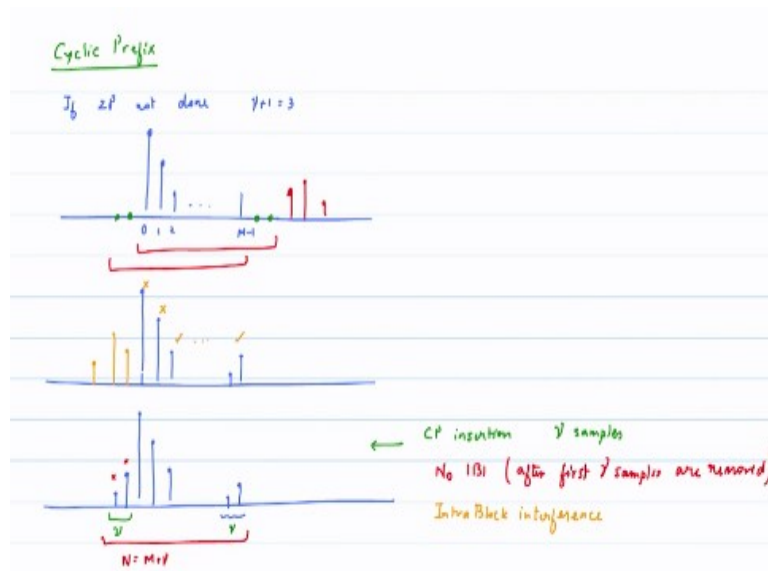


Multirate Digital Signal Processing
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Lecture – 36 (Part-2)
MCM Impairments and CP – Part 2

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So basically now we move to the second type of block transceivers where we use cyclic prefix, but this one actually requires some amount of motivation because zero padded works, what is wrong why do not you just go along with that okay. So there is a complexity in terms of the Moore-Penrose pseudoinverse that is the non-trivial operation and this matrix could be large because you know you could have thousand channels.

So which means that okay it is not going to be as straightforward to the inversion, but let us see if there is an advantage and a preferred way that we can do. So here is a question or a pictorial representation. Now if the zero padding had not been done, if zero padding not done okay what would have happened? You would have seen leakage of one block to another block. So let us take the case where $\nu + 1 = 3$ that was the channel that we considered.

Now if I had a block of M samples okay dot dot dot. So this is 0 1 2 $M - 1$. I insert two 0 valued samples then the next block begins. So this is the zero padding process. Now if this zero padding had not been there, maybe one observation that means the previous block also had zero padding right, every block has zero padding. So I can associate my block now as the

block followed by zero padding or equally I can do it as the block with zeros in the beginning followed by the zero.

Now I would like to take the second version where I have zeros in the beginning okay. Now if these zeros were not present what would have happened? I would have had this block right, I would have had this block of data and there would have been the previous block of data which came all the way up to the end because the zero padding was not there and there is a 2 tap ISI channel which means that this signal would have had suffered from ISI.

This signal also would have suffered from ISI from this point on I am okay right. So basically because of the 2 tap. So effectively the block interference was felt by the first 2 symbols okay. So the notion that says that okay you have to somehow if you want to protect the first 2 symbols from ISI, the best thing is to insert zeros; however, the option number 2 that we are going to be considering says that take this vector, this block of data, let the last 2 symbols be, last 2 samples be this.

You still have to protect your, so your, the protection will come by inserting the last 2 symbols here okay. So these are my new symbols, those are going to be inserted here okay. So which means that I still have a block which is equal to $N = M + \nu$. So this is my block. I have taken the last ν samples, where ν is the number of taps of the symbol and put it in the front. So do I have IBI? Which are the 2 samples that will have IBI?

Channel filtering is a continuous process right, it is a continuous, so this will have IBI, this will have IBI, but notice my block of data is okay right, but what about this one, the first sample here. It has ISI, but it has it from itself, from its own sample. So in one way I have mitigated the problem by saying okay if I throw away the first new samples they are the ones that have IBI from the previous block, but what I am left with is a block of M samples where there is block from interference from within myself.

But maybe that is a better way. So here is the summary of the cyclic prefix. You understand cyclic prefix. Whatever is the value of ν , I take the last ν samples of the block and then put it up front okay, last ν . So basically you still have a longer block, the redundancy is coming in not in the form of zeros, but is coming of nonzero samples which are from your own data block.

So this is CP insertion. So what we are seeing here is CP insertion. It basically means the new samples from the end are introduced at the front okay and this also means that there is no inter block interference. Once I have removed the first ν samples. So no IBI after the first ν samples are removed okay, that is the correct statement. Also it has ISI, but that is what we will say here intra block interference is present.

Intra block interference, but all the samples have intra block and all the intra block interference is from within itself, it does not come from outside okay. So this is a useful observation for us to keep in mind and also build upon. Now I need you to quickly work with me on the following example okay. I am going to ask you to do a linear convolution, but write it in matrix form.

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$$\text{Ex } \{x_0, x_1, \dots, x_7\} * \{c_0, c_1, c_2, c_3, c_4\}$$

$$M=8 \quad \nu=4$$

The diagram illustrates the matrix representation of linear convolution. A large matrix is shown with a vertical dashed line and a yellow bracket underneath. The matrix is partitioned into two parts: the left part contains the channel response coefficients c_0, c_1, c_2, c_3, c_4 and the right part contains the input samples $x_0, x_1, x_2, \dots, x_7$. The output vector y is shown to the right of the matrix, with the first four elements being zero and the remaining elements being $x_5, x_6, x_7, x_0, x_1, x_2, \dots, x_7$.

So this is an example and it is kind of, it requires a little bit of writing down, but please do that it is a helpful example. So basically I want to do a linear convolution of a sequence of 8 samples X_0, X_1 through X_7 okay. There are 8 samples. So $M = 8$, I want to do linear convolution. Asterisk is linear convolution. I want to convolve it with a channel response C_0, C_1, C_2, C_3, C_4 okay.

So this is the case where $\nu = 4$ okay, so basically we would have to write down a matrix and show how the convolution occurs and I am sure this is something that you can write down fairly easily. Let me just sort of start you off on this. The first entry is H_0 , so here is the

vector, X_0, X_1, X_2, X_7 , that is my vector. I am going to draw it a little bit larger than necessary in a minute you will see why.

So my matrix okay, so this is the not equal to yeah this is what has to multiply. The second entry in your convolution matrix will be $H_1 H_0$. Third entry $H_2 H_1 H_0$ and so forth correct and you can please go ahead and write down all the equations. Now if I had padded myself, if I had to do zero padding, I would have to pad with 4 zeros, 1, 2, 3, 4. If 4 zeros were present that means my matrix does not start from here, it actually starts from a little bit ahead.

So the matrix actually will be H_0 here, $H_1 H_0$, $H_2 H_1 H_0$ and one more row and by the time it came to this point it would have $H_4 H_3 H_2 H_1 H_0$ okay. So that is what the zero padding will do and this is you can verify that this is indeed the case okay. Any confusion? Oh I am sorry C. Sorry I tend to use H for channel so in this particular example we have been using C. $C_1 C_0$, I apologize okay. I have got lots more to change okay. Sorry about that. These are all the C's, $C_4 C_3 C_2 C_1$ now we have to eliminate these okay.

I hope this is okay now, this is $C_0 C_1 C_0 C_2 C_1 C_0$ dot dot dot okay and the last of the entries that we would be working with would be $C_0 C_1 C_2 C_3 C_4$ okay and you will basically get the Toeplitz structure and you can see the Toeplitz structure that is emerging okay. So the reason for this is not, I am sure you know how to do linear convolution that is not the purpose for this, as always there is a specific reason.

Now if I had to do cyclic prefix these zero samples would be replaced by the first one be replaced by x_7 , the previous one by $X_6 X_5 X_4$ okay. Now and I throw away the first how many elements, after I do with the cyclic prefix? I will throw away the first n_u entries. So which means that when I look at the output of this matrix product the first n_u rows are not important to me.

So where does the n_u cutoff come in, the first n_u rows are here, these are the first n_u rows okay beyond that will be the rows of. So basically these rows are something that are not of interest to me because I am going to remove that. So what I am left with is this matrix, the remaining rows okay multiplied with the cyclic prefix okay. You will see that now what is C_1 multiplying here? when I multiply with this? C_0 is multiplying X_0 .

C1 will multiply X7, C2 will multiply X6 and so on okay. Now this matrix operation can also be you can please verify this is very important, you can also implement it in the following way.

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The diagram illustrates a circulant matrix multiplication. On the left, a matrix is shown with columns labeled C_0, C_1, C_2, C_3, C_4 and rows labeled x_0, x_1, x_2, x_3, x_4 . The matrix is circulant, with each row being a cyclic shift of the previous one. A bracket below the matrix is labeled "Circulant Matrix". To the right of the matrix, the following text is written:

- Length of data seq = 8
- Length of channel = 5 + 3 zeros padded = 8
- Length of circular conv = 8

Below the matrix, a list of steps is provided:

1. Add CP (≥ 2 * ISI tops)
2. Channel → Linear Conv
3. After removing length of C
Circular Convolution

C_0, C_1, C_0 dot dot dot, all the way to C_0 here. C_1, C_2, C_3, C_4 and right at this point where I get, no I should get above C_3 . So right above C_3 I will get C_4 here C_3, C_2, C_1 okay. I am going to fill this up in a minute, this is the input vector X_0, X_1 through X_7 okay. This is C_0, C_1, C_2, C_3, C_4 . Basically you will have this as a Toeplitz matrix. This is C_3 , the first column, this column corresponds to.

So basically what you will have if I wrote down this row, you would have C_0 here C_1 here C_2 here C_3 and then C_4 actually has gone up to the top. Now if you check these equations, you will get exactly the same output as this. Now basically what we are showing is this is a compact form representation of what was the computation that is done in the previous case, only applicable when I have done a cyclic prefix right.

I have inserted the cyclic prefix and I have thrown away the first new rows, the remaining computation can be shown to be equivalent to this okay. Now if this is the case then I want to go in and look at this matrix. This looks like the pseudo circulant matrix, except there is no Z inverse sitting on top. This is actually a straightforward circulant matrix. So this is a circulant matrix.

So the important thing that we observe is that the whatever linear convolution that we were trying to do can be represented in this form where you have a circulant matrix times the data. Now if you go back and recall the process of properties of the DFT when I do pass a signal through an LTI system, I get linear convolution, but if I multiply the DFT of two the input sequence and the impulse response, what do I get? Circular convolution.

And how did you represent circular convolution, it would be in the form of a circulant matrix times this or a periodic extension of X of 7 and cyclic prefix is nothing but a periodic extension of X only to the extent that you need to obtain circular convolution. So this is actually with the cyclic prefix, the linear convolution that happens in the channel now has become a circular convolution okay, very important observation, circular convolution okay.

So what is the length of the data sequence? Length of data sequence in this case was 8 okay, length of the impulse response or the channel impulse response, of the channel, length of the channel was actually equal to 5, but when I want to do circular convolution then I add 3 zeros + 3 zeros padded okay. So then I will get this also to be of length 8. So notice that if I filled in this there will be 0 0 0 in my matrix those are the entries that we have kind of omitted.

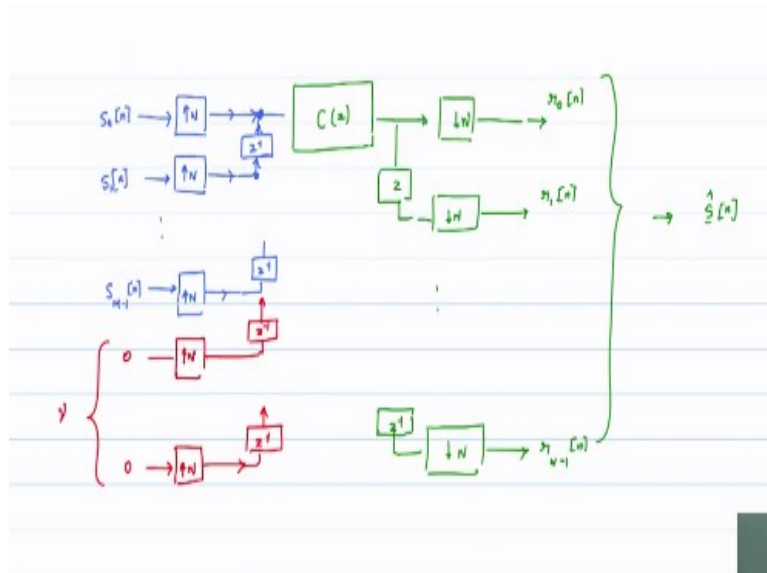
So basically and when I do circular convolution of a data sequence of length 8 and an impulse response circular convolution will give me another sequence of length 8. So the length of the circular convolution output does not increase in length. It always remains the same that is one of the properties of circular convolution. So this is circular convolution is also equal to 8 okay.

So here is a very important statement that we are going to make. Please make sure that you are comfortable with this. First step was to add cyclic prefix, how much cyclic prefix to be added this has to be greater than or equal to ν , which is the number of ISI taps, number of ISI taps, okay the cyclic prefix has to be greater than or equal to. Under that condition the length of cyclic prefix greater than or equal to the ISI the channel does circular convolution sorry linear convolution.

So in the channel we have linear convolution, but after CP the portion with CP, if the first ν terms are removed actually what we are left with this after removing the first ν or the CP portion okay because CP we said after removing the length of CP terms then what we are left

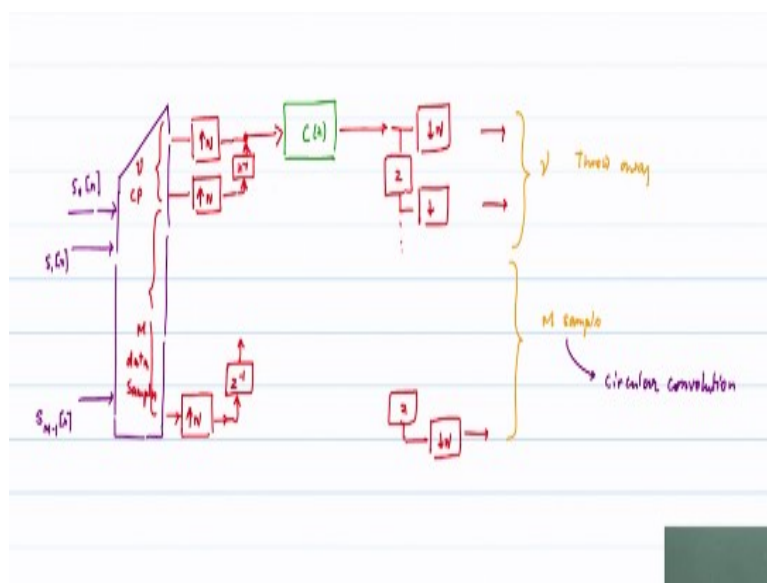
with is circular convolution of the channel with the vector, circular convolution okay. Now probably still at this point not really very clear where we are heading with, what we are discussing and how to build upon this okay.

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Now can you as an exercise I would like you to draw a structure that introduces redundancy which is of the form of the cyclic prefix. So basically you will go back to this type of a structure where the last nu elements will get added to the front. So basically remove this red portion, add the cyclic prefix which will be at the top and then. So maybe if I just indicate it to you I request you to finish the rest of this part okay. So the part that we want you to introduce is the following.

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So you have the M the original input data, that is S_0 of N which will be passed through the, wait a minute, correct, okay, S_0 of N S_1 of N , S_{M-1} of N . Now the cyclic prefix will get added at the top. So the redundancy I am kind of showing it in the other direction because these are the M samples and here are the new CP. This is the M data samples. In this case it will be S_0, S_1 all of those plus the cyclic prefix.

Once you have the cyclic prefix each of these, this is where the redundancy is coming in up sampled by N , next branch up sampled by N , combined with a delay element okay and please implement it all the way down to the last branch up sampled by N with the delay element it is helpful for us to sort of make sure that there are no confusion or doubts as to where the redundancy is coming in okay.

So and this is what will go through the channel C of Z , this is what will become it should be a linear convolution of this sequence, but at the output once I take out the first nu samples it becomes circular convolution. So again I am not indicating the noise at this point then on the other side we will do a down sampling by N , an advanced operator Z down sample by N dot dot dot and you get these outputs, advanced operator down sampled by N okay.

The first nu entries I throw away; new samples throw away. What is left with are the remaining M samples, M samples and this is what we are going to use for the processing, this actually represents now circular convolution. Circular convolution with the data, with the channel impulse response, circular convolution okay. I would like to leave you with a very interesting and a very important result and hopefully this is a good place for us to pick up in the next class, but it is an extremely interesting observation okay.

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So which requires us to go back and look at CPC. Again leave it at this point, but so if you go back and look at the CPC, we did this matrix yesterday. Previously the zero padding eliminated the last new columns, but now the CP removal part which is it going to remove? It is going to remove the first nu rows are going to be eliminated okay. So what you are going to be left with is if you notice the Z inverse terms all occur in the upper nu rows of the matrix.

So if I have removed that once again interblock interference has gone even through the cyclic prefix method, the inter block interference is gone. What we are left with is a matrix that, what remains is the one that we have to we can now say that it is a constant matrix which we now can work with. So the constant matrix is the one below the red line. It is a banded diagonal, banded Toeplitz matrix which we can then now leverage.

So I will leave it at this point just to get a feel for make sure you get the whole picture together. What is that F matrix does how does it insert the cyclic prefix, then how do you remove the cyclic prefix and then when you look at the cyclic prefix terms the CPC of Z ? the first nu rows are getting eliminated.

So then what is left with this a constant matrix times that, so which you can now say that after cyclic prefix removal it becomes once again a simple linear equation, a set of equations where there is no inter block interference okay. So we will leave it at this point, but we will pick it up from here because we are one step away from the answer that we need and we will complete this in the next class. Thank you very much.