Multirate Digital Signal Processing

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Lecture-34 (Part-3)
M-Channel Multicarrier Transceiver (Part-3)

So basically a couple of results and then we will conclude today's session. So the polyphase implementation of a multicarrier system. Multicarrier system goes back to a few slides.

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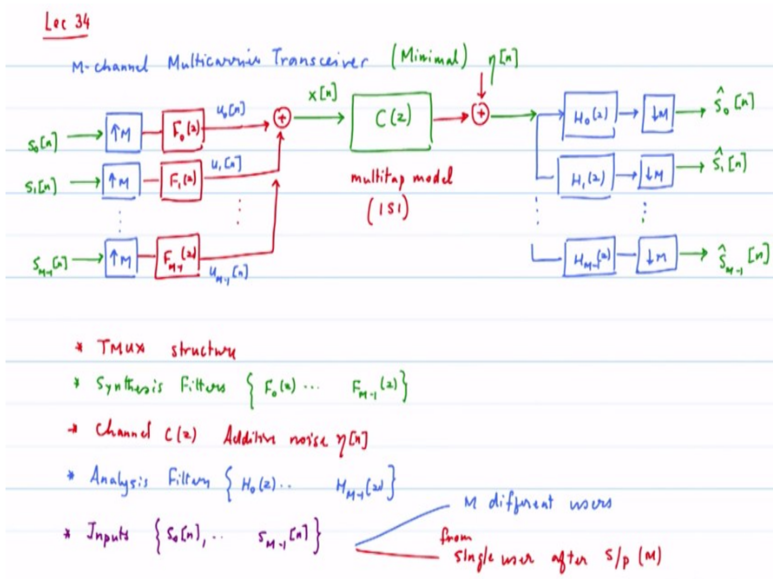

Okay this is my multicarrier system remember with redundancy we have introduced n which is some value larger than m. So I want to draw the redraw the equivalent diagram for us in the context that we are looking at.

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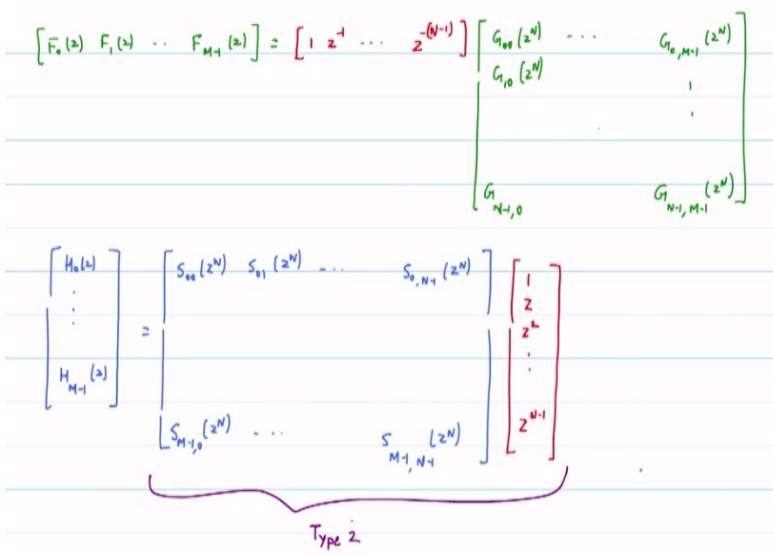

So first we start off just representing the synthesis filters. I will just write down the results because we have already have obtained these results before F1 of z, F M-1 of z. We will split this into N polyphase components. So it will be a row vector 1 z inverse all the way to Z power N-1 followed by a matrix which is an M*M matrix. I would not write down everything this is G00 z power n, G10, G M-1,0 these are all z power N all the way to G0, M-1 z power N all the way here. Okay this is the synthesis filters.

The analysis filter remember we needed it with the powers of z. So the definitions that we have used before using the shifted versions H0 to H M-1 of z. Did I use some other name N-1 absolutely correct thank you yeah it is N-1. G N-1,0 then I better write the last one also so that there is no confusion. This one should be G N-1, M-1, M-1 z raised to the power N okay these are all Z power N terms.

Now H of z this analysis filter banks with the powers of z will be S00 Z power N, S01 z power N all the way to S0, N-1 Z power N. N polyphase components the last one is S M-1, 0 Z power N… the last one is S M-1, N-1 Z power N, N-1 polyphase component of this. So basically we have obtained this one I forget 1, Z, Z square, Z power N-1. So this we have obtained using the Type 2 polyphase decomposition and the other one using the Type 1 polyphase decomposition you can fill in those data.

Okay so the final representation of this transceiver with redundancy in our context can be represented as follows.

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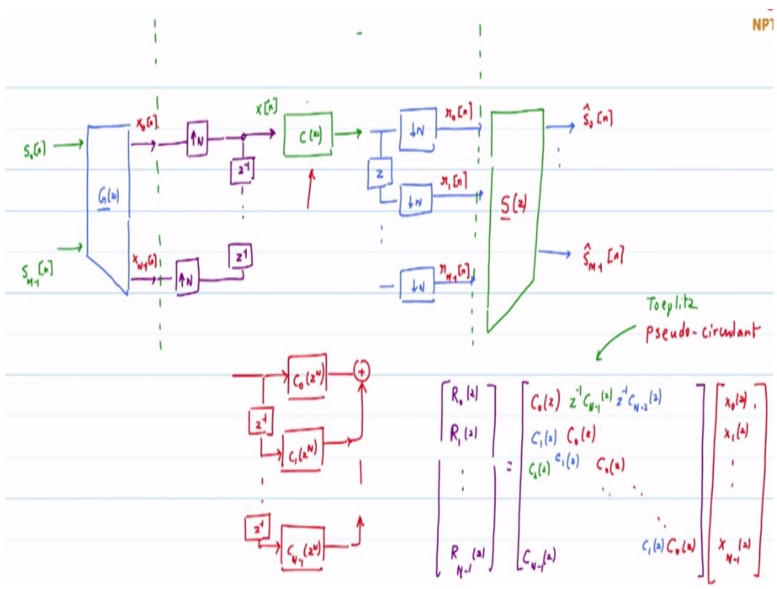

A trapezoid which represents G of z. It has M-1 inputs. S0 to S M-1 S0 of n, S0 to S M-1, S0 of n S M-1 of n that is the last input. I produce n outputs; label them as X0 of n all the way to X n-1 of n. Each of these is up sampled by a factor of n up sampled by a factor of n maybe I should have just written it as parallel to serial conversion. All of these are added with a delay chain okay maybe it is worth it because it helps us to visualize the full picture with the delay chain all of these added up these are all delay chains.

This comes out to be the signal X of n which is pass through the channel C of z there will be a noise term, but really now our analysis noise is not going to play a picture so I am omitting that, but you can perfectly okay to introduce that. At the receiver you have these parallel channels down sampled by N notice it is a advance operator Z down sampled by N such branches down sampled by N.

We will introduce a label for these it will help us in our analysis called this as r0 of n, r1 of n, rN-1 of n there are N signals which are then combined by another trapezoid basically another matrix which is not a square matrix it is a rectangular matrix takes N inputs and products M outputs. So the outputs this is S0 hat of n, S M-1 hat of n and what is sitting inside is S of z matrix.

This G of z is also matrix okay. So this is our representation. Okay now as an exercise I would like you to do the following - do a polyphase decomposition of C of z. So basically do a polyphase decomposition of C of z what will happen. In this structure you will get the following you will get C0 of Z power n. Do a Type 1 polyphase decomposition; that simple Z inverse C1 Z power N and so on until you get C N-1 Z power N all of these are through the delay chain.

Basically I am just rewriting C of z by a summation and then add up all of these okay. Now leave off this G of z draw a line here and likewise we will for the moment we would not worry about S of z draw another line here look at what is in between. You have a parallel to serial converter through delay chain using the delay chain then a channel which has been implemented in polyphase form and a serial to parallel converter at the other end.

Now is this an LTI system very important question because anything to the left of green line is LTI, anything to the right of the green is LTI. Now can this be represented as an LTI system very important question. The answer turns out to be yes okay. Why because the transfer function between X0 trace the line between X0 and S0 sorry R0. X0 goes through an up sampler and it can go through any one of these branches. You can assume that is going through all of the branches.

And at the other end it goes through a down sampler which of these will survive only C0 will survive because all the others will get killed because it is an up sampling followed by a delay followed by the down sampling. And why is that because this down sampler can move to the left of the using the noble identify it can move to the left and therefore the input, output relationship between the R0 of z, R1 of z all the way to R M-1 of z.

Okay is actually a transfer matrix where I will just write down a few entries and request you to finish the rest. The inputs are X0 of z, X1 of z all the way to X is it M-1 or N-1 it is N-1 I am sorry. So this should be N-1.

And here we will have X of N-1 of z okay between X0 and R0 the transfer function turns out to be C0 of Z why is it not C0 of Z power N because the down sampler mode and you had the noble identities. Okay between X1 and R1 X1 comes in with a up sampler followed by one delay chain passes through C of z oh it can pass through any of these branches and at the other end R1 comes out with a one advance operator.

So the Z inverse and the Z will cancel each other leaving the transfer function to be only C0 all the others would have gotten killed. So X1 to R1 is also C0 of z. Now just one more entry and then we will be done between X0 and R1. X0 goes in without any delay chain it is only the up sampler. R1 comes out with an advance operation which of these branches will survive - C1 only C1 survive so basically this becomes C1.

Now if you go in and complete the rest of this matrix you will find that all the main diagonal elements are C0 of z. All the one lower diagonal all of them are C1 okay and all the way down here C1 of z then comes C2 of z. Now any matrix that has got this is called a Toeplitz matrix okay. Now very interesting what is a transverse function between R0 and X1 will basically be the second element here right.

So if you go ahead and fill out all of these entries this last will be C N-1 of z. Now the important question is what is the transfer function between X1 and R0 X1 is basically enters through one delay element it can pass through any of these branches of C of z and then R1 is the one with one advance operator no R0 so there is no it has to just go through the down sampler.

So there is a Z inverse if it goes through any of these branches it will get killed because of the down sampling. The only branch it can go through is the last one because it will have Z power –N when I down sample, when I down sample I will get the transfer function to be Z inverse times C N-1 of z okay and then you will get a Toeplitz matrix of Z inverses. The next one is going to be Z inverse of C N-2 of z again a Toeplitz matrix.

So basically what you see as a transfer function between the X vector and the R vector is actually an LTI system. It has got a very interesting and important structure because the transfer function is a Toeplitz matrix. In addition to being Toeplitz it is also got a circulant property because what comes if you take the first column C0, C1, C2 all the way to C N-1 you will find that the next column is just down shifted by one with a circular wrap around and when C N-1 of z comes to the top it comes out with a Z inverse.

Now this type of matrices have got so basically this matrix has got a property called the Toeplitz properties all the diagonals are the same and it has got the circulant property, but circulant means the last term just appears at the top, but it appear with a Z inverse so it is called pseudo circulant matrix okay. So replace this Multirate part with G of z a pseudo circulant matrix followed by S of Z that is how far we have been able to come.

Now it turns out that we can now start to actually show what are the relationship between G this transfer function C and S of z in order for us to start being able to design very powerful types of systems and we can also show where the design elements of Multicarrier systems will come into play. So we will stop here, but we have introduced some new notation also introduced some concepts which if you can read the corresponding chapter in Yuan-Pei Lin.

It will be more helpful more or less I have used exactly the same notation so that there should not be any confusion, but we are now at a point where we can move into the design of multicarrier systems where we can eliminate inter carrier interference, intra sub carrier inference and actually be able to design very efficient systems, but the key question is I have introduced redundancy, how do I exploit?

Now what comes into play is how do I exploit the redundancy in the system that we have designed. We will stop here we will pick it from here in the next class. Thank you very much.