**Multirate Digital Signal Processing**
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**Lecture – 34 (Part-1)**
**M-channel Multicarrier Transceiver – Part1**

Okay, welcome back we will pick up from where we left off and build on it, keep in mind the building blocks that we have developed and because now, this section it is more about putting all the pieces that we have developed together.

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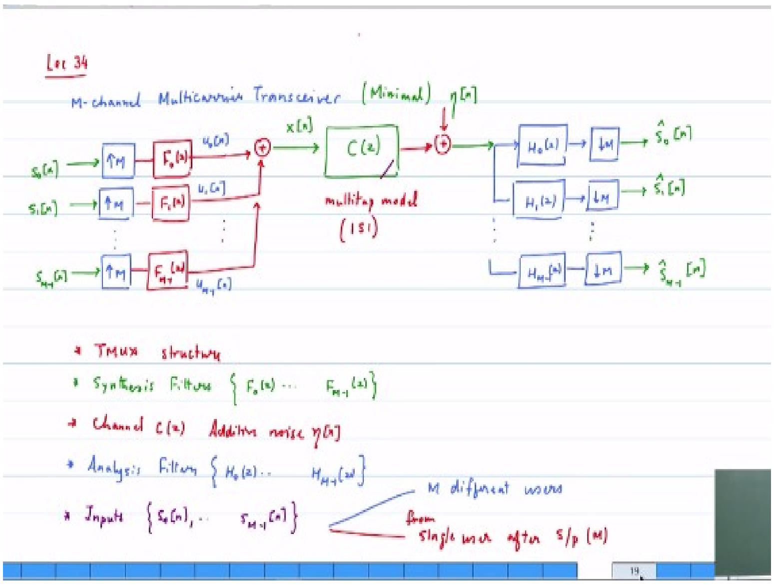


So, here is the first figure that would like you to keep in mind, so here is an m channel multi-carrier transceiver, okay and we will use the following notation where we say minimal that means, it is optimal in some sense, the fact that we are going; not going to use any additional bandwidth, no redundancy built into the system, so the structure of this is going to be as follows well, this is our base lines reference diagram.

I have m signals s0, s1 through sm – 1, we would like to process this; combine them into a single signal, pass it through the channel and then recover it, so the process of combining going to draw it as up sampling by m, no redundancy in this particular structure, this is our baseline structure dot, dot, dot each of these filtered by a appropriate filter either low pass or band pass f0 of z, f1 of z, fm – 1 of z, combine these signals by adding dot, dot, dot, okay no delays.

Because we are just basically adding the outputs of the filters, let me give some labelling, so we can analyse it, u0 of n, u1 of n, um - 1 of n; the output of this is the signal let us call that as x of n, here comes the channel, we have not made any assumptions about the channel in the general case, it will be a dispersive time varying channel, so if you want to describe it, this would be a multi tap that means, we will have C0 + C1 z inverse C2 z – 2.

So, multi-tap model; this is what will capture the inter symbol interference that will be introduced by the channel. Now, one aspect that we cannot control but we have to include or at least mention in the channel model is the noise, let's called it as eta of t, we are really not going to do much about the noise, nothing we can do, it is just an impairment that will make the affect the quality of the signal.

But other than that so, now that is the pass through the channel now, at the other end we must start the recovery process, so the first step is to pass it through a bank of filters H0 of z, H1 of z up to Hm - 1 of z, since we have to preserve the sampling rate, each of these sub channels is down sampled by a factor of m, okay and the outputs are labelled as S1 hat; S0 hat of n, S1 hat of n, Sm - 1 hat of n.

And the role of a multi-carrier or a multi-carrier transceiver is to have S0 hat to be as close to S1 as possible, S0 as possible because that is what you have done, you have been able to combine m signals, transmit it through the channel and through the processing at the other end you have been able to separate it or separate it out and then get back the original signals, so this would be the minimal transceiver for a multi-carrier system.

In a multi rate DSP context, we will call it a trans multiplexer, okay in communications or in other contexts we could call it as a multi carrier transceiver, so this is a trans multiplex structure primarily because you have the synthesis stage, where you combine signals to produce a frequency division multiplex signal and then you do the reverse in the other end okay. So, the synthesis filters, there are m filters - synthesis filter, so this would be the complete description.

You will have f0 of z all the way to fm - 1 of z, these are these synthesis filter, there is a channel which is a multi-tap channel, we represented as C of z, there is an additive noise which is given by eta of t, eta of n, I does not mix continuous time and discrete time, let me just change this to, okay everything we will represent it completely in the discrete time; eta of n that is the and then we have the analysis filters.

There also can be called as the receiver filters, analysis filters; H0 of z, Hm - 1 of z that is your analysis filter bank and your inputs to the multi-carrier system, inputs are S0 of n all the way to Sm - 1 of n and an important observation or as a comment which is useful for us to keep in mind, now these signals can come from m different users that is possible; M different users, you can think of them as signals of different users.

At the same time, you can also think of it as the user data from a single user after from a single user, after the serial to parallel conversion; serial to parallel conversion of dimension m okay, so that is also possible, so if you want to think of a high data rate system, then you would think of it as one user's data which I split, transmit through the channel and then recombine I mean, and recover it at the other end.

So, both again how you got S0, S1, Sm - 1 is not as important but again it is something that we just introduced okay.

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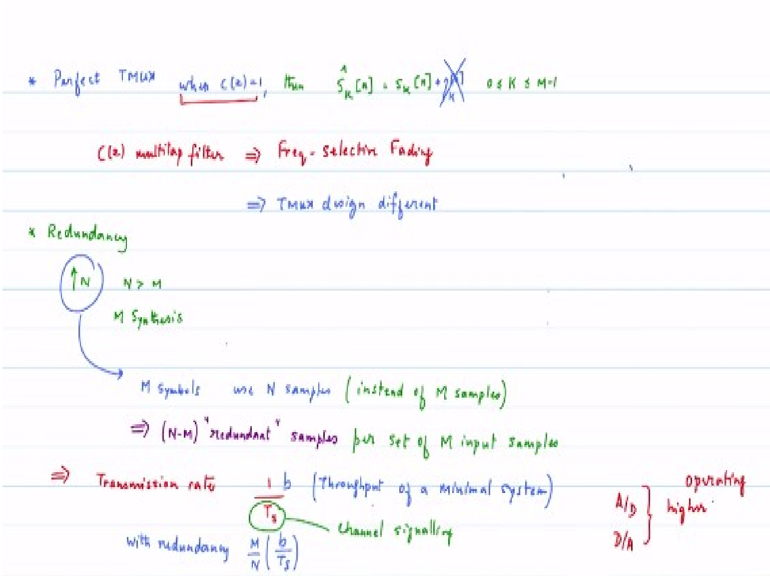


So, if you have this picture in your mind, right on the following observations or comments, we call it a perfect trans multiplexer just like we had perfect reconstruction, when you had analysis followed by synthesis, we can have a perfect trans multiplexer under the condition that C of z when C of z = 1 that means, the channel is not dispersive, it does not do anything under the; under this condition, if you can show then when Cz = 1, then see Sk hat of n =to Sk of n okay.

So, if there is no channel, if you can show that the input outputs are related in such a way that the whatever you fed in at the input came out at the output for all the channels, noise is something that we just say that okay this that is why we do not say, we can say maybe you can add it as plus noise, okay, so some nk of n but really, there is not much we can do about this, so we are saying that in the; if you ignore noise, will you get back the original signal.

So, k > or = 0, < or = m – 1, so basically it will be the input signal with some corruption by the noise but not any other type of however, this is not a realistic assumption because typically C of z is a multi-tap filter so therefore, now let me ask you one quick question; is x of n a narrowband signal or a wideband signal; wideband because you have combined many narrowband signals okay, keep in mind.

So, it is a wideband signal; wideband signal means, it will see a; because a wideband signal has got very high time resolution, reciprocal of the bandwidth so therefore, it will actually see a multi-tap filter, so C of z is a multi-tap filter and which means that the signal x of n is going to see a frequency selective fading channel, okay so that is a very important consideration and this is what makes the trans multiplexer design difficult okay.

Because you have a wideband signal and it is passing through a dispersive channel and this is when you go back and ask the question, hey wait, wait, you need to help me solve the equalizer problem because otherwise I am going to have a real tough time for a trans multiplexer, the equalizer problem, then you go back and say hey, you know what by the way if this was a set of narrow carriers orthogonal to each other, if you constructed x of n in that fashion which is what this structure can give you if you construct it correctly.

Then the C of z may not be as much of a problem because each of these narrowband signals will see a flat channel and I can design for you an equalizer, so this is where we are coming from so basically, if you just took a multi-carrier system yes, there is a challenge with the channel but if you design it in the fashion of orthogonal carriers, then we can do the take advantage of that.

Now, the filters that we have to involve in the design of the trans multiplexer that is f0, f1, fm – 1, H0, H1, Hm – 1, now these become very critical, if you have a minimal transceiver, so that is where we introduce redundancy, so we say that okay, in this minimal transceiver if you; if we are permitted to introduce redundancy; redundancy basically means that we will do up sampling by a factor of n; up sampling and down sampling by a factor of n, where n is > m.

However, there are only m, this thing synthesis filters okay, so m synthesis filters that part has not changed but we are using a higher sampling rate, so therefore redundancy has come in, this is what we have talked about so, this also means the fact that you have n is that for transmitting m symbols okay, m symbols, in a minimal transceiver you would have used m samples. Now, we are using n samples, okay so this is where the redundancy is sort of coming in.

So, instead let me just maybe make it very clear saying instead of m symbols, m samples okay so that is where the redundant transceiver is sort of building it, so this basically gives you a clue as to where the redundancy is coming in, I have n - m additional samples or let me call that as the redundant samples okay, within quotes, these are the ones that are introducing the redundancy for me for each per set of m input samples.

That means, when I want to transmit one block of m vectors, I should have used m samples but now I am using more number of that so, this can also be interpreted in the following way, it says that what is my transmission rate of my system, so the transmission rate or the throughput rate, transmission rate if it were; if it were a minimal system the number of symbols per second is 1 over Ts that is the number.

The Ts is the duration of one symbol, the number of symbols per second is 1 over Ts, now each symbol can carry B bits of information, so into B, correct and there are m channels, so mB/ T s would be my throughput that is what that would be my throughput that we would achieve okay, so this would be effectively the throughput of a minimal system okay. Now, did I get that correctly, 1 over Ts that is correct?

No, no, no, wait, wait, wait, Ts is the; I have to see which how we have defined Ts; Ts is the input sampling rate so therefore, I would, okay, okay actually on the; okay yeah, this has to be because once you go to the after combining it, here it will be it has to go at a faster rate, right because of the interpolation, it will be Ts over m, so yeah, so this Ts is at the input rate, so basically B times Ts.

So, this would be the rate at which the information is going across the channel now, if I look at the case where I have redundancy, with redundancy I will get a factor < 1 and that factor will be m over n * B over Ts, okay, so that would be the; so this Ts is on the channel, the channel signalling rate, okay so basically, I would use n symbols but I would have effectively transmitted only m information bits.

So, which means that I would get a reduction compared to the minimal system, so the redundancy basically you pay a price for it, the other aspect that we just have to keep in mind again just as a comment, your A to D converter at the receiver side and the D to A converter both are running at a higher rate because you are done a higher sampling rate, higher operating at higher rate.

So, in other words, the transceiver with redundancy has got some price penalty but again, it is one thing that helps us build the practical system in a good way, so higher operating at a higher rate okay, these are sort of general comments, so let us go back and combine the information. So, what are the types of distortions that we will encounter in this particular system, so here is a visualization of the system?

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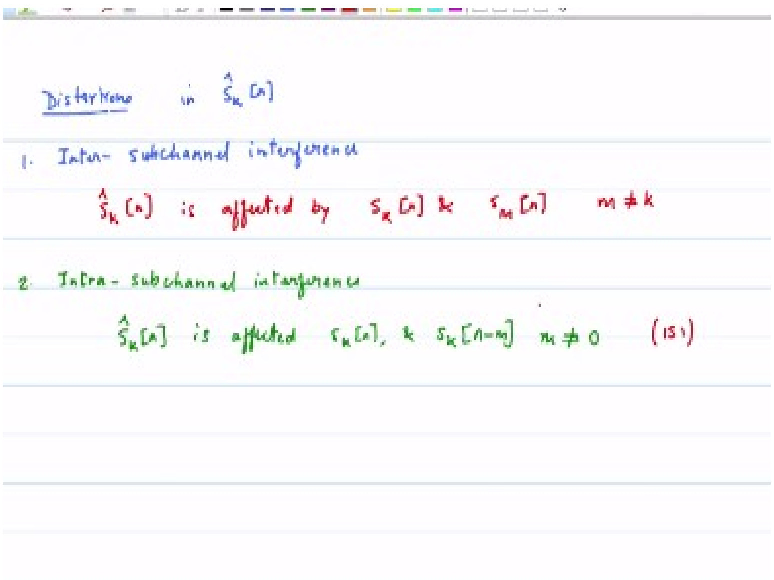


I would like you to comment and note down the distortions, so the possible distortions that are present, distortions are in the Sk hat of n, where the Sk are the; Sk hats are the outputs, so the distortions in Sk hat of n, the first type of distortion is what we call as inter sub channel interference, okay now, this means that Sk of n is bidding affected by Sk - 1 and Sk - 2 because they are the sub in different sub channels.

If they can interfere, now how can they interfere because when we constructed the combined signal remember, these are there is a mixing of the multiple carriers, so at the receiver if I do not recover them cleanly then, there could be an interference between the different sub channels okay, so basically what we say is that Sk hat of n is affected by Sk of n but that is the correct thing because that is the input and some Sm of n where m is not = k.

Some of the other sub channel signals also are showing up, the second type of impairment is what we call as the intra sub channel interference supposing, you have designed your system such that there is no inter sub channel, these different sub channels are not mixing, you could still have what is called as an intra-sub channel, so which means that Sk hat of n is affected by or is it depends on Sk of n and Sk of n - m where m is not = 0.

Some symbols either before or after okay, and this can happen if there is some kind of a filter in between the input and that output transfer function and that means, there will be some mixing of this, there will be inter symbol interference, so this is a form of inter symbol interference, so within its it can create interference, this also is a undesirable part and we want to control want to control that as well.

So, having all of these picture in mind, so let me summarize, this is the multi carrier transceiver if I want to introduce redundancy, we will change m to n, where n is a larger number and then we have to design the overall system such that we pay the additional price of redundancy but at the end of the day can we get rid of inter sub channel interference, can we get rid of intra sub channel interference and then build the system.

So, this is where the multi rate part now, we come in bring it in and then actually help us design the systems.