Multirate Digital Signal Processing
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Lecture – 39 (Part-2)
OFDM Applications – Quantization – Part2

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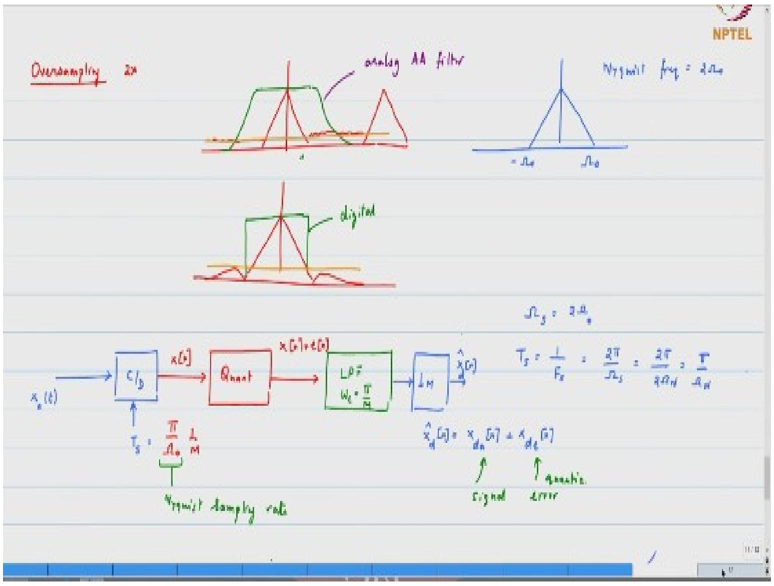

So over sampling, how can over sampling help and what are the ways in which we can leverage it, we can take advantage of it okay, so over sampling basically means that we can when you do the representation, the copies of the spectrum now come further apart right that is what is the; so let us take the a simple case that I do 2x over sampling, now this is where you will start to see the benefits of what we have done what we have discussed okay.

So, when you have quantization noise basically, it is uniform and over the entire range that we are going to be working with so, anytime we have quantization noise, we have to worry about so, the reason you would go to 2x first and foremost before if you want to do sampling would be to do anti-aliasing filtering, so you need to have an anti-aliasing filtering before you do sampling.

If you; if I were to do Nyquist sampling, my anti-aliasing filter would have to be of this form correct, one side once I have this sort of an anti-aliasing filtering, then the quantization then I will quantize but the problem is; if I do this then my next copy of the spectrum comes right there, I do not have any room to do manipulation on the quantization noise, I am stuck with the quantization and also the fact that I am going to have a very restrictive design of the anti-aliasing filter.

Now, if I say that okay I am going to allow 2x over sampling that means, these are not in the picture, this is not there, this is not there, this is not there, this is the 2x over sampling, I now have separated it out now, so the anti-aliasing filter does not have to be as stringent as before it could be something that has got a representation as you know we have designed these filters, you want to keep only the desired portion of this signal spectrum.

So, keep it flat in this portion and then have a drop-off okay, now what will this do for the quantization noise, quantization noise is going to be present across, right so basically quantization noise will get generated across so effectively, what you will see with after the signal is present is that you will see the signal, the signal is present here, there is some effect of quantization noise.

So, the effect of the quantization noise is that there is quantization noise in the desired signal band, right and of course, outside of it also the; because the anti-aliasing filter did not kill all those samples, there is the quantization noise but basically, you will see some kind of a roll off of the quantization noise, okay, so there is the noise is present even outside of the signal spectrum, okay.

Now, so this goes and now if I down sample by a factor of 2, which is what you would need to do to achieve, I have over sampled it at the; so after the anti-aliasing filtering is over sampled by a factor of 2, I have to at some point down sampled by a factor of 2, so which means that I would now apply a digital filter and then down sample by a factor of 2, this would be a digital filter; digital and this by the way is an analog anti-aliasing filter, okay.

Okay, wait, wait, the desired signal is that is a good question, the desired signal plus some unwanted signals, it is not necessarily quantization as you are correct, I have to be careful with this, so what has been allowed by the what has been allowed that I am very, very correct okay the what has been allowed by the anti-aliasing filter is some portion of the signal that is outside okay, I do not know what is outside.

I have my desired signal there could be something beyond my, the desired signal outside the bandwidth and it is that portion of it, so it should not be orange, it should be something else which is part of the continuous time signal that is present there when after quantization, quantization noise will go all the way from the up to the sampling frequency, I have to down sample and then get rid of the; you are right.

So, this is actually not orange, it is actually red, the quantization signal has to be flat, okay, quantization comes after the anti-aliasing filtering, this portion is what has; what has come; I do not know what is outside of my signal band, it may be the adjacent channel, it could be some other unwanted signal, so some other extraneous signal that is present which has come into my system because my anti-aliasing filter is not very tight, okay.

But subsequently, I will get rid of it okay, so this is a way which we originally said when we were talking only about sampling, we did not talk about quantization, we said that this was a technique to actually reduce the complexity of the anti-aliasing filter, if you recall, right that so we really did not know so but does this give any benefit at all in terms of the quantization noise and if so, what is the benefit and how do we quantify the benefit okay.

So, here is the block diagram and let us take it in a systematic manner, so I have a analog signal xa of t, which is followed by a C to D converter and this is an over sampled approach, the sampling period if you were to sort of related it to so, let me just write it down here, the sampling period must be at least 2 times omega n; omega n is the Nyquist frequency the maximum possible frequency.

The sampling period = 1 over Fs which is Fs = sigma s/ 2pi, so this becomes 2pi/ omega s, this is nothing but 2pi/ 2 omega n, so this would be pi/ omega n okay, so Ts at if you want to do a Nyquist sampling would be pi/ omega n, so this has been set to pi divided by omega n * 1 over m that means your sampling period is 1 over m of the Nyquist sampling period okay which means that you are over sampling by a factor of m.

So, this produces for us the signal x of n which would be passed through the quantizer that would produce for us the x of n + e of n, the noise; noisy signal + the error signal after which we will go into the digital domain where I will pass this through a low-pass filter with cut-off omega c = pi over m, the reason we are doing that is because we are going to do a down sampling by a factor of m, to take care of the over sampling factor followed by a down sampling by a factor of m.

And here comes the final signal which is the quantized signal okay, now I will be with you in a minute, so just for differentiating this quantized signal from the earlier type, where we did not do any over sampling just want to write it with the following subscript D that means, it has come through a process of over sampling and down conversion and xd of n can be related to xda of n that is the discrete time signal through this down sampling process + xde of n that is the error signal.

So, one is the signal component, this is the signal component and this is the error component quantization error component okay, so just some notations are we can do yeah, there was a question. **"Professor – student conversation starts"** student: What is omega N over here, is it the Nyquist rate or is it the maximum. Professor: okay, good question let us clarify that this would be in the continuous time if we were to, now this is all discrete time here, so if you were to draw it, write it in the continuous time, this signal this is omega n okay.

And maybe the confusion is coming because of this okay, whatever this value is let us call it omega0 that is probably the best okay, omega0 – omega0, the Nyquist frequency; Nyquist frequency = 2 times omega0, right so and is that what we called as omega n previously okay, so if we have called that as omega n, then just make this as omega0, whatever is the highest frequency component, it would have to be, so let us change it everywhere.

So, this would have to be omega 2 times omega0 for the Nyquist rate, okay let us not introduce omega n if you have not introduced it before and so this would be pi over omega0 times 1 over n, whatever is the; so pi over omega0 would represent the Nyquist sampling rate, okay, so this is Nyquist sampling rate okay, so given this form, the signal to quantization noise ratio, so please note what are the; **"Professor – student conversation ends"**.

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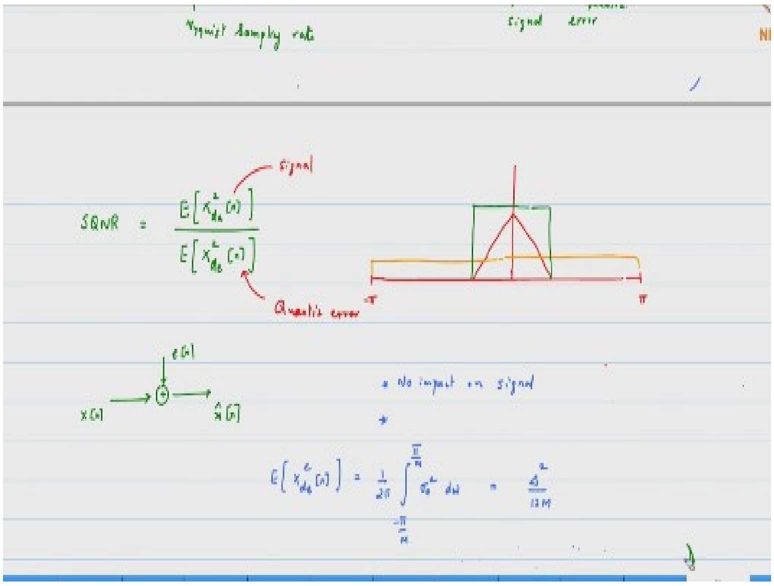

So, the signal to quantization noise ratio now becomes SQNR is expected value of x da of n whole squared that signal power divided by expected value of x de squared of n, so that is the signal component, this is the signal part, this is the quantization error part okay, quantization error okay, so here is where I would need you to visualize the process as follows, so the pictorial representation of this is the following.

I have my desired signal, I am going to plot from - pi to pi and I am assuming that the reason is your constraint to only your part of your of the range - pi to pi is because of the over sampling part okay, now this has gone through a quantizer of B + 1 bits, so the quantization error is uniform in this region okay. Now, what does the digital anti-aliasing filter do; it is going to ideally portion of this okay, did the anti-aliasing filter?

Combination of the anti-aliasing filter and analog domain and the anti-aliasing filter in the digital domain, did it do anything to your signal? Your signal went through without any issue, right no problem at all so basically, we are leveraging a key concept in the modelling of quantization noise which is as follows x of n added with e of n is what produces x hat of n, so we have modelled the quantization as a linear perturbation right basically, it is a linear.

Some other signal that got added, so when you now think of it in terms of the filtering that it has gone through, you find that there has been no; it is yes xe of n, x of n and e of n got added. But when you pass it through a linear filter, the linearity does the; it is the same as passing the x of n through the filter, then adding it to e of n passing through the filter, the property of linearity so effectively, nothing; no impact on signal okay, so a qualitative statement okay.

No impact on signal okay, so signal power has gone through untouched, so whatever was the original signal power that is what should do. What about noise power, reduced or did not make a difference, look at the spectrum, how much of the noise power is outside of the band of interest? So, in other words if you have you basically retained 1 over m of the total noise power okay, so you have actually though in the context of the; previous time when we discussed over sampling.

It was only to reduce the complexity of the anti-aliasing filter; the analog anti-aliasing filter. But now, you see that it is actually more than what you actually obtained, you actually now can go through and show the following result so basically, this is the expected value of x de of n squared will be 1 over 2pi integral - pi to pi, the range is sigma e squared and sigma e squared, wait a minute, it is actually - pi over m to pi over m that is the part that is retained, so the omega okay.

So, basically this should come out to; if you go through the process, this should come out to be delta squared/ 12 times m, 1 over m times the sigma e squared okay, so this is a huge advantage because now your signal to quantization noise ratio, the formula is slightly different okay, the formula is different, so let us quickly capture that and then and take it forward.

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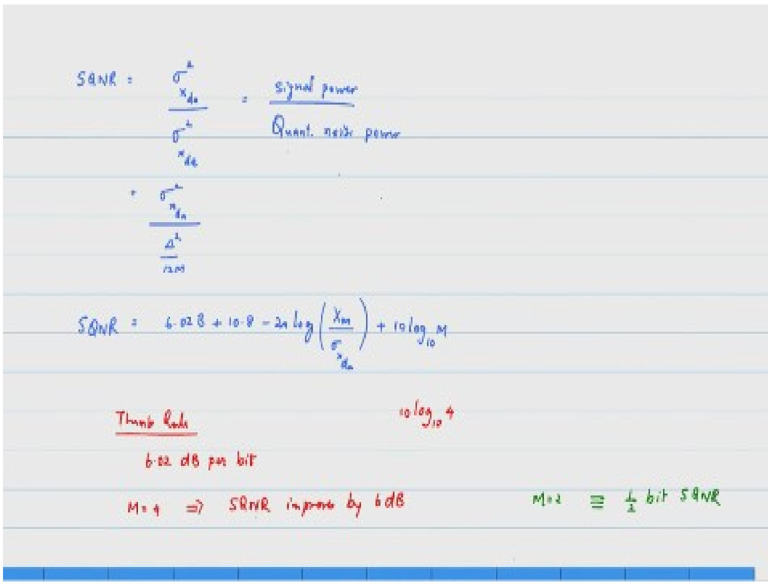

So, the SQNR signal to quantization noise ratio is basically sigma squared of x da that is the signal part divided by sigma squared of x de that is the signal power by the quantization noise power, so this is effectively signal power divided by quantization noise power okay, so one step away from a very important result, so this comes out to be sigma x da squared, I will leave that as this divided by delta squared / 12m.

So, if you go back and write it in dB, this is in dB this will be 6.02B, the same one that we got +10.8 - 20 logarithm of x of m/ sigma x da times 10 log based; 10 log 10 of m that factor of m has now entered the part okay, so by increasing your over sampling rate, you not only reduce the complexity of the anti-aliasing filter, you actually also benefited in terms of the noise, okay. So, here is a very important element, we have a thumb rule.

The thumb rule says thumb rule has from before we know, it is 6.02 dB or 6 dB per bit, right; 6.02 dB per bit, now if you go ahead and look at 10 log 10 4, basically if you have an over sampling factor of 4, what will you get, 6 dB, so which means that what I can achieve with one additional dB, I can achieve with over sampling by a factor of 4. So, of course if you already have a 16 bit quantizer and you know say I want 18 bits of precision, it is possible.

Because you can do your over sampling by a factor of 8 and then you will 4 times 4; 16, factor of 16 and you will get the required this thing, so we can also write the following, which is m = 4 implies the SQNR will improve by 6 dB, improves by 6 dB or it is the same as saying that m = 2 is equivalent to 1/2 bit improvement in SQNR okay, so you can use that as a relationship and you can work out the key results okay.

Here comes the most interesting part of this so okay, this is good enough we have done, we actually achieve but keep in mind that if you want to go from 8 bit precision let us say you have an 8 bit quantization and you want to get 16 bits of the equivalent performance, you will find that M will become very large, so you will have to have a lot of complexity. So, here is where the thought process comes in.

And we look at this figure and the question that raise is; does the quantization error need to be spectrally flat; answer is; the way we do quantization today, it is going to be that way there is nothing we can do about it, so we then ask the so, this is spectrally white, what are we asking, can we make the spectral, the quantization noise coloured, coloration of the noise. Coloration depends on some kind of memory between adjacent samples only.

When you have memory, you can introduce coloration so basically, there is if each of these samples are independent of each other, we cannot produce that so, the question is can you do something in terms of the coloration and here is a the simplest of the structures that introduces coloration and let me sketch it for you, leave it for you to do a little bit of analysis. We will pick it up from here in the first part of tomorrow's lecture, we will conclude this discussion.

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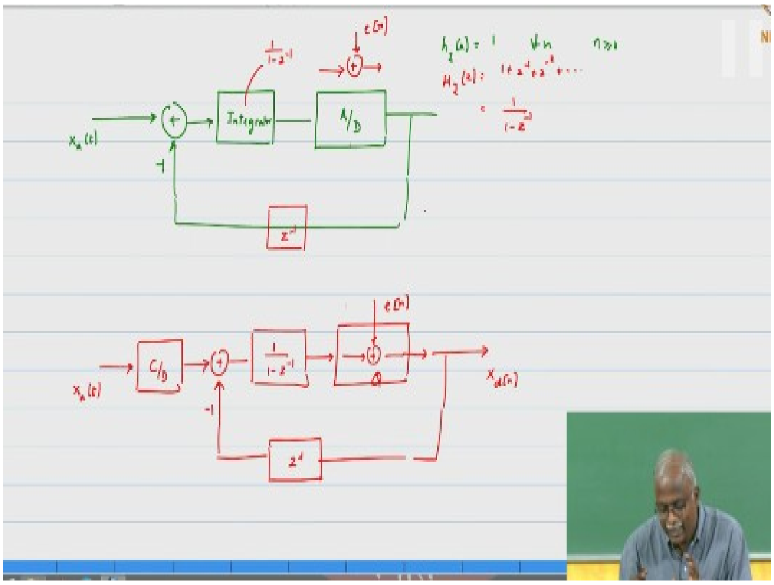

So, here is a modified quantizer, I have the analog signal xa of t, this is going to be the C to D type of quantizer, so I am going to precede the quantizer so basically, we will think of it as instead of C to D as an A to D converter so basically, it is a finite number of bits conversion apart but I am going to precede it with a integrator okay. I am going to precede it with an integrator, so what is an integrator?

An integrator is one that adds all of the past samples right, so can you write for me the impulse response of an integrator; hi of n = 1 for all values of n starting from n > or = 0, right, so basically that is a causal integrator, so if I were to ask you the filter response of this, hi of z integrator hi of z is 1 + z inverse + z - 2 dot, dot, dot this is 1 over 1 – z inverse, is it stable; no, it has got a pole on the unit circle, so it is not a stable system.

So, a discrete time integrator is not a stable system however, if you were to control the stability part, so think of this as a 1 over 1 – z inverse okay, if you need to cancel out the effect of the integrator, you must differentiate first, so how do I differentiate first? So, I take this signal, take this signal and subtract it from the difference so basically, with a minus sign, with a - 1 okay, so if I over sample my signal, 2 successive samples will look almost the same, right.

So, if I subtract out the previous sample, it looks like I am making a differentiation of a small step differentiation x of n - x of n – 1, then I pass that through an integrator and then quantize it now, you may say well you know really does this make any difference, you differentiate it and then you are cancelling it, so look at the following aspect, so if that is 1 over z inverse, think of this as a z inverse, treat this as a point where the error signal comes in okay.

So, the equivalent diagram that I want you to look at is the following x a of t followed by a C to D it is the perfect quantizer, no quantization error and then an additions part, so this is a discrete time equivalent of what we have drawn, the integrator is 1 over z inverse followed by the quantizer; quantizer, I am going to draw it as the addition of the; so this is the quantizer, I am going to add it and show it in terms of the addition of the error signal.

I am going to take this input with a delay and subtract it from my output okay, so this is the output signal, which I want to analyse and so we would now look at the; have you done anything to the what we would like to show is that the signal goes through so basically, what I want you to do is; if the output signal is xd of n, if this is xd of n, think of it as 2 inputs; one is the signal, the other one is the error.

I want to look at the transfer function for the signal part and the error part and then we will take it forward from there but the key thing to note is that this is an attempt to colour the noise and the coloration of the noise basically, says I want to do something, if I can; I cannot do anything to the energy of the noise, the total power will remain the same, so if I can do something like this, redistribute the noise in the quantization.

Then, what I can do is actually, when I do the digital filtering, I can throw out more than the what I could do before, so that half bit improvement can now become 1 bit improvement, 2 bit improvement, 3 bit improvement depends on how you have shaped the noise okay, so that is what I will just show you the noise shaping part but basically, it is a very, very interesting mechanism where you start thinking saying I do not want flat, which I must introduce some sort of memory between the successive samples.

How do I do that? That is what we are going to be looking at tomorrow okay and all this is possible because you have the flexibility of the sampling rate okay, thank you very much.