor, you can find out the exact entries in the base matrix from the standard description. Okay, so that is something you can write a program for, if you like, it is possible. Okay, I am not going into the details of how that specification happens in the standard, I will encourage you to read it on the standard document, I will provide a link for that is part of the course material, you can go look up the definition, there is lot of detail there, will skip that, but I will provide the base matrices themselves for every possible base metals matrix, base graph 1 or 2 and the expansion factor I will give you the base matrices directly, so you do not have to do that, but nevertheless it is good to go look at the description and figure out how the base matrices are specified in the standard.

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Example: Base matrix entries

- 10 rows and 20 columns of BG2
- $iLS = 3, j = 4, Zc = 48$
- $a=3$

42 x 52

6 E = used in encoding

24	14	23	37	-1	-1	47	-1	-1	8	0	-1	-1	-1	-1	-1	-1	-1	-1	-1
5	-1	-1	12	19	12	19	8	29	31	-1	0	0	-1	-1	-1	-1	-1	-1	-1
8	35	-1	46	47	-1	-1	-1	43	-1	0	-1	0	0	-1	-1	-1	-1	-1	-1
-1	41	6	-1	36	28	28	14	12	37	1	-1	-1	0	-1	-1	-1	-1	-1	-1
8	16	-1	-1	-1	-1	-1	-1	-1	-1	-1	5	-1	-1	-1	-1	-1	-1	-1	-1
41	42	-1	-1	-1	26	-1	27	-1	-1	-1	1	-1	-1	-1	0	-1	-1	-1	-1
27	-1	-1	-1	-1	7	-1	31	-1	30	-1	17	-1	-1	-1	-1	0	-1	-1	-1
-1	7	-1	-1	-1	13	-1	9	-1	-1	-1	6	-1	37	-1	-1	-1	0	-1	-1
3	43	-1	-1	-1	-1	-1	-1	-1	-1	-1	8	-1	-1	-1	-1	-1	-1	0	-1
-1	2	-1	-1	-1	-1	-1	-1	30	-1	40	35	-1	-1	-1	-1	-1	-1	-1	0


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Okay, so here is an example of a base matrix, so I am not shown entire base matrix here, the BG2, if you remember, there is 42 by 52, I am only showing 10 rows and 20 columns. Okay, so after this, below this and to the right of this, there are a lot of more entries in the base matrix I am skipping all that, this corresponds to ILS equals 3, J equals 4 and Zc equals 48, so this will correspond to a some A value, I believe it is a 3. Okay, so I think this is A equals 3 right, so 3 into 2 power 4s, 48. Okay, so this is how the entry looks, so Zc is 48, so the base matrix entries will go from minus 1, 0, all the way up to 47.

Okay, so you can see this 47 here, this 26, 36, 46, all sorts of entries here and, so what I have shown here is the E part, the matrix E and this will be the O part, this will be the A part and all that, I am drawing some special attention to E here I tell you why and then you can see the identity part coming here, right, so this is the identity part. Okay, you can see the zero along the main diagonal and then minus 1 throughout, remember the zero gets replaced by the identity matrix. So overall after expansion you will get an identity matrix here, this O will be a big zero here and then, this entries are all over the place.

Okay, so I mention this double diagonal structure in E is right, so this is diagonal, this is diagonal, everything else is zero and then, this also has a peculiar sort of form. Okay, so you will have this 1, minus 1, 0, 1, so this plays an important role later on, we will see that in the encoding with play a role. Okay, so we will come to that and we see it. Okay, so this is how it looks right, I mean this nothing much more to talk about here, so this things are specified the precisely in the standard and like I said, we are not going to see the full description, but this is how the matrix looks, for any other case, the entries are specified, you can go find them and write it out and look very closely, you will get numbers like this, the only practice this E part I will urge you to look at the E part in everything and this first column will also have a peculiar structure.

So later on I will talk about why this is true? Why this is useful, this double diagonal structure, so this is sort of useful in encoding. Okay, so this structure useful in encoding, this double diagonal structure, I will mention that, later on in a lecture.

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Okay, so this is some one example. I think this is a last slide. What I want to do, is a show you, so I promised that will show you the base matrices themselves, so here is a folder where I have written a program to generate all the base matrices in the, in our standard so you can do that and then, this is just written down here for every single possible J and ILS and then I am just going to show you some of the base matrices that are there, so you can pick anything else, so, for instance, let us say we pick this one, okay, so shall rather large base matrix, maybe I can blow it up a little bit not sure, if visible this is.

Okay, so, like I mentioned, I have written a program to look at the standard and then pull out all the base matrices and so this is all the base matrices as text documents, so let us pick out one such base matrix, this is got an expansion factor of 208 and you can see how the matrix looks, so this is a base the graph two, once again, 42 by 52 and you can see the dense part here at A, B and C and the E part here, right, and then the long identity part here and then the all zero part here. Okay, so this is how the base matrices look and you could write this as well, but I am not sure for this class it is so crucial for you to, you know, write the piece of code for generating this base matrices, so what I would do to help you out is to provide the base matrices themselves.

So I will create a google drive folder and share it on the course page, you will get access to all this files and you can look at the base matrices and pulled out the base matrix for any particular specification, like I said, it is not a big secret, is there in a standard except that you have to read through and write a small piece of program, to create the base matrices, I am helping you out, there with providing the base matrices myself.

Okay, so the main focus of this course is on building decoder as for this course. Okay, so like I said, how do you make a decoder? How do you describe and write down code Matlab code for a decoder, for this kind of parity check matrices, like I said, this is just specification of the parity check matrix, a from here, how do you build decoder. Okay, so that is going to be the main challenge as far as this is courses and let us focus on that mostly, okay. Instead of worrying about how to create the base matrices.

Okay, so this brings me to the end of this lecture, so from the next lecture onwards will start looking at decoders for LDPC code. Okay. Thank you very much.