**Multirate Digital Signal Processing**
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**Lecture – 23**
**2-Channel QMF Filter Bank Design**

Good morning and welcome to lecture 23.

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And today we will be continuing where we left off in the last lecture, lecture 22 and the lecture 22 we had spent a fair amount of time looking at the 2 channel filter bank. The historical development, the polyphase decomposition and also we started to talk about how to use the linear phase filters and today we will pick it up there and we will design the 2 channel filter bank using the linear phase constraint.

But before that let us begin with a quick review of the topics that we have covered in lecture 22.

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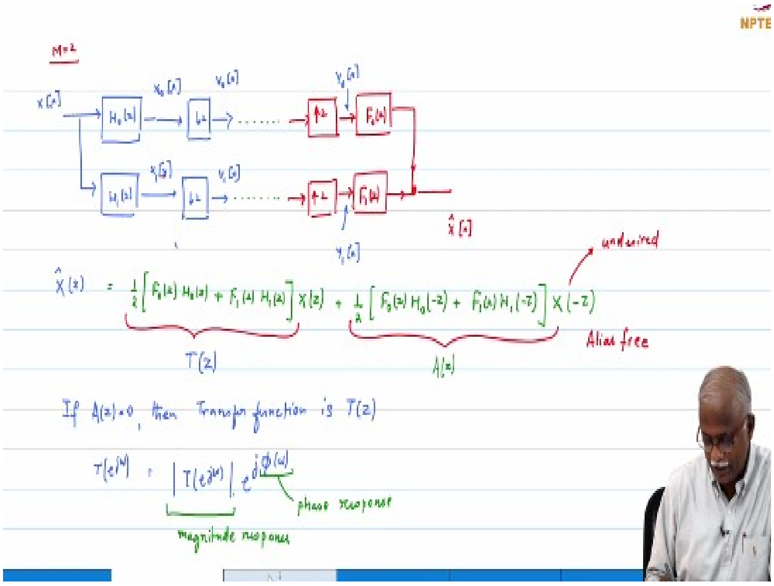


I would like to begin by reviewing the 2 channel filter bank showing that we can write the overall response between X input and output in this fashion where X hat of z can be written as T of z times X of z+ A of z times X of –z and X of –z representing the term which contains the Aliasing. So this you can view as the undesired component the X of –z. If A of z becomes= 0 then we get rid of the aliasing term and therefore we call it Alias free.

So A of z=0 is Alias free in which case we are left with only the transfer function T of z and T of z can be represented in terms of a magnitude response and the phase response so Te of j omega can be written as mod of Te of j omega e power j phi of omega where this is the magnitude response and we have the phase response a combination of these 2 and this we have looked at in the last class.

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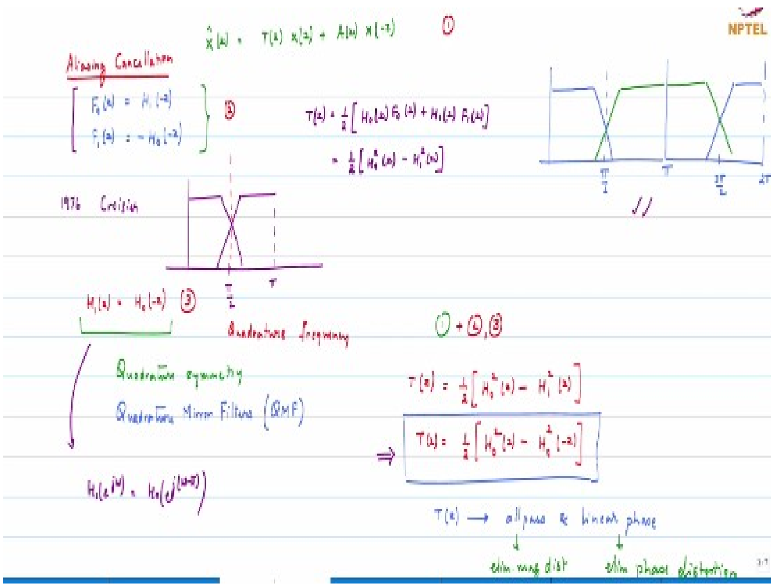

Aliasing cancellation the condition that guarantees that aliasing will be cancelled where was derived in the last lecture. Again we will use this in today's lecture as well F of z=H1 of –z and F1 of z=H0 of z. So this is the combination we call as the aliasing cancellation constraint. So equation number one is the overall transfer function if we combine it with the Aliasing cancellation condition of 2 then we get rid of A of z.

We are left with only the T of z and today we will look at what are some of the aspects that we will be using to design the filters. So just for completeness let me write down the expression for T of z. So Aliasing is cancelled we are left with T of z= 1/2 of H0 of z- or + H1 of z F1 of z applying the constraint that we have shown for the Aliasing cancellation. This will become= H0 squared of z- H1 squared of z okay.

Now as we discussed in the last lecture 1976 was when an additional observation and a constraint was introduced that of the Quadrature Mirror Symmetry. So Quadrature frequency represents pi/2 and we design H0 and H1 to have symmetry around the quadrature frequency. So these are called Quadrature mirror filters H0 and H1 if they satisfy the symmetry is called Quadrature Symmetry.

So the Quadrature Symmetry is given by H1 of z=H0 of –z. In other words, it is a shifted version of the original filter if you want to write it in the in terms of the Fourier Transform this can be expressed in the following manner. H1 e of j omega= H0 e of j omega- pi okay. So that is the expression that we want to keep in mind it is a shifted version. Now if this condition is satisfied we now have condition number 3.

And with this we now have the condition of T of z is now modified to get the following expression where T of z= 1/2 of H0 squared of z- H0 squared of –z. So basically the entire condition expression for the transfer function is now expressed in terms of a single filter H0 of z. So now we would like to look at in the last lecture we pointed out that if T of z happens to be all pass.

Then we have eliminated magnitude distortion, but there is phase distortion is present. However, if T of z is linear phase then we would have eliminated phase distortion. So in today's lecture we are going to look at how we can eliminate phase distortion in the overall transfer function so that is our goal for today and we will develop it.

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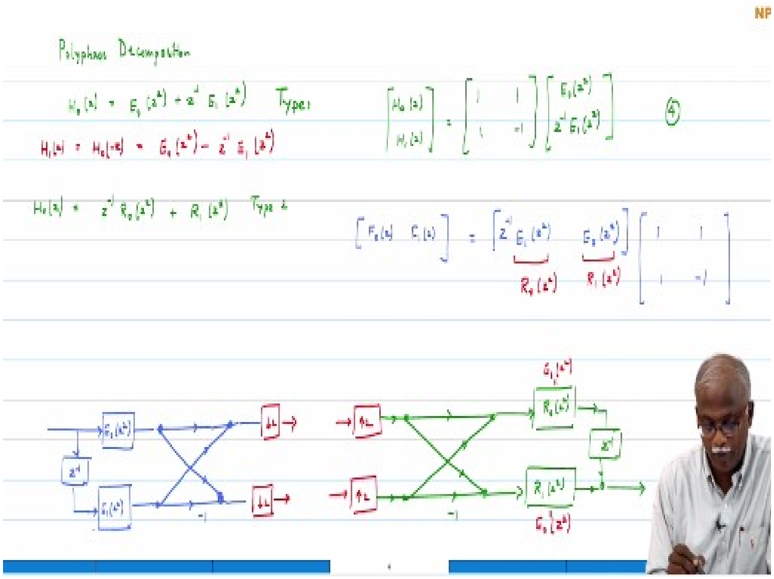


But before that let us quickly look at couple of other results that were obtained in the last lecture just for completeness. If you decompose the analysis filters in terms of the polyphase components.

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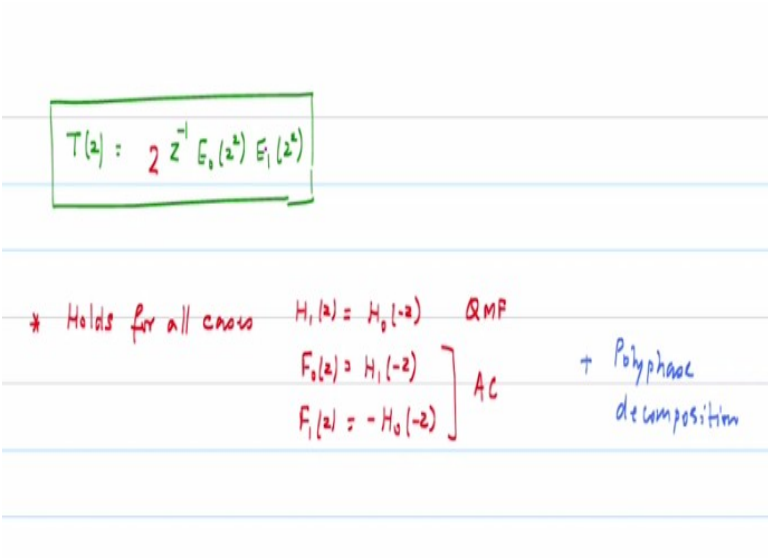


It was shown that the overall transfer function can be written in terms of T of z= 2 times z inverse E0 of z squared E1 of z squared. Now the conditions that we have imposed are that the Aliasing cancellation conditions are present or satisfied the Quadrature mirror conditions are satisfied and we have done polyphase decomposition to get this expression.

Now we want to go back and look at the case that transfer function T of z is a linear phase transfer function. So in the last lecture we talked about what would be the implications if E0 and E1 were delays what would be the implications in terms of the prototype filter and we saw that making E0 and E1 as delays will not give us good low pass filter. So let us for the moment we will put aside the magnitude distortion constraint.

And let us see how we can satisfied the linear phase constraint so that will be the focus of our discussion today.

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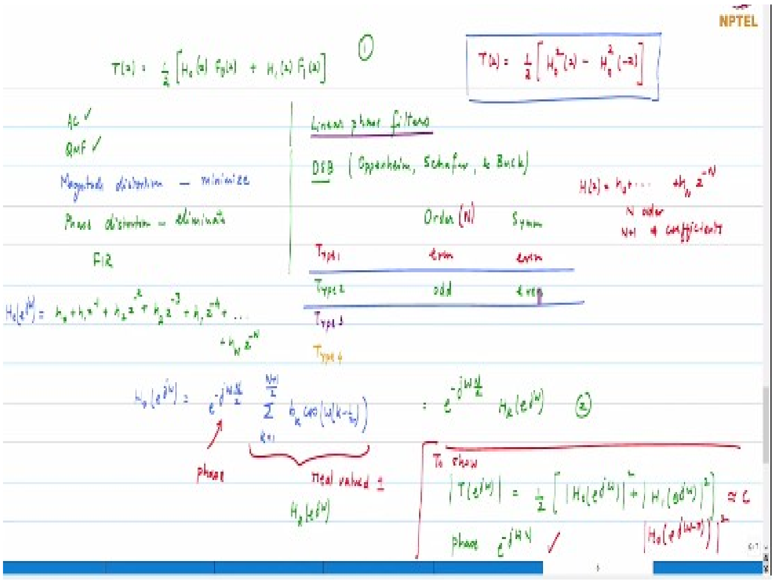


Overall transfer function is given by half H0 F0+ H1 F1 Aliasing cancellation constraint satisfied, QMF constraint satisfied then it will reduce to the transfer function that was presented in the earlier it will be this transfer function that we will now have to deal with okay. So let us just state that for completeness. So this is the transfer function after we have satisfied these conditions so this is the condition that we have to satisfy.

In the reference book that has been suggested for the course Oppenheim, Schafer and Buck there is a section where we talk about the design of linear phase filters that there are 4 types of filters as we mentioned in the last lecture. If the filter order is N we can write H of z as h0+ h1 z inverse h2 z-2 all the way h of N z-N. Notice that the filter order is N and filter order is indicated by the highest power of z and the number of coefficient h0 to hN will give us total of N+1 coefficients.

So Type 1 linear phase filters has got even order and it has got even symmetry. Type 2 linear phase filters have got odd orders and even symmetry. Now let me just maybe it is useful for us to just take a simple example of a filter that satisfies the Type 2 constraint because that will be useful for us in our discussion today.

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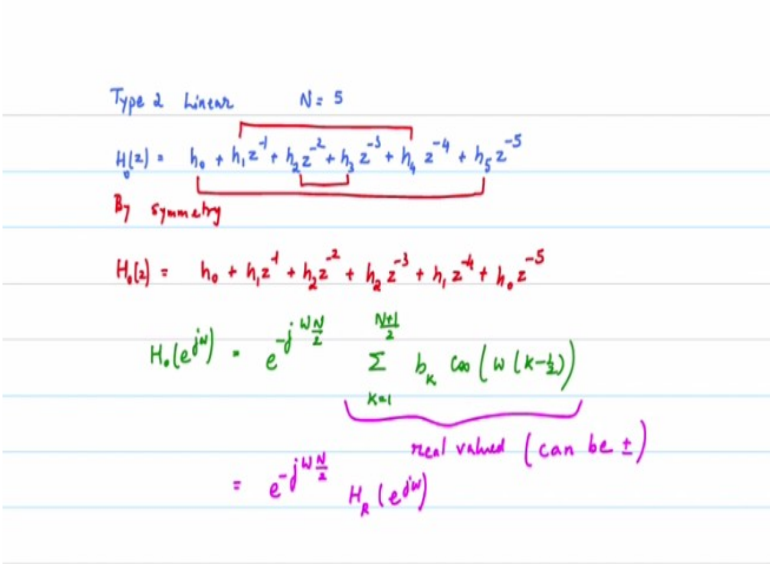


So Type 2 linear phase filter a Type 2 linear case filter let us take the case where the filter order=5. In that case we will get a filter coefficients which are H of z= h0+ h1 z inverse h2 z-2+ h3 z-3+ h4 z-4 and h5 z-5 okay h5 z-5. Imposing the symmetry constraints by symmetry where are the symmetry conditions where are the symmetric coefficient h0 and h5 will be the same h1 and h4 will be the same h2 and h3 will be the same what we will get is a filter which is h0+h1 z inverse+ h2 z-2+ h2 z-3+ h1 z-4+ h0 z-5.

And because of the symmetry if you write down for your transform and express it we can write this the frequency response H0 e of j omega this is h0 of z. The symmetry conditions and if you look at the Fourier transform H0 e of j omega we can show that this is of the form e power-j omega N/2 summation k=1 through N+1/2 b subscript k cosine of omega k-1/2.

Again this is the result from Oppenheim and Schafer that is the reason why I am not deriving it in this class. I request you to kindly look it up. Now this is a real valued function let us make a note of that. Real valued function it can be positive or negative because it is a real valued function above it can be more than 0 less than 0 can be positive or negative and we can denote this in terms of a real valued function.

So basically we can write this as e power –j omega N/2 H subscript R which indicates that it is a real valued function of e of j omega which is representing the sum of cosine term. So this is the part that I just wanted to highlight this is related to what we have been developing. This is Type 2 linear phase this was what was developed in the last class just sort of refreshed your memory in terms of what has been developed and what we can do.

So now what we would like to do is apply this Type 2 linear phase expression with our overall transfer function that is our task for now. So apply it to the overall transfer function let us start on a fresh page.

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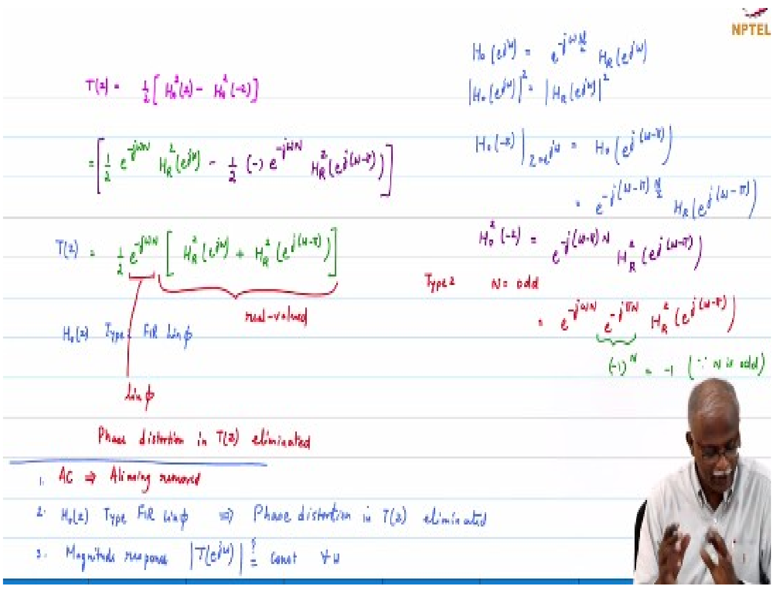


We have the overall transfer function T of z=1/2 of H0 squared of z- H0 squared of –z. We have H0 e of j omega given by e power j omega N/2 HR of e of j omega. Let us just get the expression and then substitute. So H0 e of j omega magnitude squared will be= e power j omega N/2 that term will go away because we are taking the modulus. This will be HR e of j omega magnitude squared. Now H0 of –z evaluated on the unit circle H0 of –z evaluated on the unit circle can be expressed in the following manner.

This can be shown to be= H0 e of j omega- pi. H0 of e of j omega-pi. So now substitute the expression for H0 this will be= e power – j omega- pi omega – pi times N/2 times HR e of j omega –pi okay that is the expression that we have. And if you take modulus squared we can then show that what will be the expression that we will have. So now what we need here is H0 magnitude squared, but what we need is H0 squared.

So let us substitute and obtain the expression. So this will be half H0 squared so that will give us e power –j omega N times HR e of j omega squared that is the first term. The second term will be= we are going to move this little bit to the right so that we have more space to express or give the expression. Now we write down the second term – 1/2 of H0 squared of –z. So if you want to write down that step H0 squared of –z that will be= it will be the squaring of this expression e power – j omega – pi times N HR squared e power j omega- pi okay.

So that is the expression, but note an important one we have assumed Type 2 linear phase. So therefore N is odd. So if you now expand this expression this will be e power – j omega N e power –j pi N e power – j pi is -1 and the term here is HR squared e power j omega –pi. Just a quick simplification of this term this is nothing but -1 raise to the power N where N is odd=-1 since N is odd.

So therefore if you now want to substitute back into this expression what you will get is – sign from the -1 e power – j omega N HR e of j omega –pi whole squared or write it here H squared okay. So this is the overall transfer function that we have it can be simplified and we can obtain some very interesting insights into the expression that we have just now derived. So we can take the 1/2 and e power – j omega N outside.

What is left within the bracket is HR squared e of j omega+ notice that the – and – will cancel each other giving us a + sign HR squared e of j omega – pi whole squared okay. So this is the overall transfer function if H0 happened to be a Type 2 linear phase transfer function. So let us just now assume we have assume that H0 of z is a Type 2 FIR linear phase transfer function then in that case the overall transfer function T of z comes out to be of this form.

Very important to note that this is a real valued and what we have here is a term that is linear in terms of the linear in terms of the phase linear phase. You can see that the phase grows linearly as omega increases. So what we now have obtained is a transfer function for the 2 channel filter bank where the overall transfer function is linear phase and therefore under this condition the phase distortion in T of z has been removed.

Phase distortion in T of z is eliminated. However, we are still left with the transfer function HR squared e of j omega+ HR of squared e of j omega- pi. Now that could have some magnitude distortion. So how will we deal with that? So here is a summary statement if you will. Aliasing cancellation implies Aliasing is removed or eliminated okay so that is point number one.

Aliasing cancellation constraint will remove Aliasing; that is number one. Second we have made the assumption that H0 of z is a Type 2 FIR linear phase. This implies that phase distortion is eliminated phase distortion in T of z. Phase distortion in T of z it has been eliminated or removed eliminated. So what we are left with is only the magnitude of the transfer function so which would be the magnitude response will be T e of j omega.

The magnitude response we would like it to be as close to a constant as possible as close to like an all pass filter. So this is= constant for all values of omega and we will put a question mark or how close can we satisfy this condition. So that is what we are going to be looking at and trying to understand and then try to implement. So let me summarize what we have said so far.

We started out with the 2 channel filter bank. We said that we can enforce the Aliasing cancellation constraint to eliminate Aliasing which leaves us only with the transfer function T of z. Now T of z has got a magnitude component and a phase component. The magnitude component we showed can be eliminated if the overall we wrote the polyphase components we showed how the magnitude components could be magnitude distortion could be eliminated.

But that lead us to filters that are not of very good quality. So therefore for now we said let us leave aside the magnitude distortion let us look at the phase distortion. So going back to the original structure applying the QMF constraints we now said and the fact that the analysis filter H0 will be linear phase transfer function we said that we will pick the Type 2 transfer function and then showed that this would be the case of the overall response.

And we now have the fact that Aliasing has been removed. We now have H0 of z as a Type 2 linear phase transfer function, Type 2 FIR linear phase transfer function so we showed that the phase distortion has been eliminated and now we would now like to see how we can eliminate the magnitude distortion that magnitude distortion as well. So the task for the rest of the lecture is to develop a system where we can design the filters to minimize the magnitude distortion.

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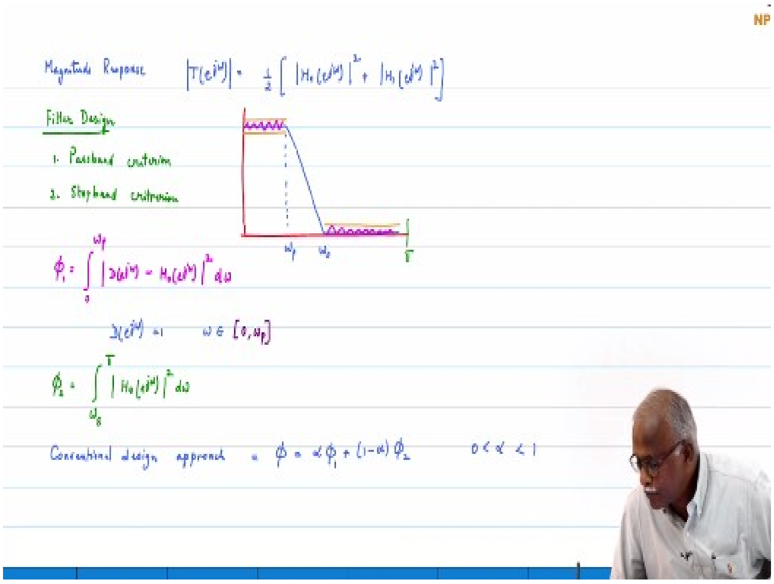

So the magnitude distortion or the magnitude response. Magnitude response we can describe or write it down as T e of j omega magnitude= 1/2 H0 e of j omega magnitude squared+ H1 e of j omega magnitude squared. So that is the response we are going to design it with the Type 2 linear phase filters and now we go back into the filter design component which I am sure you would have studied in digital signal processing.

But we will just highlight the parts that we would need for today's discussion okay. So typically in the design of a filter, design of a FIR filter we would introduce 2 criteria one is the pass band criterion and there would be a stop band criterion. Now let me just sketch the requirement and then we can proceed with the aspects of the design. So this is the design of FIR linear phase real coefficient filter.

So we have the filter response to be as flat as possible under we reach omega p. This is the pass band edge omega p and then therefore have a transition band between omega p and omega s and then from this point on you want the ideal transfer function to be close to 0, but in practice what we would find is that you would have to permit a band around the during the pass band edge and also the stop band edge.

And we would like the filter response which is the actual filter response to be contained within this and as close to this as possible and this is what we would like to design this is the classical FIR filter design where you specify the pass band edge and the stop band edge okay. So the way we would design it is we would first of all compute the pass band error criterion let us called that as phi 1.

This is integral between 0 and omega p where the deviation of the desired response of the filters response from the desired response desired response I am going to write it as De of j omega – the response of the filter H0 e of j omega magnitude squared d omega. Now in the pass band we could set De of j omega=1 for omega belonging to the pass band from 0 to the pass band.

So this is in the pass band region okay. This is basically what is the how much deviation are we seeing from the ideal response from the pass band gain of 1 then we have a second transfer constraint on the stop band edge stop band performance. So this basically corresponds to computing the total energy in the stop band. We want that to be as small as possible so that the filter will attenuate any signal in the stop band.

This is given by starting with the stop band edge all the way to pi of the magnitude response of the filter. H0 e of j omega magnitude squared d omega. Now conventional design approach says that we must minimize the combination of phi 1 and phi 2 conventional design approach says we minimize the combination of phi 1 and phi 2 let us call it alpha times phi 1+ 1- alpha times phi 2 where alpha is a real valued constant which is between 0 and 1.

So therefore you have given weighting accordingly you can give half and half which means you get equal rating or you can give 0.7 and 0.3 any combination we can do. So if you are more interested in the pass band give more emphasis on the pass band error or if you are more interested on the stop band give more emphasis on the stop band side. Now this is conventional low pass filter design.

How is it different when we want to design a filter for a 2 channel QMF bank so that is the task so is before us today.

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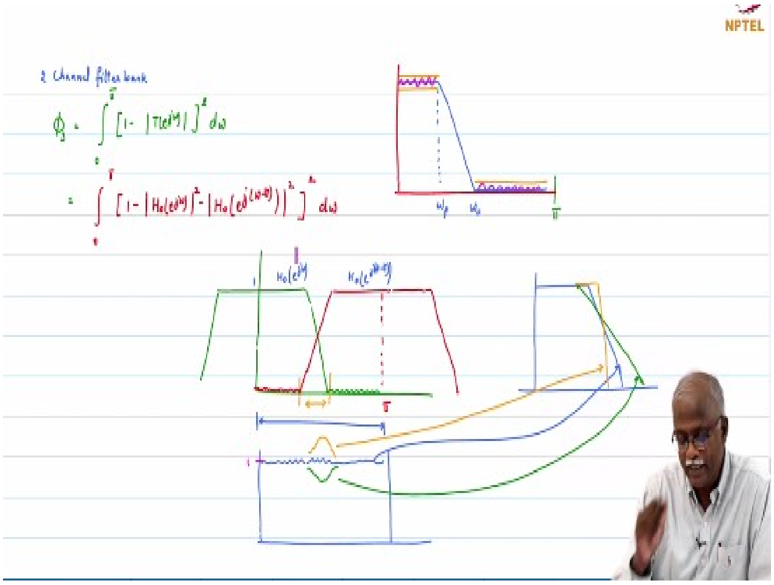


So we said that in the case of a 2 channel filter bank we are trying to minimize the magnitude distortion and therefore the condition that we would like to minimize let us call it as by in terms of a third objective criterion which is given by in the entire band from 0 to pi in the whole band. We would like the transfer function to be as close to a constant as possible. So this would be= 1- the magnitude response the magnitude response that we are talking about is mod Te of j omega squared okay.

So this is the magnitude response and we would like to be= we would like to this to be as close as 1 as possible so basically want to minimize this. So 1- Te of j omega whole squared d omega. Now this we can write it in terms of the components of Te of j omega this can be written as 0 to pi 1- mod of H0 e of j omega magnitude squared – H1 e or let us write it in terms of H0 itself. H0 e of j omega- pi magnitude squared and the whole squared time d omega.

So this is the overall objective function phi 3. Now what I want you to visualize is the following. If you go back and look at the design of the filter we have this as the constraint for the classical filter design. Let us take a closer look at that okay. So we have here the classical filter design, but now can we on to this one can we map the constraint for what we call as a flatness constraint.

The condition that we are trying to minimize the magnitude distortion completely. So here is what the response or what we are trying to achieve what it looks like. So we have H0 e of j omega which has a certain response. And we then have H0 e of j omega shifted by pi and this frequency is pi and so this is the complete system scenario. Now between 0 and pi between in this band of interest between 0 and pi we are measuring the deviation from a constant.

So we are taking this is H0 this is H0 e of j omega-pi shifted version this is H0 e of j omega. We want to take magnitude squared and then add them together. Now if this is close to 1 magnitude squared just look at the magnitude squared response between 0 and pi. This is going to be close to 1 in this region. It is going to be close to 1 in the stop band region. Now where things are not so certain is in the transition band.

So the question mark lies in what happens in this band. It could happen that you could have a flat response here, but it could also happen that you may have a bump upward bump. And it becomes flat after that or you could also have a scenario where there is a depending on how much overlap you are allowing there could be a bump. So in some sense you could visualize it like this if you had the blue response you had a flat constraint.

Now if it turned out that you had the green response comes when there is a wider transition and therefore there is a dip in the so the transition let us say is a little bit wider than so the transition is wider. Now when do we get a bump when the frequency response is possibly are sharper overlapping more. So therefore I will use the orange okay all right. So this is the let me change this too.

Let me draw it with orange. So this could be the case when you get a bump here. So orange corresponds to the case with orange. The blue line corresponds with the one with the blue which is almost flat over the entire response. And then you have the green case where there is a little bit of a bump. Yes, we do not ahead of time how to design the pass band and the stop band, but this is what we will do via the optimization method.

So here is the optimization step or the step of that we have to do the competition. So notice that we have to apply the following condition. We want to have a very good flatness constraint over the entire frequency band. And that will be to a large extent satisfied if you have a good stop band condition because if there are large ripples in the stop band you will notice that the flatness constraint will also get affected.

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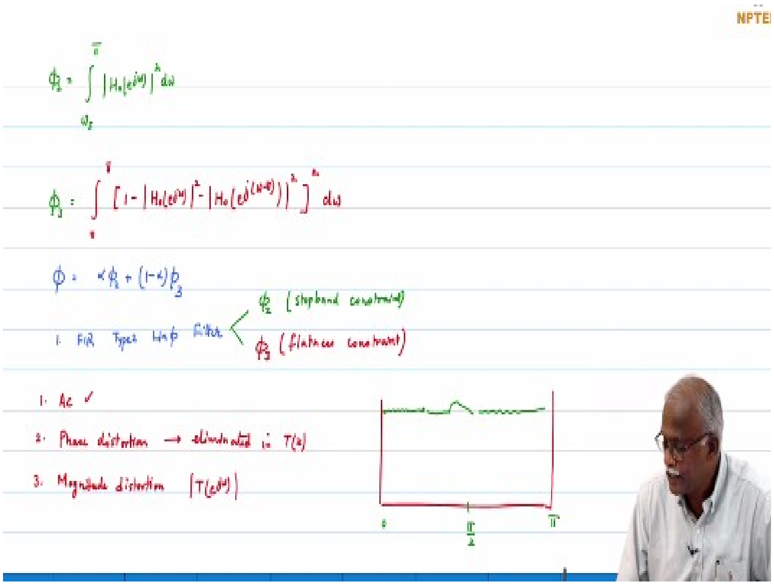


So therefore we now have 2 of the constraints. Phi 2 is given by 0 from omega s to pi the stop band energy omega s to pi mod of H0 e of j omega magnitude squared d omega and phi 3 is the flatness constraint that we have discussed in the last slide. And let me just copy that and capture it in the current slide for completeness okay. So here it is the okay so we have these 2 constraints. Now the question is do we still need the pass band constraints.

It turns out that the pass band constraint is not needed anymore for the following reason because if you visualize the ripple here. The ripple is of this form this ripple is nothing, but the stop band ripple of the original filter. Notice the stop band when you shifted it by pi now this happens overlap with the pass band. So as long as we have optimize the stop band of the filter H0 the shifted version will ensure that this is small in this region and therefore the pass band has got very little ripple is as flat as possible.

So the overall weighting function that we now need to employ is does not require the pass band, but however needs the stop band and it requires the flatness constraint. So alpha times phi 2+ 1- alpha times phi 3 or you could write it the other if you want does not make different. So basically it is a weighted combination of these 2 and this is what will help us satisfy the condition that we are looking to have. So therefore what are we doing? We have designed FIR Type 2 linear phase filter with both the stop band constraint phi 2. So this would be the stop band constraint and we also have the flatness constraint. The flatness constraint comes from phi 3 this is the flatness constraint combination of these have been used to get the desired filter flatness constraint okay.

So when we use this we have Aliasing cancellation guaranteed. We then have the phase distortion is completely removed because this is a linear phase filter phase distortion eliminated, eliminated in T of z.

Now what is the only thing left is magnitude distortion. Magnitude distortion is the magnitude response of Te of j omega and how much it deviates from a constant that more or less comes from phi 3 and this is what the response will look like. You have a very flat response in the region of the pass band then may be a small bump where the transition band occurs. So if this is pi/2 this is where the transition band will occur this is pi this is 0. So around here maybe a small bump and it remains flat for the rest of it.

So basically we have something that is approximately flat it is not exactly flat, but it becomes approximately flat. So this is a very powerful result which we are able to leverage for our designs.

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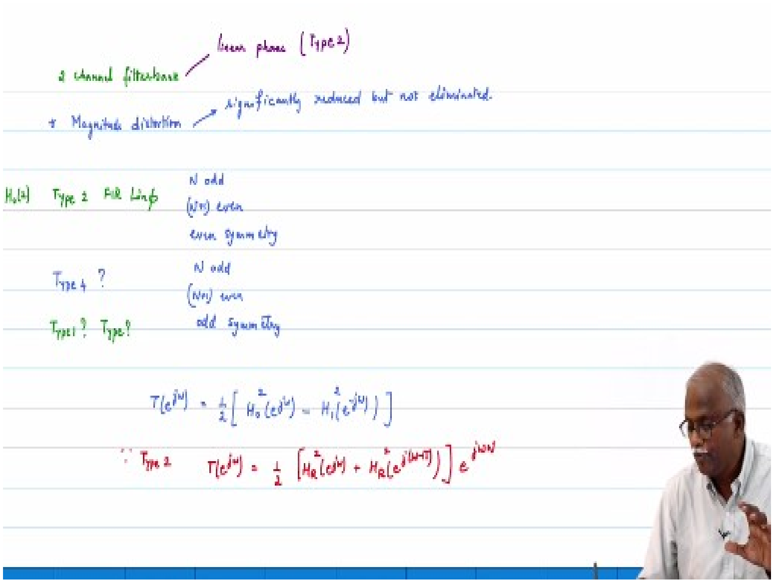


So the 2 channel filter banks using the linear phase filter was a very popular method filter bank using the linear phase Type 2. This was very, very popular because it gave us a way to design filters that did not have Alias thing, did not have phase distortion and had very minimal magnitude distortion which we could then optimize with respect to the condition that we have okay.

So now we can summarize the total results that we have in the following manner. The 2 channel filter bank using the linear phase Type 2 filters we are able to eliminate magnitude distortion sorry eliminate phase distortion and Aliasing cancellation and we just have residual magnitude distortion as was shown in the previous graph. So we now have a method where we can design the 2 channel filter bank to a large extent satisfying the requirements.

But let us make the note that magnitude distortion has been significantly reduced is significantly reduced but not eliminated because we have to still satisfy the flatness constraint and the flatness constraint is only approximately satisfied it is not exactly. So significantly but not eliminated okay. So that is the situation with the 2 channel filter bank. So we are still interested in methods that can completely eliminate the magnitude distortion.

Now that is something that we will take up in the next lecture, but before that I just want you to think about the constraints that we had posed. We had posed the constraint that it must be a Type 2 on the constraint on H0 of z. It must be a Type 2 FIR linear phase okay. Now Type 2 we know has got odd order N is odd the numbers of taps is even and it has got even symmetry okay.

Now there is a Type 4 FIR linear phase which has got N odd which has got N+1 even and it has got odd symmetry. Now could Type 4 have been used why not Type 1, Type 2 okay. These are very, very crucial questions in our understanding of the design process. Notice our overall transfer function was of the form Te of j omega= 1/2 of H0 squared e of j omega- H1 squared e of –j omega okay.

So this overall expression very, very important for us to keep in mind because this we simplified and then obtained it because of Type 2 assumption because of Type 2 we got it of the form Te of j omega= 1/2 of HR e of j omega squared+ HR of e of j omega- pi whole squared. So we were able to reduce it to this form e of j omega N and then when you took the magnitude response you got the expression.

So this is very, very important because only then we can get the flatness constraint. You will find that if you chose other combinations we will not be able to satisfy the flatness constraint that is an important element that I wanted us to keep in mind okay. So the summary that we can then put forth is that we have been able to design a 2 channel filter bank where we have Type 2 FIR linear phase. And we have been able to capture the flatness constraint, we are able to eliminate the Aliasing also the phase distortion.

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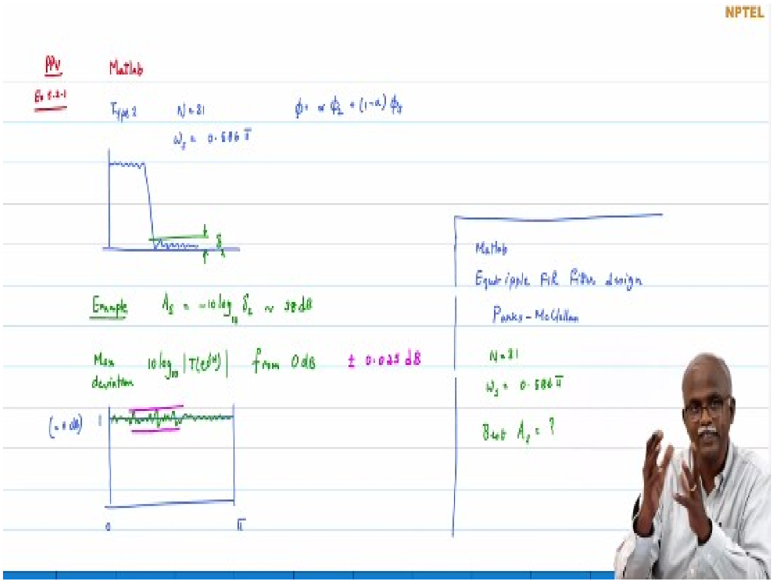

Now what I would like you to do is try the design example that is presented in the text book PP Vaidhyanathan in section 5.2.1 example 5.2.1. This is a design example where we have you can use MATLAB to design the filter that we are looking for. Remember it is a Type 2 linear phase filter. So we are choosing N=31 and stop band edge omega s as 0.586 pi and if you can then look at the design the filter through optimization and then show what is the stop band attenuation that you have for H0.

So basically you would have to design it with the overall constraint as alpha times phi 2+1-alpha times phi 3. This is the filter response we are looking at what is the maximum minimum stop band attenuation AS. In the particular example that has been given to us you can please try it out and verify that you are getting something which is very similar to this one.

So delta 2 is the maximum deviation so this is delta 2 AS=-10 times log base 10 of delta 2 and this is approximately of the order of 38 dB for the filter that has been designed for the 2 channel filter bank and the deviation from a constant. So if you look at 10 times log base 10 of magnitude Te of j omega we would like it to be close to 1 as close to 1 as possible. So the deviation the maximum deviation of this expression from 0 dB what we want as a flat purely flat constraint from 0 dB.

So in other words the same structure let me draw this, this is 0 this is pi. Ideally we would like it to be a constant at 1 which is=0 dB. Now when we plot the response of Te of j omega there will be some variations from here and probably the maximum variation occurs around pi/2 and then you have this. So basically we are measuring the maximum deviation from 0 dB either on the positive side or on the negative side.

And this comes out to be+ or – 0.025 dB. So we can see that from this particular design example that you can get filters with good stop band good stop and attenuation and also satisfying the flatness constraint to a very, very large degree. Now it is very insightful for us to also compare this particular design with the traditional linear phase FIR design. So if you are able to use MATLAB and are able to use the Equiripple filter design FIR filter design.

This is also known as the Parks–McClellan filters so a filter design. Please see what is the condition that we can achieve if you pose the constraint on this where the filter length=31 and omega s is of the order of the same as before 0.586 pi. And what is the best value of the stop band attenuation that you can get what is the best stop band attenuation. So you can just get a feel for if you did not have the flatness constraint what would the filter design look like if you had the flatness constraint, what will the filter design look like.

And therefore we are able to get the response. Now there is a very important observation that we can make and the observation can be summarized in the following manner. The observation is that the pass band and the stop band edge when we are designing the filter bank are not independent of each other.

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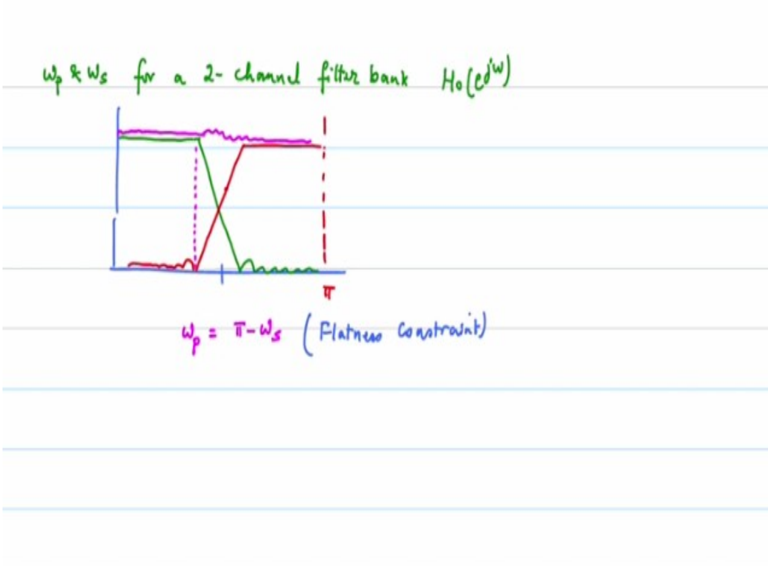


So here is the observation omega p and omega s for a 2 channel filter bank H0 e of j omega.

So H0 e of j omega is a filter being designed for this. Notice that there is a stop band edge that we have to worry about. So flatness constraint and then a stop band edge and then there will be some ripple. Now because of the constraint that we have the shifted version. The shifted version will be basically shifted by pi and this is pi and this will have the same stop band edge.

Now the portion where and the squares of these two must be a constant right the squares the sum of these 2 must be a constant must be as close to a constant as possible okay. Now because this is already in the stop band the pass band is guaranteed that is why we said we do not need an additional pass band constraint. So if the stop band is the frequency at which the frequency response becomes very small then by symmetry.

We observe that the pass band will also be related to the stop band or in other words omega p will be= pi- omega s basically the frequency at which you start where the stop band begins is related to where the pass band ends because of the flatness constraint. So this is because of the flatness constraint. So again this is just an observation it is good for us to know that how will the pass band and the stop band be related in the filter they will have symmetry with respect to the Quadrature frequency because of the flatness constraint.

Now one last aspect just like you I had asked you to analyze the answer the following questions. Why Type 2 why not any of the other types why not other things. So here is an example which I would like you to look at and also try to analyze so that we can then get a better understanding.

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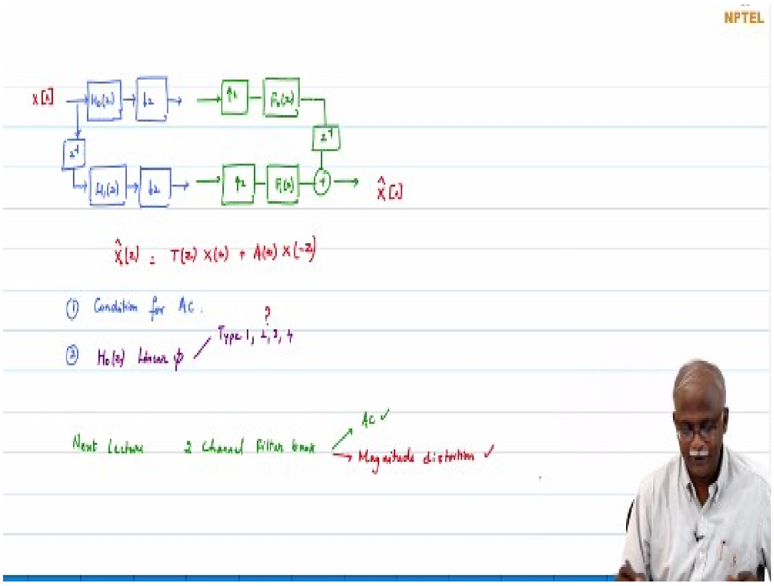


Look at the following structure. It is a 2 channel filter bank with a slight difference from the one that we have studied earlier H0 of z down sampled by a factor of 2 on this branch we have a delay z inverse followed by H1 of z followed by a down sampling by a factor of 2 okay and then of course we will have the up sampling and the synthesis filters. Let us draw the complete picture then picking it up from here we have up sampling by a factor of 2 up sampling by a factor of 2 look at the synthesis filter F0 of z up sampling by a factor of 2.

And synthesis filter F1 of z and the 2 signals then combined via delay added together producing the output okay. So this is output as X hat of n input as X of n. What I would like you to do is look at the overall transfer function write down X hat of z in terms of T of z times X of z+ A of z times X of –z just like we did before and then obtain the conditions for Aliasing cancellation.

What are the conditions that must be satisfied conditions for Aliasing cancellation? So this is the task number one. Now if we want to eliminate the phase distortion we will want to go for a linear phase. So H0 of z has to be linear phase. Now is it Type 1, Type 2, Type 3, Type 4 which of these types will work. Will Type 2 work for this case also? Type 1, 2, 3, 4 which one of these will work? And it is instructive for you to go through this example.

And then try to answer the question how is the linear phase property being utilized, how is the flatness constraint being achieved and thereby overall system which has got no Aliasing, no phase distortion, but only minimal amount of magnitude distortion which is obtained because of the design methodology that we have used. So we will stop here for today's lecture.

And in tomorrow lecture or in the next lecture we will then begin or we will start to look at ways of eliminating the magnitude distortion. So next lecture so we would look at the 2 channel filter bank. We of course want to Aliasing cancellation to be done, we want magnitude distortion elimination. Just like we have eliminated the phase distortion in today's lecture can we look at the magnitude distortion elimination?

So that will be the second what will be the next lecture that we will be picking up in tomorrow lecture. Thank you very much.