Multirate Digital Signal Processing
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Lecture - 24
Study of All-Pass Filters

Good morning. Today's lecture we are going to spend a substantial amount of time on the all-pass filters and the reason allpass comes in is because we are now we have studied the looked at the option of eliminating phase distortion in a 2-channel filter bank and we said let us go back and relook at the problem of eliminating magnitude distortion in a 2-channel filter bank.

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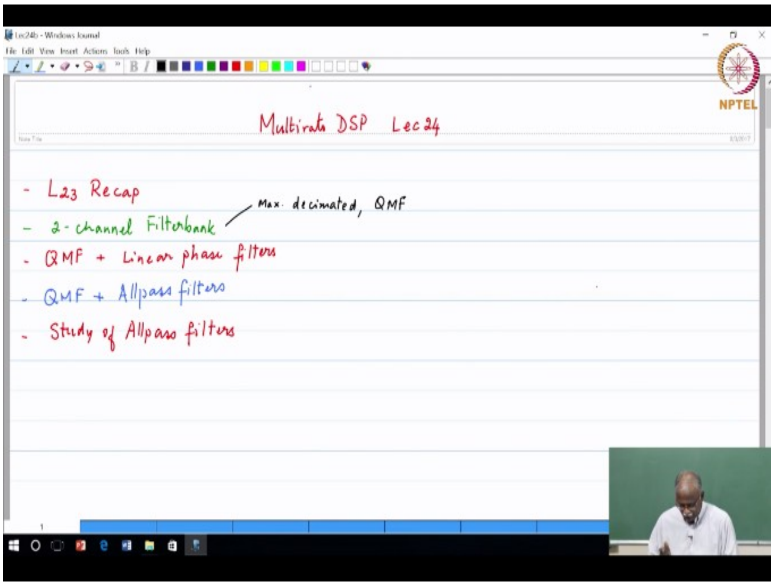

So just to quickly summarize the key results from the previous lecture, we are looking at the 2-channel filter bank, add to this that it is a maximally decimated filter bank. It is a maximally decimated 2-channel filter bank and also add to it the fact that we are using quadrature mirror filter property so that we have to design only one filter from the entire discussion.

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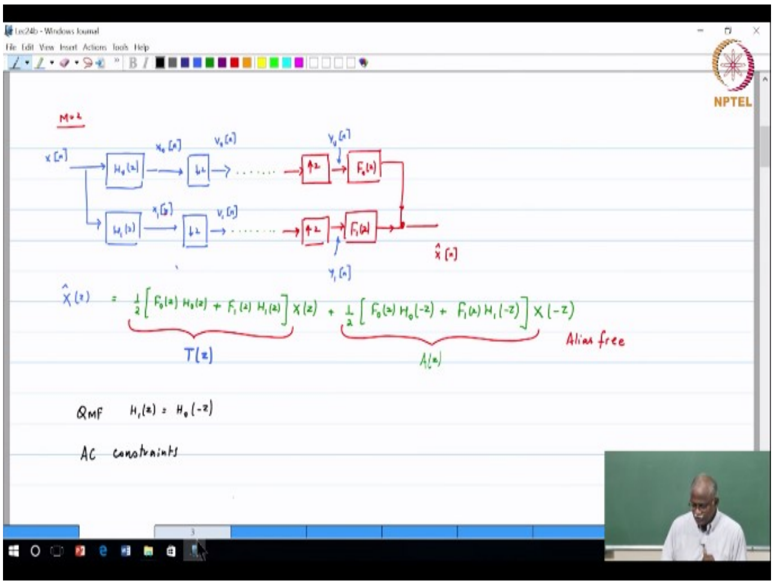

So the 2-channel filter bank with the constraint that the QMF constraint H1 of z is=H0 of –z. This is the QMF constraint; on top of that we have the aliasing cancellation constraints as well, so you can add to this the aliasing cancellation constraints.

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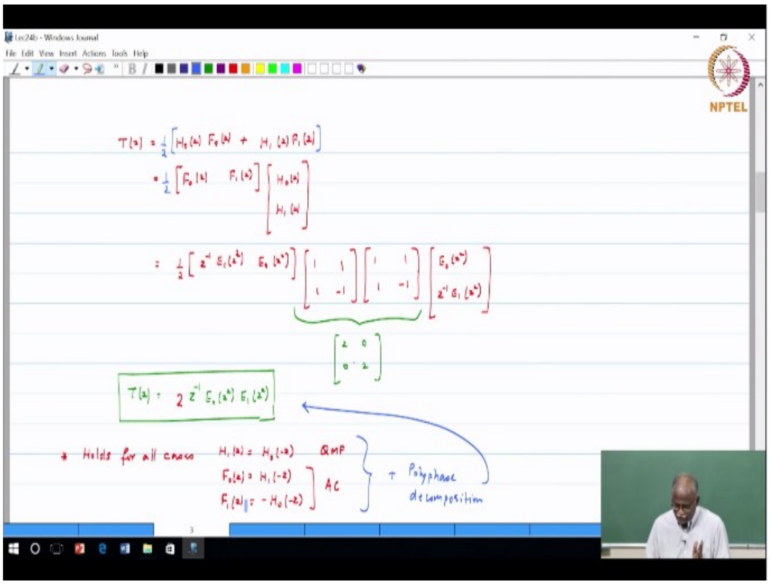

Then, this collapses to a transfer function which we can work with. Now if you do the polyphase decomposition, this is the expression that we get, 2 times z inverse E0 of z squared E1 of z squared.

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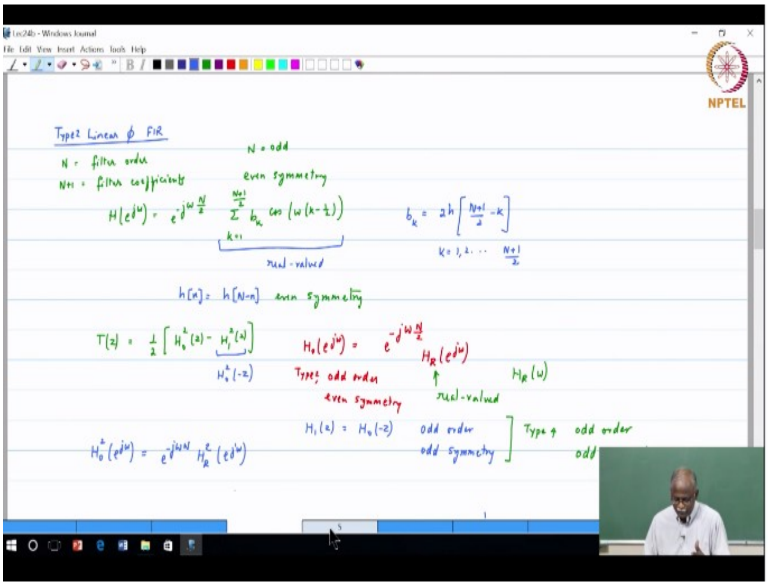

If it is not the polyphase decomposition, let say you are looking at the phased elimination part, we said we look at a type 2 linear phase filter.

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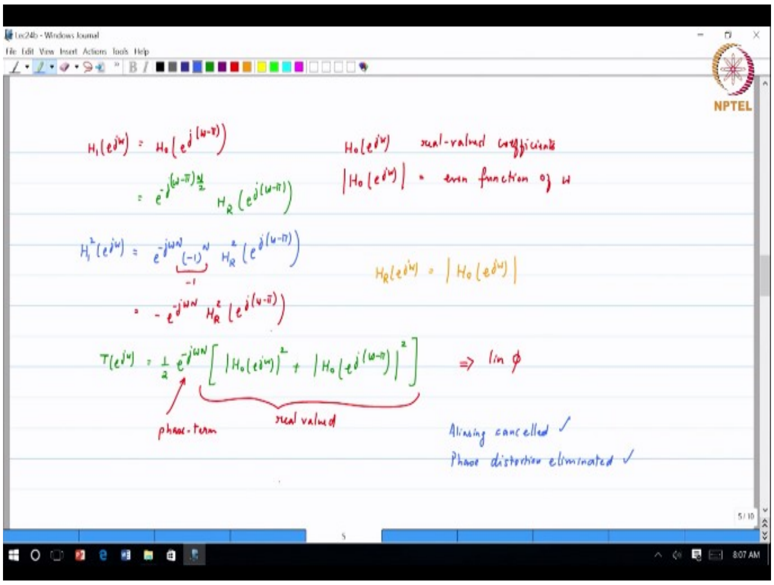

And the expression for the transfer function T e of j omega came out to be 1/2 e power -j omega N H0 magnitude squared H1 magnitude squared or H0 e power j omega -pi magnitude squared. So basically that overall transfer function has got linear phase, magnitude phase distortion is eliminated, aliasing eliminated, we just had to deal with the magnitude distortion and make sure that is as close to constant as possible.

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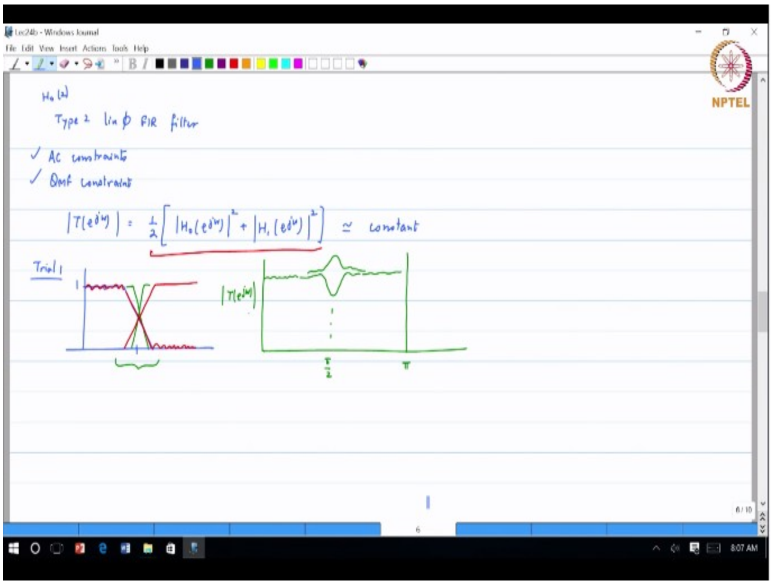

Therefore, we would get a minimum magnitude distortion as well. We spent a bit of time looking at the design of the of a 2-channel filter bank and we said that depending upon how we choose the stopband, stopband plays a very important role and stopband will more or less then decide where the crossover occurs and then you may get in when you plot the overall transfer function as long as your T your stopband energy is small you are going to be small in this region and also in this region.

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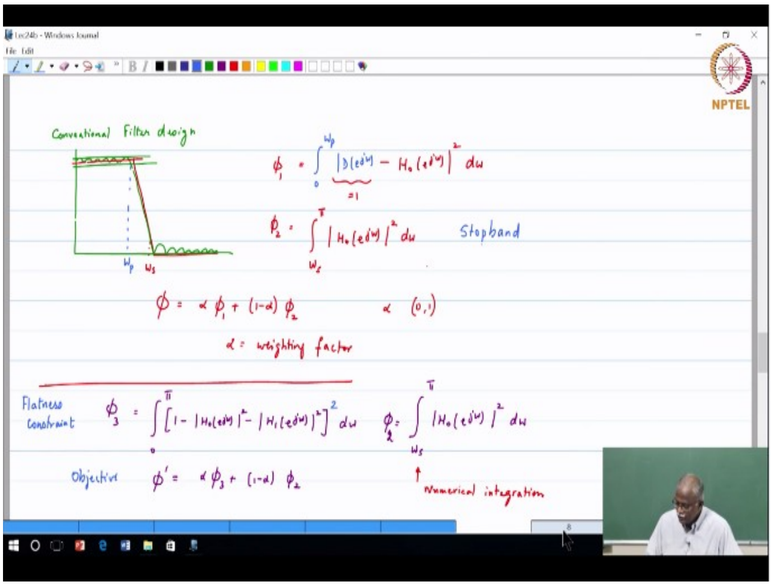

The only uncertainty is in the overlap where the transition band occurs and that is what we try to optimize in the discussion yesterday. So the stopband energy, so this is the expression for the stopband energy that is what tells us that we will get a good low-pass shape. The question that was asked yesterday is how does that alone guarantee that the passband is good, that alone cannot guarantee the passband is good.

So actually the combination of the flat constraint, let us call this as the flatness constraint. The flatness constraint of T e of j omega, the combination of the flatness constraint and the stopband energy we argued actually gives us the constraint. Actually, there is one error that was pointed out, what you have within this bracket, the square brackets is a term that can go positive or negative.

So actually you have to square it before you integrate so that you get a positive quantity and therefore it becomes a quantity that can be minimized. So make sure that the square is inserted, there is objective function, objective function that we are going to is alpha times phi3 times 1-alpha times phi2. So the problem statement if I were to rewrite this supposing you were to sort of say it a fresh.

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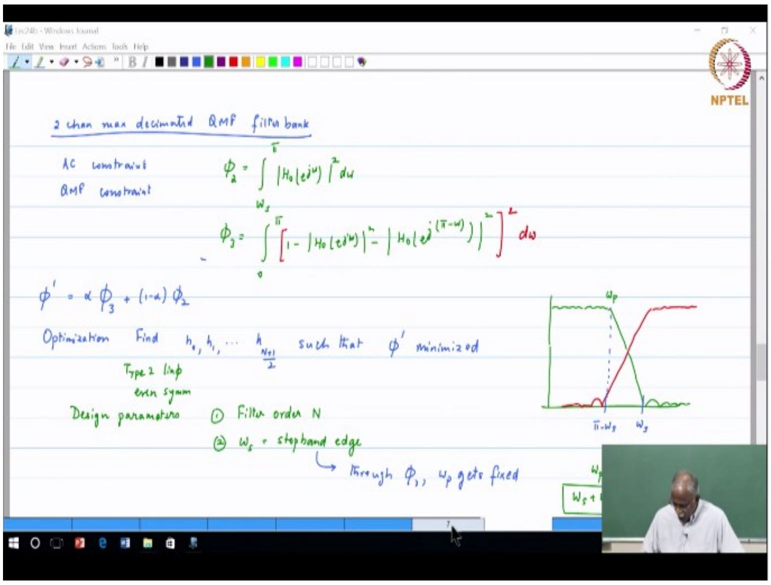

So the 2-channel maximally decimated filter bank, maximally decimated QMF filter bank okay, we have come to the stage of designing the filters. So we will apply the aliasing cancellation constraint, apply the QMF constraint and then the two functions that we are specifying are phi2 which is integral of omega s to pi. This is the stopband energy H0 e of j omega magnitude squared d omega.

Phi3 is over the entire range from 0 to pi 1-mod H0 e of j omega magnitude squared-mod H0 e of j pi -omega or omega -pi does not matter whole squared and this one whole squared okay. So let me just change the colors that you would not miss that. So there is an outer square times d omega and the overall objective function is alpha times phi3+1-alpha times phi2. We said we will call it as phi bar that is the objective function.

And the optimization problem now states that the optimization task is to find the coefficients of the filter H0, H1, all the way to H of n+1/2. Keep in mind that we are designing a type 2 linear phase, linear phase with even symmetry. So that is why we need to worry about only half the coefficients. The other half will be the same in reverse order and find these coefficients such that the objective function phi is minimized.

So actually is a numerical optimization problem that we will set okay. So what are the design parameters? What can you vary or what do you have to specify? The design parameters are only 2. Number 1 is the filter order, filter order N that tells you how many coefficients are going to be optimized. The second design parameter that we have to specify is omega s which is the stopband edge.

Because in your objective function that is the only one that is not yet specified, all the others then get specified, stopband edge okay and implicitly once you fix the stopband edge and you have constraints phi3, through phi3 this once you specify omega s omega p gets fixed because of the flatness constraint that we have specified gets fixed okay and interesting to observe the following may be already sort of noted that.

If you sketch the H0, it has a crossover around okay I should always show the ripples because these are not ideal filters. So some ripples in the passband and then that okay that is the and if I draw the shifted version, it is going to look identical because it is the original filter shifted, so let us mark off the what has been specified. This is omega s okay, so what is this? That will be pi-omega s.

Notice that because of the flatness constraint, these two will have to align right because that is the point at which your stopband of the red filter is starting, that should align with otherwise the flatness constraint will not be satisfied. That edge where the green or the passband edge of the green filter that is omega p for the green filter okay. So omega p has to line with pi-omega s because of the symmetry constraints or in other words omega s+omega p are respect to pi okay.

So there is a constraint that omega p gets fixed without being actually being explicitly specified because of the symmetric constraint, the flatness constraint and you can also look at it pictorially okay. Now let us go back and relook at the problem where you decided to use it as all-pass filters, take it I am sorry okay you will have to specify yes of course yeah good point.

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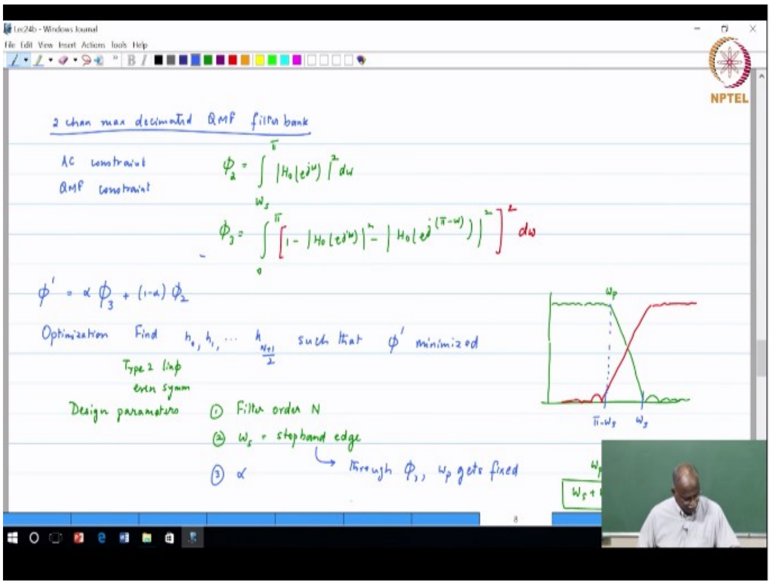

So maybe we add that as the third one because you want to give a certain weightage, so you can add that as I should have mentioned that alpha is also something that you have to specify okay. Now if you now look at the polyphase decomposition okay so the phase elimination part is done, the design of the filter bank is done, we will do a MATLAB exercise just to make sure that we actually can verify the flatness constraint okay.

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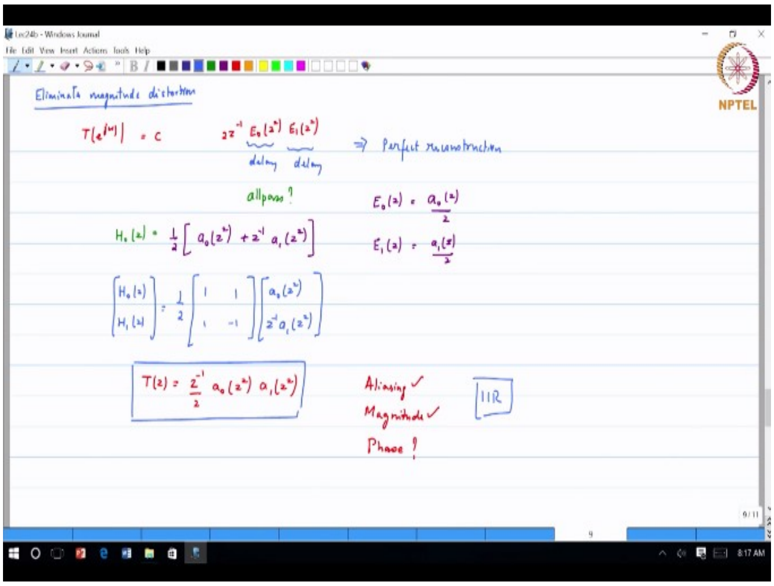

The next part where we were trying to eliminate the magnitude distortion, so eliminate magnitude distortion, you are back to a problem that we have studied before but hopefully with a little bit more insight that we can derive this time, eliminate magnitude distortion the polyphase components, the overall transfer function will be 2 times z inverse E0 of z squared E1 of z squared.

The last time we forced them to be delays, it is not necessary. We said that they could be all-pass filters and if they are all-pass filters then the transfer function comes out to be T of z=z inverse/2 a0 of z squared A1 of z squared a0 and a1 are all-pass function. So therefore you get perfectly flat T of z response and all you need to now worry about is how do you get good filters and in the process what has happened to the phase okay.

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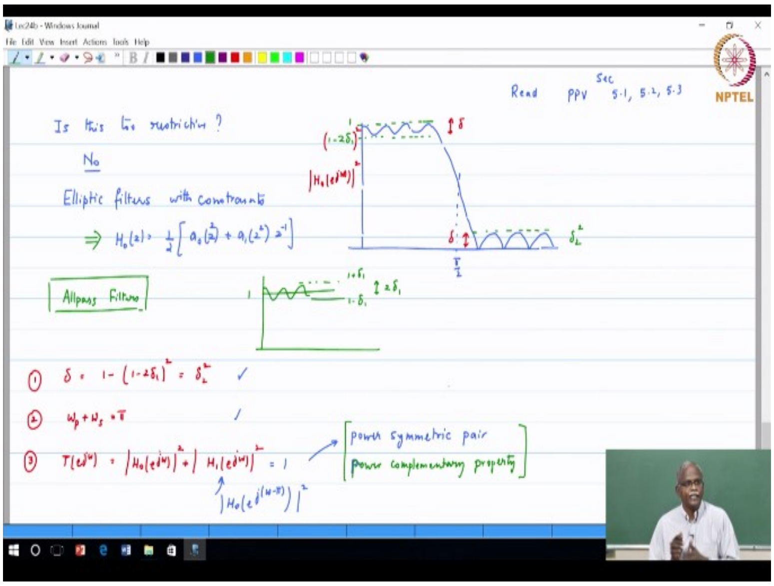

So that is the part at where we have stopped. We said that elliptic filters with certain constraints actually do satisfy this property. Let me just make a couple of statements about the elliptic filter. Again, encouraging you to read the following sections PPV, sections 5.1, 5.2, 5.3, those are these sections which we are covering today and intuitively this is what is happening as far as the elliptic filter is concerned.

Elliptic filters belong to a unique class of IIR filters which have a very distinct type of property. Both there are ripples in the passband, ripples in the stopband and those ripples are Equiripple, so elliptic filters have got a very unique property. So if you were to sketch an elliptic filter, you will see something of this type. I have sort of exaggerated the ripple and you will see something of this type.

So that is a very interesting, now typically when you design a filter low-pass filter you specify this to be equal to 1 and your ripple goes on either side and basically your upper point you call it as 1+delta1, the lower point as 1-delta1 and the total ripple as 2 times delta1. Now we are going to do something slightly different. We are going to normalize the peak value to be equal to 1 okay.

If you normalize the peak value to be equal to 1, then this point now becomes 1-2 times delta1, you have kind of have normalized it, so again the reason you are calling it 1-2 delta1 is evident from the convention, usually it is 1+delta1 and 1-delta1, you have called this. So if I were to plot the magnitude squared H0 e of j omega magnitude squared okay. If I were to plot that this would be square okay.

So that is the and so for the squared response if I call this as delta that is a total deviation for the squared response. What will this be? This typically would have been called delta 2, when you square it, it will become delta2 squared that is the stopband energy and because of the elliptic filter property and if we specify that this is also equal to delta okay so what you are trying to force your elliptic filter to have the following constraint.

That your ripple for the squared response, the ripple in the passband and the ripple in the stopband are the same. Again, these are design constraints that can be implemented, so not restrictive in any way. We just want to make sure. Now very soon or you know in a minute you will see why we are trying to go through this exercise but if you do satisfy this.

So then the constraint that we have now imposed on the elliptic filter design is delta=1 – (1-2 delta1) whole squared, 1 is the upper line. The lower line is 1-2 delta1 squared, the deviation is this is also equal to delta 2 squared. So delta1, delta 2 from the definitions and we have done some scaling. So therefore this is it and this is constraint number 1 for the elliptic filter. The second constraint is again probably no surprise omega p+omega s=pi okay, omega p+omega s=pi.

Now I forced that constraint as well and then a third constraint that will come about is T e of j omega=magnitude H0 e of j omega magnitude squared+H1 e of j omega magnitude squared. Again, the QMF constraint says that this is actually H0 e of j omega-pi or pi-omega magnitude squared that is the same and this being equal to 1 okay. Now under these constraints we are guaranteed that the polyphase decomposition of this elliptic filter will result in all-pass functions okay.

There is a fair amount of theory to prove that. Again, what we are going to do in our class is to actually assume that the all-pass functions exist and then move forward but for the complete picture these are and you can kind of get a feel for it. What we have said is there is going to be a quadrature symmetry. So basically this point is going to be pi/2 okay. These ripples are of the same level.

So when I flip it and add them together then there is a chance that where there is a dip if I have a peak then actually I will get a constant. So again the way we have constrained the design or of the elliptic filter kind of says that okay this is something that was straightforward. This is something that is can be imposed. The tricky part was how do you get the complementary property.

So any pair of filters H0 and H1 that satisfy this kind of a property that their magnitude squares add up to a constant actually belong to a class of filters called the power symmetric pairs. So H0 and H1 must come out to be a power symmetric pair because of the QMF condition there is also sometimes this is also referred to as a power complementary property okay, so the powers of H0 and H1 are complementary so they can add up to a constant.

So power symmetric or power complementary property, these are all things that we are going to just sort of engage with but I just wanted to give you a heads up on what is the what enables us to satisfy this okay. Now given this set of conditions, the 1 and 2 are straightforward, third is a property that we would have to be careful to deal with but this class of power symmetric filters exists.

So given this the elliptic filter design with these constraints will be decomposable in the following way. So given that statement we will come back to this once we have studied all-pass filters. Now we are going to spend the next portion of the lecture and a bit depending on again your familiarity with allpass, we will spend some amount of time.

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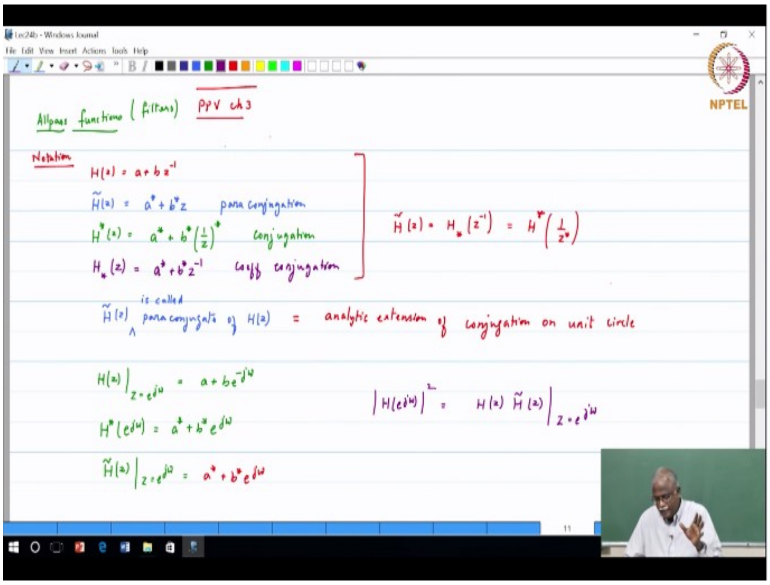

So the first thing in our study of all-pass functions, you can also call it all-pass filters but all-pass functions or filters. By the way, this is also covered very nicely in Vaidyanathan's book chapter 3. There is a section on all-pass filters. Again, I definitely would encourage you to read that okay. The first one I would like to introduce to you is notation. There is a lot of notation that is that may be somewhat new.

I just want to introduce that so you are comfortable with it. If I have a polynomial H of z a+b z inverse where a and b can be complex okay. I am going to define the following H of z tilde as I am going to define it in the following way, it is a conjugate+b conjugate times z. This is called para conjugation. Now where does this come from, how do you get something like this maybe if you have not seen it before, it does not look intuitive at all.

And why do you have to give it a name for example. So now let us anchor para conjugation in terms of the operations that we are familiar with. Now instead of H tilde if I had asked you to do H conjugate of z okay conjugate the function. So anything that is a complex number is has to be conjugated. So if you do that then what you get is a conjugate+b conjugate. I am going to write z inverse as 1/z, z is a complex number.

So therefore it is 1/z conjugate, so this is conjugation okay and yet another notation that is going to be introduced is when I have a polynomial with complex coefficients, if I put the conjugation side that asterisk at the bottom that says conjugate the coefficients, leave the z terms alone, do not conjugate those. So then what you get is a conjugate+b conjugate z inverse okay.

So this is coefficient conjugation, only coefficient conjugation okay. So 3 things we have introduced. One is para conjugation where you conjugate and change z inverse to z, conjugation of the expression where all terms including the z term gets conjugated and then you have the coefficient conjugation okay. So now using these as the components, I would like you to just verify that H tilde of z can be written in 2 ways.

The first one is coefficient conjugation, only thing that does not changed is z, replace z with z inverse, do the coefficient conjugation and replace z with z inverse or if you are going to do the traditional conjugation where everything gets conjugated then what we have to do is you will end up with 1/z star, do not want that, replace that with 1/z star, replace z with 1/z star, so basically 1/z will become it will become z and z star star will become z itself.

So basically you will get what you are looking for okay. So there are 2 ways of expressing it. Again, this is just to make sure that you are comfortable with the notation that we have introduced. Now here is the important part, H tilde of z is called the para conjugate of H of z is called or is referred to or is defined as the para conjugate of H of z okay. Lot of this comes from the theory of analytic functions and complex variables.

And again you may have been exposed to it, I will just I am using only that parts that I need to need for us okay. Now why are we even defining the para conjugate? So when you look at this function on the unit circle, so H of z evaluated on the unit circle e of j omega you can say that this is equal to a+b e of -j omega okay. Now if I conjugate H conjugate e of j omega then what do I get? I get a star+b star e of j omega okay.

Now please evaluate H tilde of z, z=j omega you actually get the same thing. You get a star+b star e of j omega okay. So the reason we have defined the para conjugate is because this is the analytic extension of conjugation on the unit circle that is as simple as that, z plane has a unit circle, conjugation is defined on the unit circle, everywhere else you have to define an analytic extension of conjugation which is what.

So this is the analytic extension of conjugation of on the unit circle, extension of conjugation on the unit circle okay. Again, this is something that you would have studied as part of your analytic functions and properties even otherwise Vaidyanathan's book gives an excellent overview of the those results that we need from complex variable theory okay. So one of the things that we are often interested in the design of filters in our case.

Once you start trying to impose these magnitude constraints is magnitude H e of j omega squared and this can be written as H of z H tilde of z, z e of j omega because what basically the writing it mathematically saying that something is analytic extension of conjugation on the unit circle sort of takes it to the next level by saying instead of just writing H conjugate e of j omega, often we are interested in magnitude squared which means it is product of H and H star everywhere else on the unit circle.

Other than the unit circle, you will have to represent it as H of z and H tilde of z okay. So that is a useful representation for us and we would like to sort of keep that picture in mind. Now why did we do this and you know what are the benefits of this, lot of the insights comes in a very, very short time.

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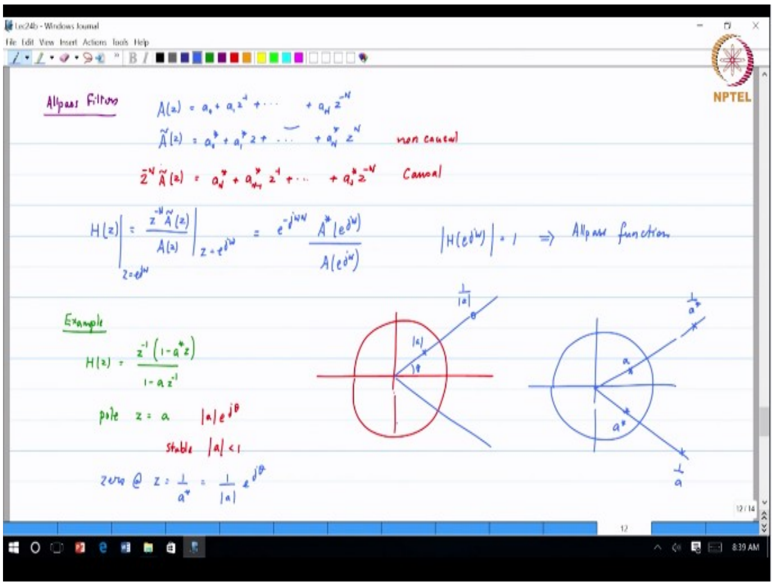

Let us pick up from there. So focus now on all-pass filters and would like to sort of use a result right at the beginning right off the back. Now if I have A of z as some polynomial, a0+a1 z inverse dot dot dot a N z power -N okay. A tilde of z is given by a0 conjugate, a1 conjugate times z dot dot dot+a N conjugate z power N alright. So this is non-causal okay but if I do z power -N times A tilde of z.

Then what do I get? Get aN star, aN-1 star times z inverse dot dot dot+a0 star z -N which is causal okay. Now here is the key result. Now if I tell you please look at a transfer function which is of the following form. A of z z inverse A tilde of z okay and evaluate it on the unit circle, z is equal to e of j omega. So this evaluated at z=e of j omega. This comes out to be e power -j omega N.

A tilde is nothing but a conjugate e of j omega; denominator is a e of j Omega okay. Take the magnitude equal to 1. So this is the general form of an all-pass filter okay. So the reason we have gone through this A tilde business and the analytic extension all of that is just sort of we get very, very compact expression, understanding of what the structure of an all-pass filter is going to be.

Because now you can verify that magnitude H e of j omega is equal to 1 and this is the basic property of all-pass functions. So in other words the transfer function H of z is a ratio of the analytic extension of a basically it is some polynomial and it is para conjugate, ratio of the para conjugate. Once you visualize it like that and you know that on the unit circle these are conjugates of each other.

Then, the all-pass property falls into place, the beauty of it is it also tells us the structure of the of the system. So what do I mean by that? Where are the poles and zeros located? So let us take a very simple, first order example. **"Professor - student conversation starts."** I am sorry z power –N thank you, yes z power –N. **"Professor - student conversation ends."** Okay. First, a very, very simple example okay.

I want to look at a first order system a of z is 1-a z inverse okay, z for order is equal to 1, so I have to multiply the numerator by z inverse by times A tilde of z 1-a conjugate times z okay. Now tell me where the poles and zeros are located. I look at the denominator first, there is a pole at z=a okay. If I can write it as modulus of a e power j theta, so I got a magnitude and so if I were to sketch this that is the unit circle.

Modulus of a if assuming that this is a stable allpass, so stable implies mod a has to be less than 1 and some angle theta. So again just for argument sake if this is the angle theta then well maybe I should draw at the other side okay. If this is angle theta that is the radius mod a, so this is where the pole is located okay. Now go back and look at the 0, we find that there is a 0 at z is equal to 1/a star, z=1/a star, so that will be equal to 1/mod a times e of j theta.

So it is along the same radial direction but at a distance of 1/a, 1 over mod a okay. Now I am sure you are familiar in the sketching of poles and zeros. So if I have a complex pole located at a, this is a conjugate, this is 1/a and this is 1/a conjugate right. Those are the locations of the so that is how you would look at the poles and zeros. So yes wherever there is a pole you will also have a 0 at the conjugate location, reciprocal conjugate location.

That is what this says and again in chapter 5 of Oppenheim and Shaffer you looked at frequency response right. How do you look at frequency response? Distance from the zero, distance from the pole, take the ratio of that. So basically when they are at reciprocal points, those distances the ratio is always maintained. So therefore wherever you go on the unit circle, the ratio of the distances is maintained and therefore you get a constant value.

That is how an all-pass function is structured, so the para conjugation property actually gives us a lot of insight into saying that because of the para conjugation relationship between the numerator and denominator of an all-pass function then we can say that all poles and zeros have to be located in this fashion, only then the all-pass function will be satisfied okay. So we are starting to get some interesting insights.

Again, let us quickly move forward because this is not our main goal. Our goal is to use all-pass functions.

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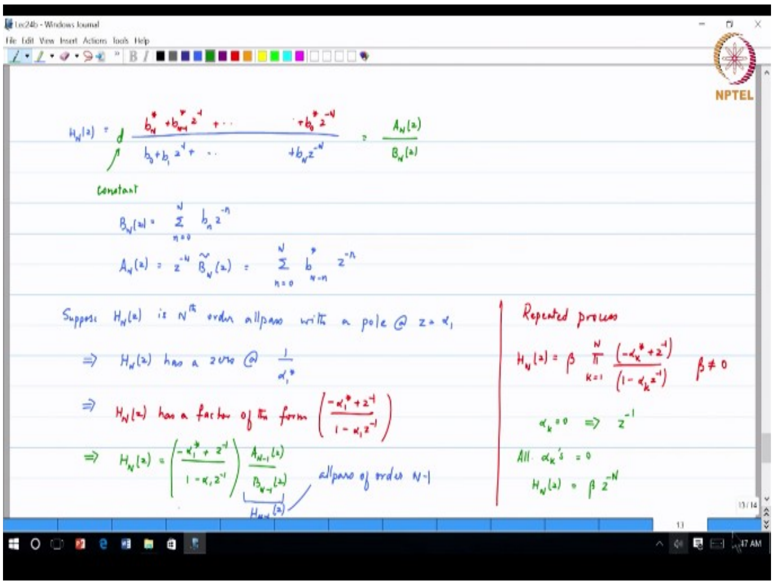

So in the general case HN of z can be would be written as the Nth order allpass, can be would be written as b0+b1 z inverse dot dot dot+bN z power –N. Numerator will be bN star+bN-1 star. So the general form of all-pass function we write it in this form and then now we would like to analyze this. So let us write down BN of z as the polynomial, summation n=0 to upper case N z power -n b of n z power -n and An of z is the para conjugate with the appropriate shift z power –N BN tilde of z okay.

And that can be written as summation n=0 to N b of N -n conjugate z power -n okay. Now here is the argument that we make based on what we discussed for the earlier case. Because of the para conjugate nature, we make the following statement. If H of HN of z, suppose HN of z is an nth order allpass that is what we have constructed it to be is an nth order allpass with a pole at z=alpha1 okay.

Then, it must have a 0 at 1/alpha1 conjugate, it must have, only then the all-pass property will be satisfied. So this also means that there is a HN of z has a 0, has a 0 at 1/alpha1 star. It has to have that is the structure of the allpass. So this also can be combined to make the following statement that if I pull out this particular pole and this particular 0 then HN of z has a factor of the form 1-alpha1 z inverse that is the pole, -alpha1 conjugate+z inverse that is the 0.

There is a factor of this type, very, very important. This means that an nth order allpass can be factored into a first order allpass -alpha1 star+z inverse divided by 1-alpha1 z inverse. Both numerator and denominator has got a first-order factor out. So then this must be well let us call it as AN-1 of z, it is a polynomial of one order less divided by BN-1 of z one order less. If I call this as HN-1 of z, now HN-1 of z also is an allpass because HN of z is an allpass.

So then this becomes an allpass of order N-1. So all-pass functions have got a very beautiful property. You can actually factor them into lower orders and preserve the all-pass property and it turns out that the stability part also can be preserved going into the factorization. So again it has got some very, very interesting and powerful applications in the context of signal processing.

Our goal is to sort of pick what we need and move forward okay. There are a couple of special cases just as an observation you can write down. If so the repeated process, so repeated process of factorization says the following that HN of z can eventually be written into a scale factor beta again that is not significant for us, the rest of it is a product of first order all-pass functions, K=1 through N, denominator is 1-alpha K z inverse.

The numerator is -alpha K conjugate+z inverse okay, notice the para conjugation structure you know comes in and works beautifully for us okay. So this is the general form and of course beta cannot be 0, it has come some constant scale factor. Now an important observation on this structure, once you say that any all-pass function can be factored in this form if a particular alpha K happens to be zero then what do you get? Is it still an allpass? Is that factor still an allpass?

Yes, still an allpass, it is equal to z inverse. So z inverse is actually a delay, is an all-pass function and sort of it collapses to a delay so the case where you have all alpha K =0, all alpha K =0 then you get HN of z is equal to the scale factor times beta times z power -n that is fine. So z power -n is actually a special case, you can think of a delay as a special case of an all-pass function okay and hopefully that is a useful and an interesting insight that you get from there.

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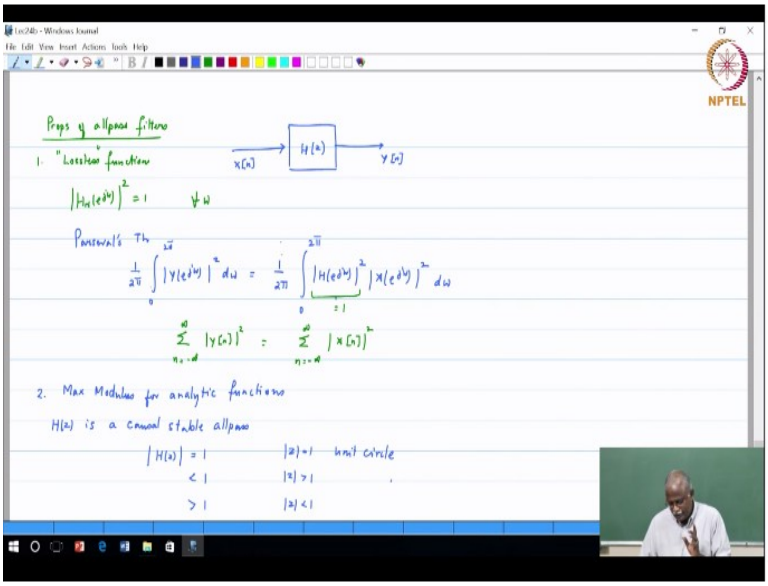

Now before we leave the discussion of allpass, a couple of quick properties I would like you to verify or a certain two properties which come to play in our discussion, properties of all-pass filters, all-pass filters belong to a unique category of signal processing functions which are called lossless functions okay. So lossless the property of losslessness okay what is that? What we say is let us take HN e of j omega magnitude squared equal to 1.

We have taken the constant equal to 1 for all value of omega that is the definition of an all-pass function. Now if I feed in any signal x of n to this filter all-pass filter H of z out comes y of n then by Parseval's theorem, I know that the input output power relationship is given by 1/2 pi integral 0 to 2pi or –pi to pi mod Y e of j omega magnitude squared d omega is equal to input writing the Parseval's relationship, it is mod H e of j omega magnitude squared X e of j omega magnitude squared d omega right.

That is input-output relationship for LTI systems for using Parseval's but this one is exactly equal to 1, so therefore the energy of Y is equal to the energy of X or if you write it in the time domain, this will be n=-infinity to infinity mod y of n whole squared, on the right hand side we get n=-infinity to infinity mod x of n whole square, energy is preserved that is why it is called lossless function okay.

The second part that second property that is very important, again I will state it without proof but please do read up or look up, again the result is what is important for us. This is called the maximum modulus theorem, maximum modulus theorem of analytic functions, complex variable theory okay. Now if H of z is a causal stable allpass, causal stable okay causal stable that means there is no powers of z.

All the poles are inside the unit circle, causal stable allpass, then one thing we know mod H of z, no mod H of z is equal to 1 on the unit circle, this is the unit circle okay. Next part is the non-intuitive part requires proof but I am stating it without result. Please do look up Vaidyanathan’s book. This is less than 1 for mod z greater than 1, greater than 1 for mod less than 1 okay. That is using the maximum modulus theorem for analytic functions.

This is a result that is derived. Now I want to just do a sanity check, where are the poles of H of z? Inside the unit circle. So there will be points where the magnitude of H of z is going to blow up which means it will be greater than 1. So that satisfies, inside the unit circle there will be poles which will cause it to blow up, so basically it is not only at the poles but everywhere inside the unit circle because of analytic continuation it has got greater than.

So basically the maximum modulus occurs on the contour is mod z=1 or minimum value okay. So basically and once you proven that property then everywhere outside has to satisfy the other constraint which will be mod of H of z is less than 1 okay. This again is a useful result but the result is more important than the result.

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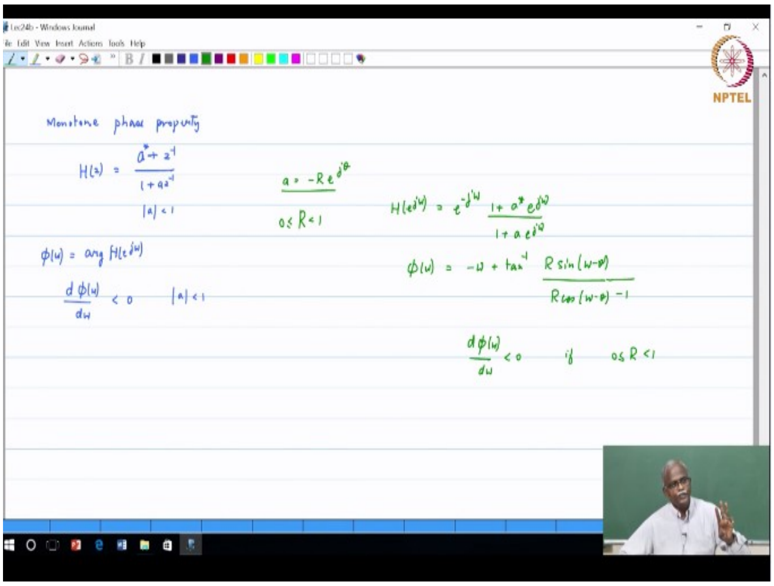

So the last one I would like you to verify is something called the monotone property. I will write it down quickly; we will stop with that. Monotone phase property okay, so if I have an all-pass function H of z which is given by 1+a z inverse a conjugate+z inverse that is a first order stable allpass. Basically, I will say that mod a is less than 1 that is the location of the pole and now if you were to write H e of j omega, argument of H e of j omega and call that as phi of omega.

And show that d phi of omega/d omega is always less than 0 provided mod a is less than 1. That means it is a stable allpass. If I look at the argument of this, it is a first order allpass. It is monotone decreasing; slope is always negative okay. Easier if you use the following convention again you can show it, please do show it, it is not a difficult task and in fact probably in DSP you already done that.

So please use the following notation, it helps if a =-minus R e power j theta and R is<1 >or =0 <1, so if you be using this notation then you can show that you can write H e of j omega in the following way e power -j omega 1+a star e of j omega, basically this is the notation where we use the para conjugate/1+a e of j omega and from this it is easy to show phi of omega is=-omega+tan inverse of R times sin omega -theta.

So basically you are writing R a e power j theta, this divided by R cosine omega –theta -1 okay. So please write down the expression, basically differentiate phi of omega and show that it is d phi of omega/d omega is<0 if R is >or= 0 < 1 which is that is under this condition you can show that. So this is monotone phase. So 3 properties of the allpass, one is the losslessness. We have proved maximum modulus; we have stated without proof.

Third one, we have stated it with the request that you should prove it okay. Thank you. We stop here and pick it up. **"Professor - student conversation starts."** Correct. Thank you.

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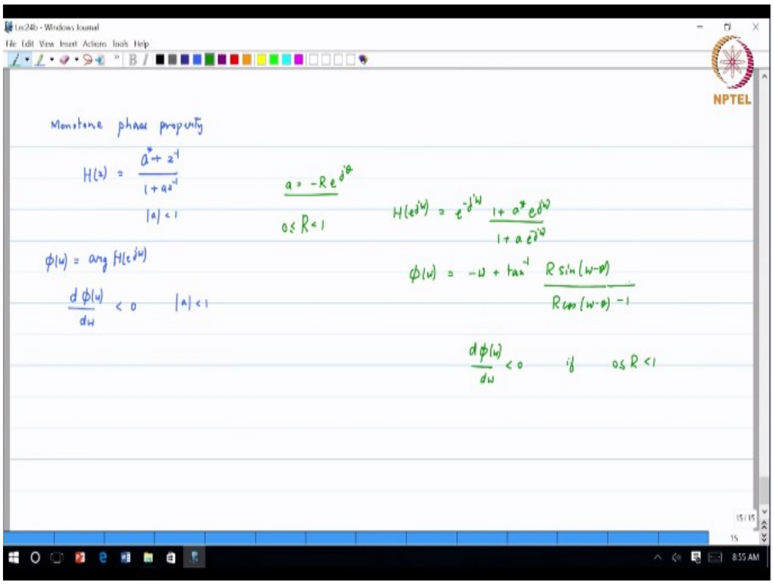

Yes, e power put it in wrong place, correct, thank you. **"Professor - student conversation ends."** We stop here okay. So please read sections 5.1, 5.2, 5.3, this gives a good platform for us to complete our discussion on all-pass functions but we are going to now go into a discussion on the structural all-pass properties. How do you derive a structure that will guarantee the all-pass property even under quantization? Thank you.