Multirate Digital Signal Processing
Prof. David Koilpillai
Department of Electrical Engineering
Indian Institute of Technology - Madras

Lecture – 30 (Part-2)
Capacity of Wireless Channels - CSIT

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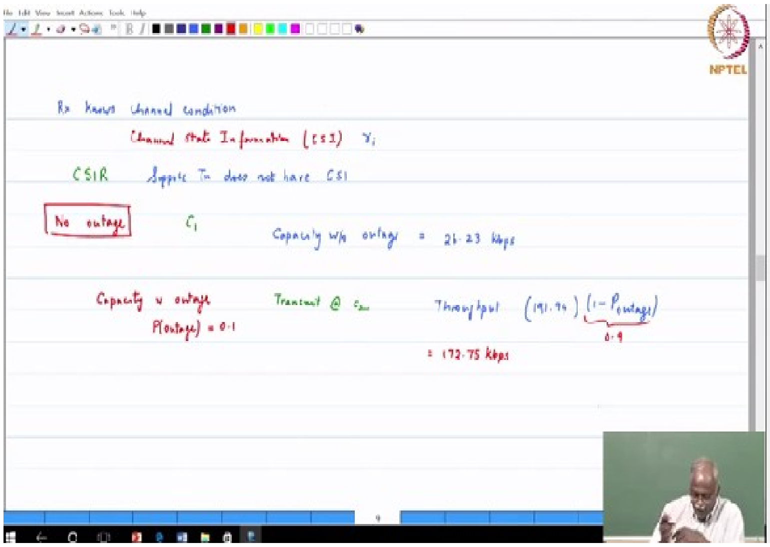

Rx knows channel condition

Channel State Information (CSI) γ_i

CSIR Suppose Tx does not have CSI

$\boxed{\text{No outage}}$ C_1 Capacity w/o outage = 26.23 kbps

Capacity w outage Transmit @ C_{out} Throughput $(191.94)\underbrace{(1-P_{outage})}_{0.9}$

P(outage) = 0.1

= 172.75 kbps

So now we then start asking the question, okay, what can we do if the transmitter has knowledge of the channel state information. That is a very important question.

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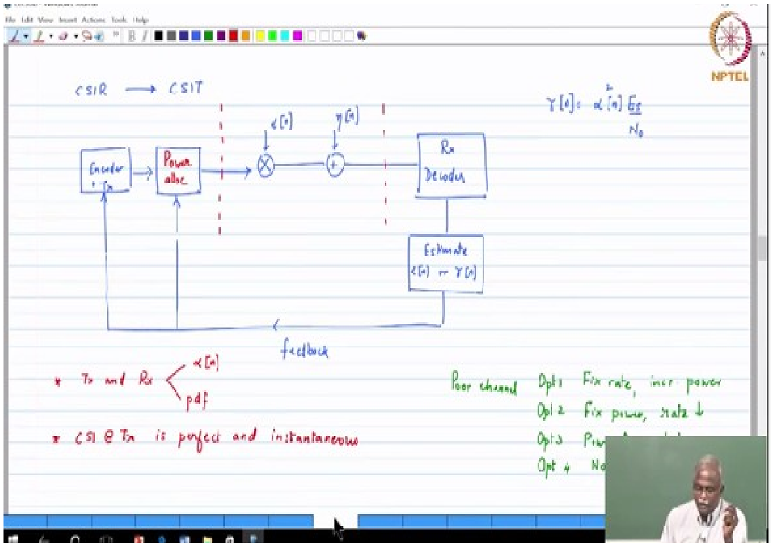

CSIR ⟶ CSIT

$\gamma[n] = \alpha^2[n]\frac{E_s}{N_0}$

* Tx and Rx < $\alpha[n]$, pdf

* CSI @ Tx is perfect and instantaneous

Poor channel
Opt 1 Fix rate, incr power
Opt 2 Fix power, rate ↓
Opt 3 Pw
Opt 4 No

So basically who has information about the channel state? The receiver. The receiver has to give the information to the transmitter, okay. So basically at the receiver, you estimate what is your channel SNR or channel state and that information will get fed back. For the moment, just ignore the box which is indicated as power allocation, okay. So just look at, so once you get this information, you can then fine tune and pick the appropriate channel coding mechanism.

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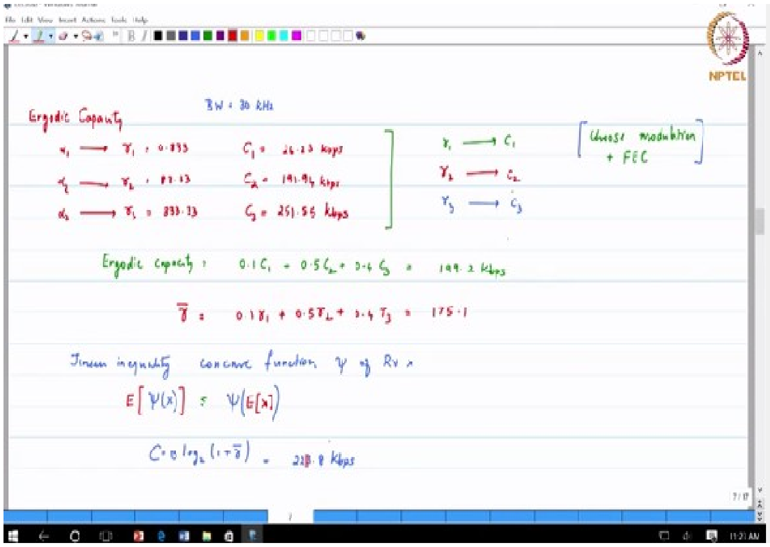

Ergodic Capacity

BW = 30 kHz

$\alpha_1 \longrightarrow \gamma_1 = 0.833$ $\quad C_1 = 26.23$ kbps

$\alpha_2 \longrightarrow \gamma_2 = 83.33$ $\quad C_2 = 191.94$ kbps

$\alpha_3 \longrightarrow \gamma_3 = 333.33$ $\quad C_3 = 251.55$ kbps

$\gamma_1 \longrightarrow C_1$

$\gamma_2 \longrightarrow C_2$

$\gamma_3 \longrightarrow C_3$

[Choose modulation + FEC]

Ergodic capacity = $0.1C_1 + 0.5C_2 + 0.4C_3 = 199.2$ kbps

$\bar{\gamma} = 0.1\gamma_1 + 0.5\gamma_2 + 0.4\gamma_3 = 175.1$

Jensen inequality concave function ψ of RV x

$E[\psi(x)] \le \psi(E[x])$

$C = B\log_2(1+\bar{\gamma}) = 223.8$ kbps

So for example, if I knew that it was gamma 3, I will not transmit with the encoding scheme for gamma 2. I will actually choose the gamma; so this is a way by which we actually want to get the best out of the system. So basically you feed this information back to the transmitter and then the transmitter gets, calculates which of the schemes that we can transmit and then we can transmit. So now given that I have CSIT scenario, how do I achieve capacity?

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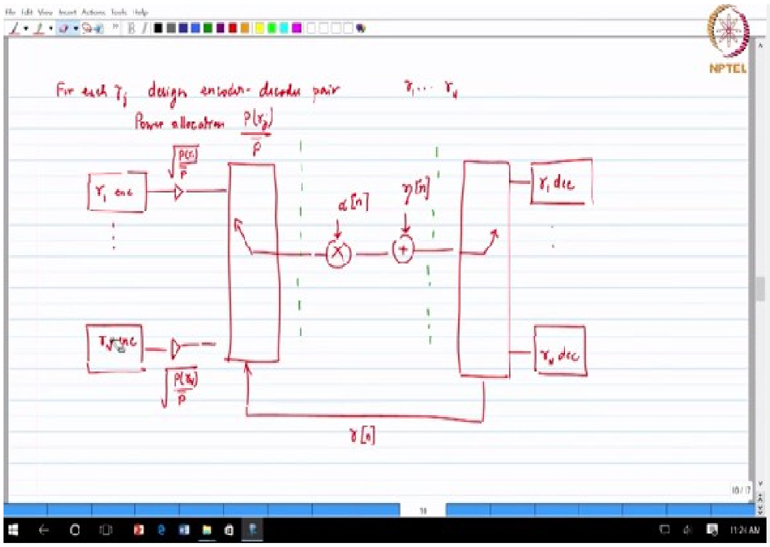

This is a graph. Again, this is there is Goldsmith. So I will just explain the graph. So whatever is the SNR, let me quantize it of the channel conditions in the SNR, we look at the case with 3 SNRs. It can be some n SNR quantized values. For each of those SNRs, you choose the optimum modulation and coding scheme. At the receiver, you couple it with the corresponding demodulation, the decoding scheme.

So what is fed back? The instantaneous SNR is fed back. If it is gamma 1, you choose the upper encoder and then it will go through and at the receiver, you will choose the corresponding decoder. If it is the best possible channel is gamma n, you choose that. So basically what you are doing is, you are constantly switching between the different modulation and coding schemes so that you can get the maximum out of the channel, okay.

So this is how with feedback, we could get very good performance, okay. Now is this part what we have indicated, is that clear? Basically how do we achieve the capacity information in a channel where the SNR is changing? Yes. **"Professor - student conversation starts"** How does the arrangement over the (()) (02:49). Okay. **"Professor - student conversation ends."** So basically you will pick up the received signal.

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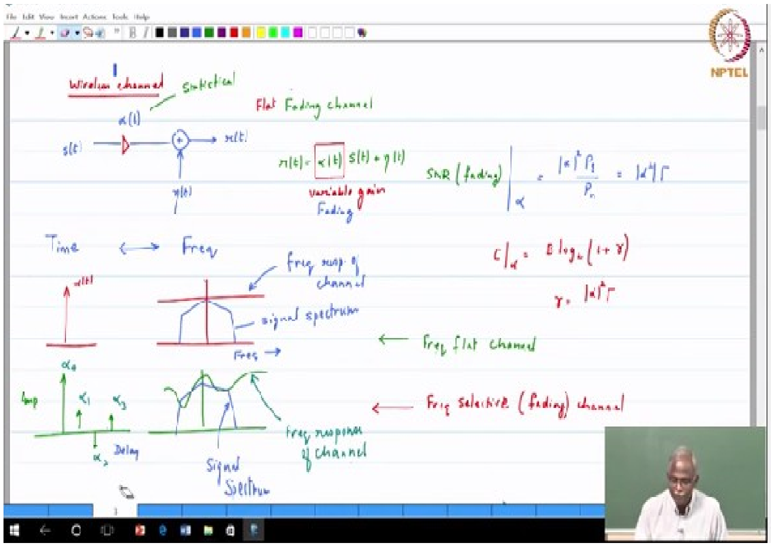

And one of the things in a coherent receiver, so if you go back to our original scheme, one of the things that you will have to do in a coherent receiver is to estimate alpha. So that is your complex gain term. So a coherent receiver must estimate alpha. Once you know alpha, the channel state information is known. Because basically it will be proportional to alpha square, right. So the receiver based on the measurement of alpha, we will be able to get an idea of the channel state information.

That would require you to have a coherent receiver. But coherent receivers automatically will know what the channel conditions are. See, for example, if alpha is very small, then you will know that you are in very bad channel conditions because the noise level, very interesting thing, the noise level does not change. So maybe we will just take a 1 minute tour to answer this question.

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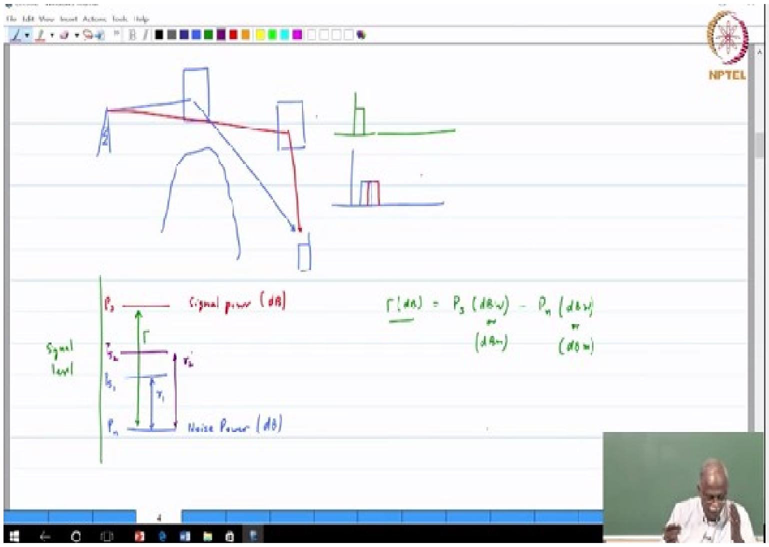

So supposing I am drawing this one. So this is signal level, okay. So I am just drawing different levels. So the level that we are always worried about is where is the noise power, where is the noise floor. So this corresponds to Pn. Now depending upon the, assuming there was no fading, let us assume that this was my signal power, Ps, okay. So the SNR basically, if I wrote all of these in dB, if I showed all of these are shown in dB, this is also shown in dB, then the difference between these 2 is actually my SNR, gamma. That is gamma.

So basically, gamma in dB is Ps in dBW or dBm, does not matter which one you choose, watts of milliwatts. You will have to choose the noise power also in dBW or dBm. The difference between the 2 will be a dimensionless quantity because it is power/power, dimensionless, that is dB. And that is what is the SNR. So now what we are saying is that there is a nominal level that the signal, that the receiver expects, that is Ps.

Now if all of a sudden there has been a significant fading that is happening and your signal power actually drop down to Ps1, okay. So this was gamma; instantaneously, this is gamma 1, okay. So you can see, what happens is the noise power does not change, right. Nothing happens to noise. The only thing that can get affected is your signal power. So how do I know the channel condition and how do I know SNR?

All I need to do is measure the signal power. Once I know the signal power, I know the noise power is always constant. So I can tell you what the channel conditions are. So to answer your question, the scenario that we are dealing with is that in an AWGN channel, it would have been a fixed gamma. Now in a fading channel, sometimes it is at this level, sometimes it is at a different signal level, call it Ps2, that corresponds to gamma 2.

And each of these will have different impacts in terms of the performance and that is what we are trying to capture by saying regardless of what the channel conditions are, if I am able to feed that information to the transmitter, the transmitter sends the information in the most robust and optimized way for that particular channel condition that is possible, okay. So may be one more piece that I would like to highlight even before to go on to this. So let me just add to this the following comments or observations.

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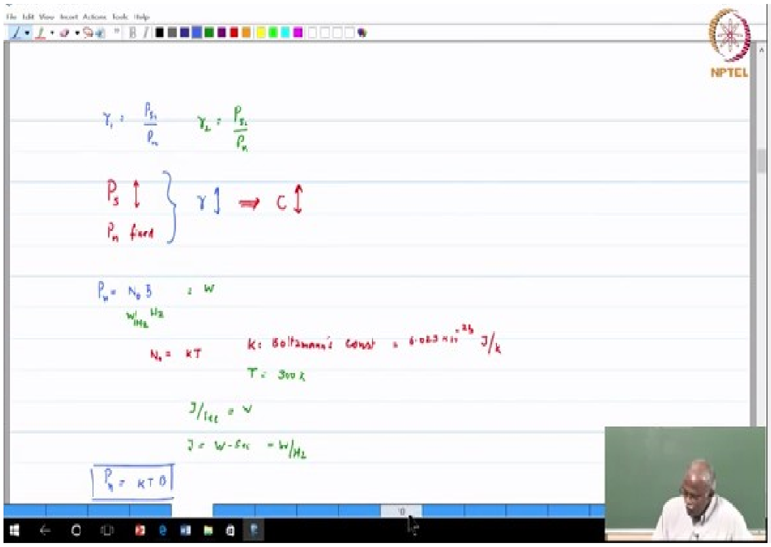

Again since the question came, I think it is a good thing to clarify. So let us say I have gamma 1 which is Ps1/Pn, notice Pn is not changing. Then I have gamma 2 which is signal power 2 Pn because of the fading, okay. What is happening is that the signal power is going up and down. I am using this notation. It is my own notation, not decimation or up sampling, it is basically saying the signal is going up and down.

But Pn is fixed, okay. So this means that this combination basically means that your instantaneous SNR is going to go up and down and if your SNR is going up and down, your capacity, this also means that your capacity is going to go up and down which is what we know from the calculations and this is what we are trying to capture, okay. Maybe just 1 last observation just so that we link completely with the communications background part.

So how is Pn measured? Why is it constant? Why it does not change? Pn corresponds to N0*B, correct. Basically it is N0/2*2B but N0B is what gives you. So N0 is your noise spectral density, okay. What are the units of noise spectral density? **“Professor - student conversation starts”** Watts per Hertz. **“Professor - student conversation ends.”** Watts per Hertz, okay. So this is W/Hz.

Band width is in Hz. So the combination will give you Watts, okay, power, this noise power will be there. Now actually what does noise spectral density actually depends on? I think that is also good for us to know? See noise spectral density, N0 actually depends on k*T, where k is Boltzmann's constant, okay. And again just for completeness, let us just write down the values, 6.023*10 power -23.

This would be Joules per Kelvin, that is what Boltzmann's constant is given as, okay. And we always take the ambient temperature as 300 K. So k*T is what specifies the noise floor, okay. So I am sure you would have done this calculation and you are familiar with it. So the problem is actually I want W/Hz, okay. So basically k*T will give me Joules, okay. So the Joules per second=Watts.

So Joules if I write it as Watts seconds, this can be written as W/Hz. So okay, everything is clean. Basically k*T actually gives you a quantity which is in W/Hz and you multiply it by the band width, you will get watts and therefore, we are able to get. So the noise power, so Pn in general for any receiver is calculated as k*T*B and sometimes we make it little easier just by specifying N0, but you do not need to do that.

You can actually compute what will be the noise floor and then calculate the noise power and this is how it is done, okay. So at the end of the day, what we are interested in is the fact that the SNR is fluctuating and therefore, capacity is fluctuating. So therefore, it is a challenge for us in terms of achieving what we want to achieve, okay. So let us quickly summarize what we have said so far that this is the scenario that we have and that means our information about the channel state has been transmitted from the receiver to the transmitter.

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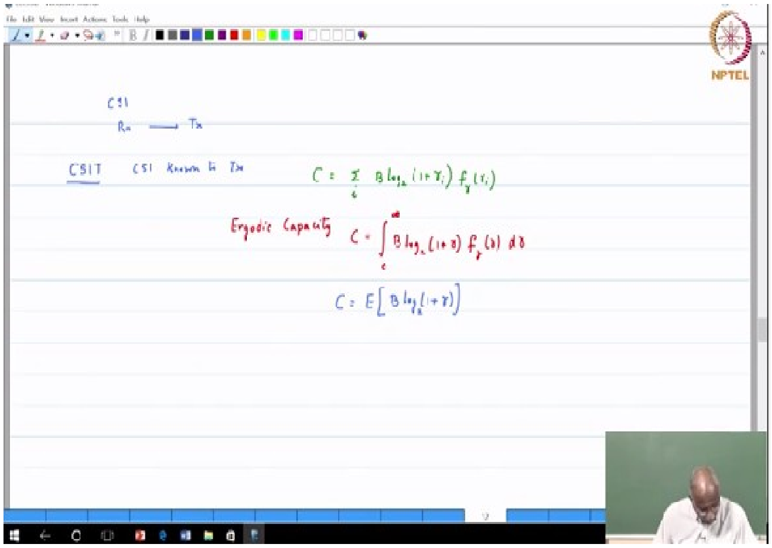

CSI

$Rx \longrightarrow Tx$

CSIT CSI Known to Tx

$C = \sum_i B\log_2(1+\gamma_i)\, f_\gamma(\gamma_i)$

Ergodic Capacity

$C = \int_0^\infty B\log_2(1+\gamma)\, f_\gamma(\gamma)\, d\gamma$

$C = E\left[B\log_2(1+\gamma)\right]$

So CSI has gone from the Rx to the Tx. So we are in the regime of what we call as CSIT, that means channel state information known to the transmitter, okay. So what is the capacity of the channel, when it is known? So the capacity under this condition would be summation over i, the different SNRs, B* logarithm base 2 1+gamma I multiplied by, do not forget this, the probability that you get that SNR, F gamma of gamma is the SNR, okay.

So this is actually your Ergodic capacity and the Ergodic capacity can be achieved only if the transmitter has knowledge of the channel state. Otherwise, you will get outage, you will have to design for the worst case channel or you know you will do. So this capacity is only meaningful if the transmitter has information. Now in the general case, if you did not quantize this, you actually wrote it down as a continuous, that the SNR is not a discrete random variable but a contribution random variable, not a problem.

We can write down the Ergodic capacity as an integral. So Ergodic capacity for a fading channel in which we know the SNR distribution, can be given to be C=integral 0 to infinity, SNR goes from 0 to infinity, B*logarithm base 2 1+gamma, F gamma of gamma d gamma. So basically the summation became an integral, okay.

So in the general case, we say that the Ergodic capacity is defined as the expected value of the capacity of the channel which is given by logarithm of logarithm base 2 of 1+gamma, expected value, that is exactly what it is, we have basically have gone wrong but this Ergodic capacity is meaningful only if the transmitter has knowledge of the channel state and that is the only time under which we can get that, okay.

So let us now quickly calculate what it is that we can achieve when we have this information and then build on that. So basically what this one says is that I can feedback the information, I can choose the appropriate capacity and I can then transmit it. So here is a summary statement of what we have done so far. What we have done so far, basically says that find out what the SNR is and then compute the capacity assuming this information is been communicated to the transmitter, we can then achieve the Ergodic capacity.