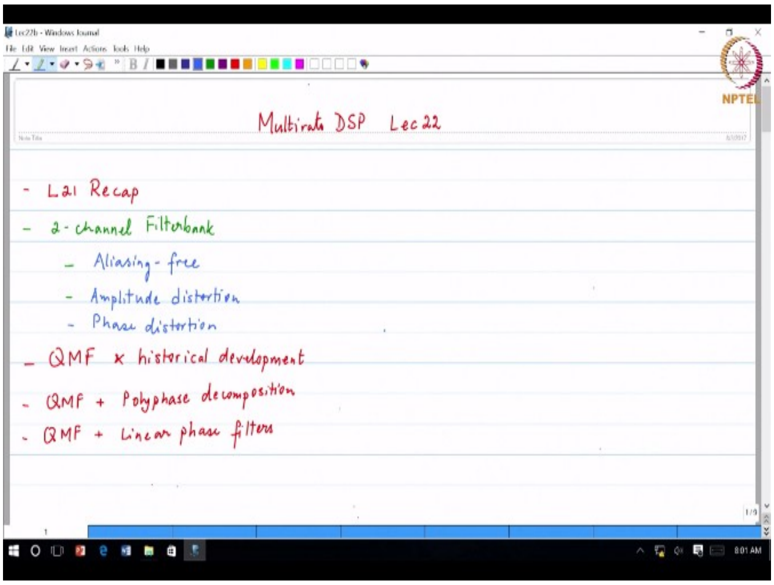
Multirate Digital Signal Processing
Prof. David Koilpillai
Department of Electrical Engineering
Indian Institute of Technology – Madras

Lecture - 22
Introduction to Quadrature Mirror Filters (QMF)

(Refer Slide Time: 00:16)

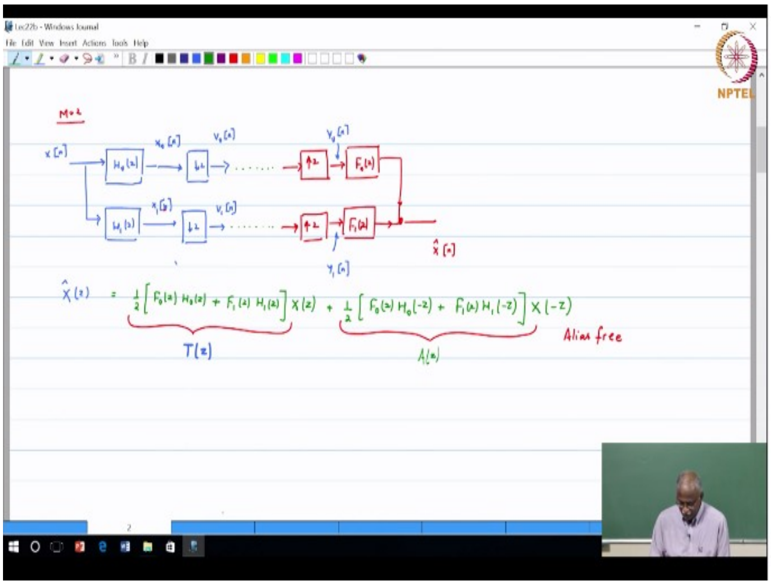


Good morning. Let us begin lecture 22. We pick up from the last lecture where we talked about the 2-channel filter bank, alias free filter bank and today we will continue on the discussion. I would like to cover some aspects of how do we eliminate amplitude distortion, how will we eliminate phase distortion. We will also look at how the historical development happened which came first, how did people actually solve this problem of removing all 3 forms of distortion.

As I mentioned in the last class, the tools that we have developed in the material covered so far one of them is polyphase decomposition. We just want to know whether polyphase decomposition is going to give us any insights into the tools or the solution that would eventually be able to satisfy all of these 3, removal of all 3 types of distortion. We also know that we have linear phase filters and the use of linear phase filters removes phase distortion.

So again we will explore both of those in today's lecture and then see how we can leverage these results for our purpose.

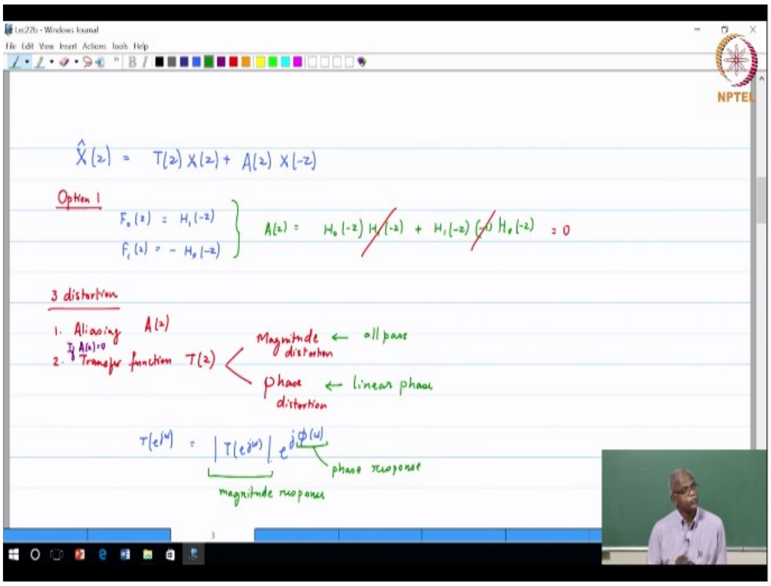
(Refer Slide Time: 01:18)



So the framework is the 2-channel maximally decimated filter bank. Again, the term maximally decimated means that your total sample rate, number of samples per second after the splitting into the sub signals, subchannel signals should be the same as the input rate sampling rate, sample rate. So you can see that both of these are down sampling by a factor of two, so therefore you would have the same input and output sampling rate.

Now we showed for this case that the output reconstructed signal or the recombined signal to be equal to a transfer function which we can write as T of z, T of z is the signal transfer function and the next one is the aliasing transfer function A of z. Now if A of z is set to 0 then we get alias free condition.

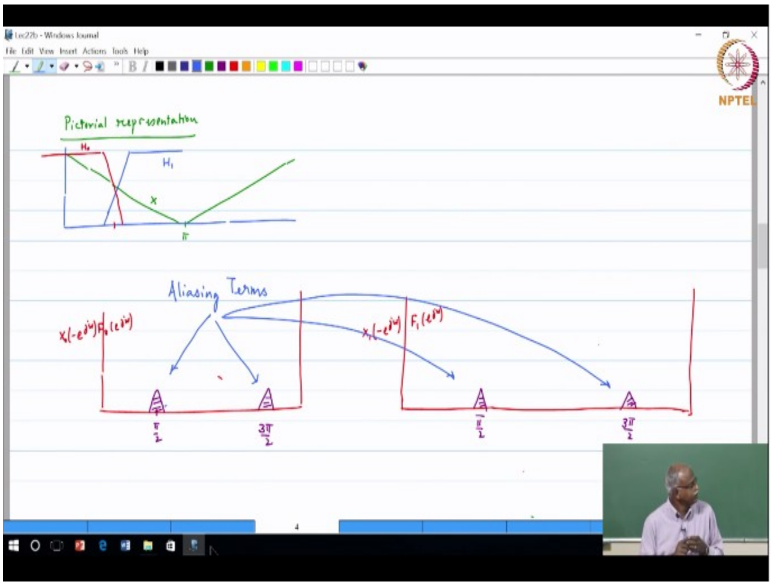
(Refer Slide Time: 02:15)



And that is something that we had already looked at because there was one choice of transfer for the choice of filters that showed that the aliasing can be canceled and again this is something that you can try out. Now if we have succeeded in setting A of z=0, then we can now start focusing on the transfer function. So if A of z is=0 then we focus on the transfer function T of z.

And say that any other transfer function has got a magnitude response, has got a phase response and the magnitude distortion if you want to set eliminated, the magnitude response must be a constant which basically says it must be an all pass filter. Phase distortion says that this transfer function must have linear phase and therefore in that condition then we would be able to eliminate the phase distortion as well.

(Refer Slide Time: 03:04)

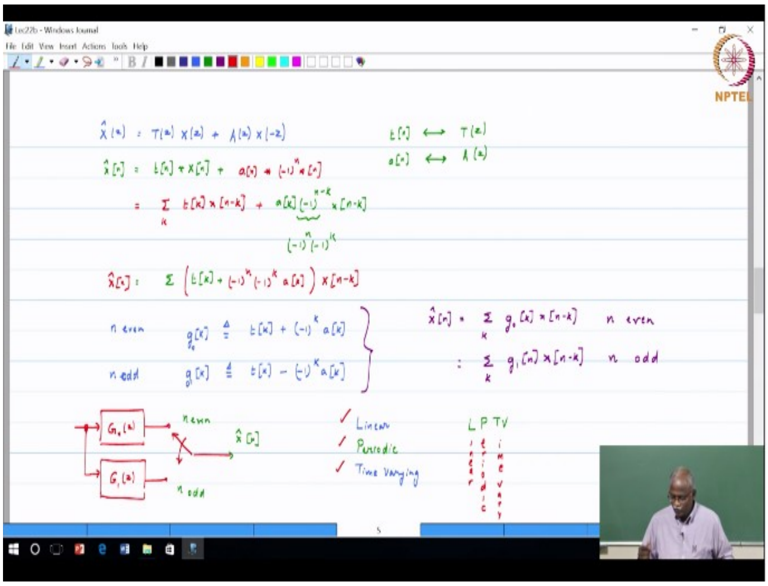


We did spend a fair amount of time going through the drawing, how if this the green line was the input signal, H0 and H1 were the 2 filters that were available to us. Then, we went through and showed that the aliasing terms which come from the two terms that are in the aliasing transfer function basically give you some nonzero value around pi/2 and around 3pi/2 and similarly the other transfer function other component also has got the same.

And looking at the spectral occupancy if these have more or less the same magnitude response and opposite phase they would cancel each other and that is what is the underlying principle okay. So some elements that I just wanted to emphasize on. So we can get rid of the aliasing that is point number 1 and the choice of the filters that will eliminate aliasing one of the options is here.

This may or may not be the only option, so this is a sufficient condition for the elimination of aliasing and when we eliminate aliasing we get a transfer function LTI system and we can take care of the magnitude response and phase response. Now I would like to spend a few minutes on this transfer function so I think it is a very important element in our development of the theory of the filter banks.

(Refer Slide Time: 04:34)



So X hat of z is=T of z times X of z+A of z times X of –z right, that is a input-output relationship for the two-channel maximally decimated filter bank okay. Now supposing we had the following that T of z corresponded to an impulse response t of n basically it let us say that the time domain representation of the transfer function is characterized by an impulse response and in the same way a of n was the inverse z-transform of the transfer function A of z, supposing these existed okay.

And now I would like you to write down the time domain equation for this. x hat of n will be T of z X of z that means it will be a convolution of t of n with x of n. This part is straightforward, t of n convolved with x of n okay my X okay hopefully that does not look like a star also okay. This is x of n okay second term. Please tell me what this is? a of n convolved with -1 raised to the power n x of n correct okay good very good.

So now please help me write the actual convolution sums. This is summation over K going from -infinity to infinity, t of K x of n-K+it will be a of K-1 power n-K correct because I have x of n-K and this will be x of n-K okay. So I am going to split this in the following form -1 power n -1 power K -1 power -K is same as -1 power K, so that is the thing. So this I am going to now do a combining of terms.

It is going to be t of K+ -1 power n okay -1 power K a of K this whole thing into x of n-K. Probably wondering where this is heading towards, I think insight is going to come in just the next line. So let me take the case of n even. In that case, I can write down a new sequence g of n, g0 of n which is defined as since n is even it will be t of K -1 raised to the power n is +1+ -1 raised to the power K a of K.

Similarly, for n odd we will get another sequence g1 of n is summation t of K- -1 power K a of K okay. So if n even n odd we get different sequences. So we can now combine these results into a very insightful result which says x hat of n can be written to be summation K from -infinity to infinity, g0 of K x of n-K if n is even. It is equal to summation over K g1 of n x of n-K if n is odd.

Do you agree with that? Basically, you get a different sequence when you are dealing with time n is even and the time when n is odd, sorry, sorry this is g0 of K I am sorry absolutely correct yeah this is g0 of K okay. So in other words I am glad that you brought it up because for n is even we have to look at the impulse response being g0 of n and n odd we have to do another impulse response.

And so the representation of this comes out to be the following. I have two filters G0 of z which corresponds to the transfer function of g0 of n. The second filter g1 which corresponds to the g1 of n impulse response, G1 of z both of them producing the output and it is as if the output sequence kind of goes back and forth you know if it is and it chooses the upper branch if n is even, chooses the lower branch if n is odd.

And this is x hat of n okay. So I have not done anything about alias cancellation. I have just let say let aliasing be there and I have analyzed what is my input output relationship okay. Now here is the key summary statement. Is my overall system linear, input and output a linear relationship? Yes, whether its time is even or odd linearity will satisfy okay. So linearity you can put a tick mark okay.

Okay second one is that is it time invariant? No, it is time varying. So that also you can tick off, time varying okay. There is one more element which is important in the time varying aspect. See time varying just says that every instant of time it is a different impulse response but this one does not have a different impulse response for every instant of time. It is actually a periodic variation.

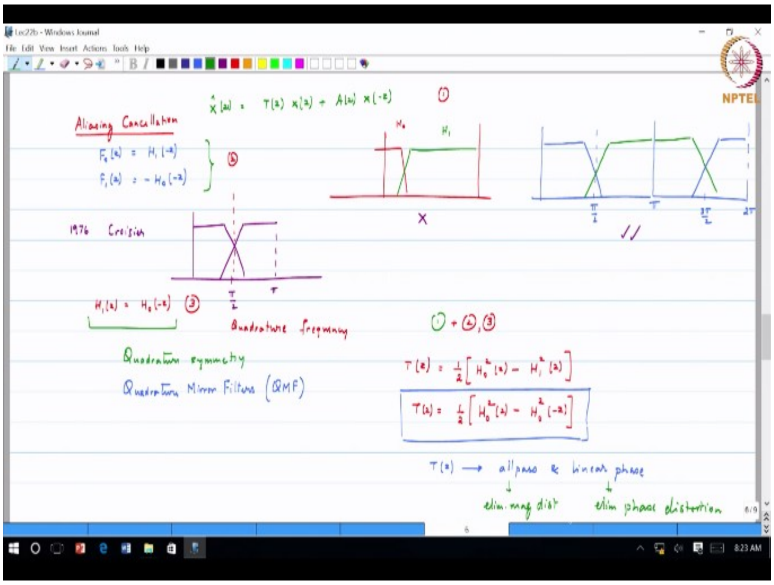
So this has also a third element which is that it is the impulse response is periodic okay. So that also is part of the description. So this input output 2-channel system where we talked about a maximally decimated system has the following. It is described in the following way. It is linear; it is periodic and time varying. See we were used to LTI, linear time invariant but now this is 3 components, this is linear, periodic and time varying okay.

So that is that is what it is and this is insightful for us because even if you go to an M channel system, you can visualize it as yes it is linear, it is going to be time varying but it is not arbitrarily time varying, it has got a periodic structure to it which we once you know that there is some structure to it, we can always exploit. That is the whole premise of signal processing.

That once you detect a certain structure in your signal, you can then take advantage of that. So a lot of the theory behind this says okay now we move from LTI systems to a broader class of systems which are linear time varying systems but they are not arbitrarily varying but they are periodically varying and that is a very interesting and useful class of systems. Again as opportunity comes you will have an opportunity to work with that okay.

So now we go back to our original plan of canceling aliasing, removing all of the things that are unwanted. So that is our focus for the rest of the lecture.

(Refer Slide Time: 13:38)



Now aliasing cancellation let us go back and summarize whatever we know about aliasing cancellation. Aliasing cancellation says that we have to choose the filters in the following manner. Choice of filters F0 of z=H1 of –z, F1 of z=-H0 of -z okay. So these were the 2 conditions that were required to be satisfied. This is an important condition and if you want to just sort of keep reference, the overall equation is X0 of z is=T of z times X of z+A of z times X of –z, so let me call this as equation 1.

This as equation 2, now there is one more element that sort of became apparent to us from our discussion when we were doing these graphical representation that was the following. In order for us to have minimum amount of aliasing that means the H0 of z, H0 of z could have been like this but in that case if this is H0 then H1 would have to be much wider because if you remember we said that you cannot have any portion of the spectrum that is not covered by either of the filters.

Because you must preserve the information, so this would have to become H1. Then, the lower branch will have a lot of aliasing. So which means that you know that is undesirable. So obviously when you say that okay minimize the aliasing but you must also prevent that there is no you know there must be full preservation of the signal then you say that okay H0 must have a pass band edge which is close to pi/2.

Must be close to pi/2, so if that is pi/2 because then you would have minimum amount of aliasing. Then, the other filter would also look like this because that is those are the relationships between these 2 filters okay. So what we observe through intuition is that this is pi okay and if you want to draw the full form this would be all the way to 3pi/2 and then the other image of H0 would also show up okay.

So this is 3pi/2 and here we have 2pi okay. So this is the so obviously this is not a desirable form, this would be the desirable way you would like to have the filters cross over at a convenient point. Now this was obviously observed very early in our in the study of filter banks. So the first reference comes in 1976, I think I may have mentioned the name already, it was by Crosier.

And he said that yes the filters must have must crossover around pi/2 and he went on to say that there are certain advantages, if these are not arbitrarily chosen but since they have to have a certain symmetry around them. He said why not introduce them to be actually shifted versions of each other okay. So if this is pi/2, this is pi okay. I am drawing only from 0 to pi. The observation that he suggested was the following.

The observation was why not choose H1 of z to be=H0 of –z. Now does that in any way limit us in terms of the generality of the solution or constraints? Now why would you want to do that? This one means that you have to design only one filter because in the other case you have to design H0 and H1. Now you would design, now let us just sort of go back and plug in and see what does this?

So by the way pi/2 has got a name, it is called the quadrature frequency. Quadrature frequency because it is 1/4 of the sampling frequency, quadrature frequency and now if you actually draw think of a mirror at the quadrature frequency, H0 and H1 are actually mirror images of each other because that is what happens when you design one to be the other and they are satisfying this condition.

So this quadrature symmetry, so this basically along with the other conditions enforce a quadrature symmetry and this class of filters when you have 2 filters which satisfy quadrature symmetry came to be called as quadrature mirror filters okay. So quadrature mirror filters and again the historical development was from an angle of let me get rid of aliasing, let me simplify the filter design and so the symmetry came about.

And so this actually came to be used very widely quadrature mirror filters and if you look at any of the filter bank literature, you will see this acronym QMF and you know where it exactly comes from. Quadrature mirror filters started from 2 channels where you have a certain symmetry around the quadrature frequency and of course the aliasing cancellation constraint as well.

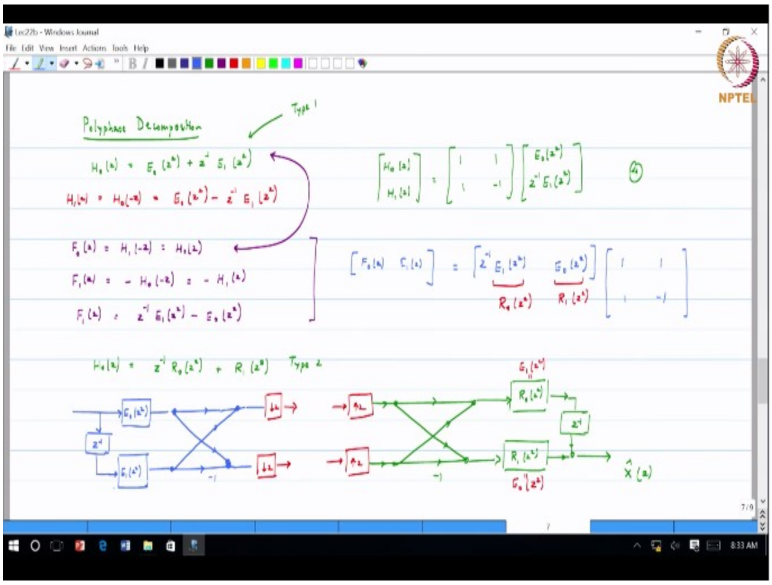
So now if you combine equation 1 okay and you have this is now equation 3. So 1+2, 3 add I want to get rid of aliasing with this constraint. Please substitute, you can verify the T of z, the transfer function will come out to be 1/2 of H0 squared of z-H1 squared of z okay and H1 squared is H1 of z is H0 of - z so this can also be written as 1/2 of H0 squared of Z - H0 squared of –Z. So this is an observation that came from introducing the quadrature mirror filters.

So if I design it to be a quadrature mirror filter to begin with and choose the synthesis filters according to equation 2. Then, I get a form that tells me that the overall transfer function now depends only on H0 because H1 and H0 are related to each other, F0 and F1 are related to H0 and H1 and therefore ultimately you have written the entire expression in terms of H0. Now the goal is that I have to design H0.

But ultimately what do I need to, what would I like to satisfy? I would like to satisfy T of z to be=T e of z. I would like it to be all pass, all pass and linear phase or something close to that okay. All pass and linear phase is a contradiction; you cannot satisfy both of those in the same except for trivial cases. If you take a delay, it would satisfy right. So but all pass so why would I want all pass because I want to get rid of magnitude distortion.

So eliminate magnitude distortion and the linear phase is to eliminate phase distortion okay. So that is what and again it has given us an expression but really has not told us how to solve this problem okay. So this is where we are, so we said okay. From now on, we will work only with QMF situation because that has simplified our problem to one level but how do I get the solutions that are useful for us in terms of the design of the filters.

(Refer Slide Time: 22:08)



So we go back to the tools that we have already developed polyphase decomposition and ask if this was going to give us any insights in terms of the design of the 2-channel filter bank and probably the course of this discussion you also get a feel for that this is a somewhat of a involved problem. It is not a trivial problem, it is not something that we can solve straight away, we have to basically plug along.

And you will see that there is a fair amount of effort and attempt to actually solve this particular problem. So H0 of z, H0 of z I am going to downsample by 2, so 2-channel polyphase decomposition, E0 of z squared+z inverse E1 of z squared okay. Now the QMF condition says H1 of z will be=H0 of –z, so that gives us a fairly interesting polyphase decomposition.

It is not a new polyphase decomposition, it is actually the same as before with only a minus sign, -z inverse E1 of z squared and that is because of the symmetry and the shift that has happened okay. Now combining these 2 equations, we can write down the analysis filter bank H1 of z, H0 of z in the following way, 1 1 1 -1 and the polyphase components E0 of z squared z inverse E1 of z squared okay.

So that is equation 4 if you wish to write it down okay. Now I want you to write down for me the synthesis filters. So very quickly let us just pencil them down and then write it in matrix form. F0 of z from aliasing cancellation of choice H1 of –z. So that is straightforward. H1 of -z is actually equal to H0 of z. So H0 and F0 actually are one in the same filter because of the polyphase decomposition and the QMF constraint.

They actually map back to the original one, F1 of z is –H0 of –z. This is same as –H1 of z, H1 of z we already have written down. So this will be equal to z inverse E1 of z squared- E0 of z square okay. That is F1 okay. Please follow along, if I have made any sign mistakes or something just flag it down immediately okay. So combining these 2 equations, I would now like to write down the synthesis filters.

Remember I told you analysis filters always written as a column vector, synthesis filters always written as a row vector. The reason is you would combine it with a column vector to produce a single combined signal okay. So this is F1 okay F0 of z F1 of z, so it is written as a column vector. Let us write down the expressions. So we would now write it down in the following way F0 and F1 if you look at it.

It has got 2 polyphase components. I am going to write it in the following manner, z inverse E1 of z squared and E0 of z squared. I will explain in a minute why I have chosen this. If you write it in this form notice that the matrix that does the combining of the polyphase components comes out to be 1 1 1 -1 okay and that is an orthogonal matrix. So I would like to preserve that because that was there in analysis filters.

I want to retain it in the synthesis filters as well. Now the reason for doing this also highlights one point that we had mentioned earlier. We said that remember that this is type 1 polyphase, this is type 1 polyphase decomposition type 1. What was the type 2 polyphase decomposition? What would it have been? It would have been z inverse R0 of z squared+R1 of z squared right.

The delays are basically the delays are in a different order as in the type 1. So this is type 2 polyphase decomposition if you go back and look at your notations. Again, we do not use it too often, so we did not specify too much which basically means that this is being labeled as R0 of z squared and this has been labeled, did I get that correct? Yes, and this has been labeled as R1 of z squared okay.

So basically if I do that correctly yes yeah, so I get the correct representation okay. So now the key element for us is to use this interpretation. So what I would like to do is redraw the figure for you. So the 2-channel polyphase decomposition can be to 2-channel maximally decimated filter bank with polyphase decomposition can be drawn in the following manner, E0 of z squared E0 of z squared z inverse E1 of z squared.

Now comes the 2 x 2 DFT butterfly okay basically the crisscross operation. The lower branch comes with a -1, these all the arrows moving to the right. The lower branch alone comes to the multiplication of -1 okay outputs and here you would have the downsampling by a factor of 2 okay. Now on the other side, it would be upsampling by a factor of 2 followed by the synthesis filters.

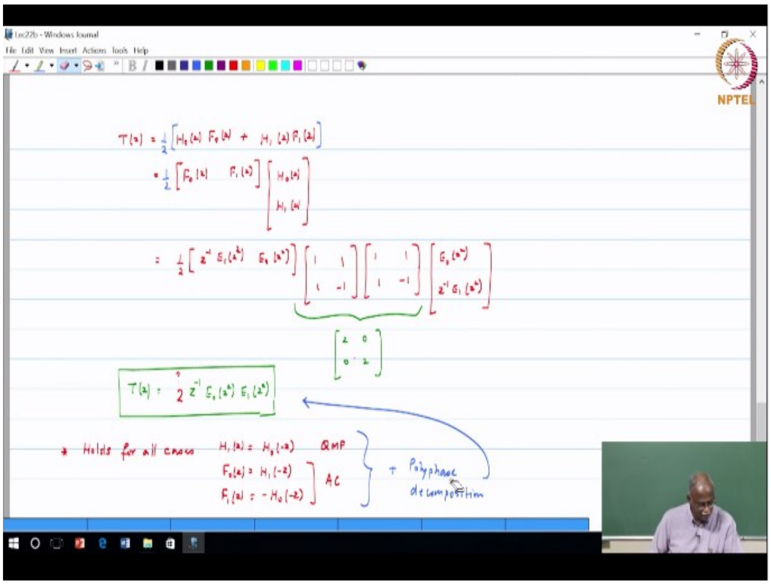
The synthesis filters if you look at the implementation also have a crisscross butterfly. The same structure, all lines moving to the right. The lower branch having a multiplication factor of -1, upper branch is R0 of z squared. Keep in mind this is same as E1 of z squared. The lower branch is R1 of z squared and then combined with a delay element z inverse added together output, all the arrows moving to the right and this is X hat of z.

And this is the same as E0 of z squared okay, these are equal okay. So this is the overall structure that we have obtained okay. Now what you could do is move the downsamplers and the upsamplers and get a certain amount of insight. What you will find is that these 2 matrices will cancel each other because they are orthogonal matrices and of course it will give you a diagonal element or diagonal, it basically becomes our scale factor of 2.

Because when you multiply these 2, you get only diagonal elements with elements equal to 2. So you will get a multiplication factor of 2, upper branch will be E0 of z, on this side it will be E1 of z, this one will be E1 of z, this one will be E0 of z okay. So you look at both of them and you say that well what did I get at the output is something very interesting. Upper branch and the lower branch give me the same thing.

And of course there is the demultiplexing and multiplexing. Now well that still puts you in a multirate environment and it is okay wait wait that is okay that is an insight but let me just go back.

(Refer Slide Time: 32:03)



And say that under this condition if we went back to our original equation, the original equation if you recall said that T of z what was it equal to H0 of z F0 of z+H1 of z F1 of z. We are guaranteed that aliasing is canceled here right. So why worry with this downsampling or upsampling. So I want to sort of work with the non-multirate framework where they say okay T of z is this.

So this can be written as F0 of z F1 of z the synthesis filters as a row vector with H0 of z H1 of z as the column vector and if you now write down the expressions what you will get is 1/2 z inverse E1 of z squared E0 of z squared a 2 x 2 matrix 1 1 1 -1 followed by another 2 x 2 matrix 1 1 1 -1 and this one would be E0 of z squared z inverse E1 of z squared okay. Now these two is what I mentioned, they will cancel each other.

You will get 2 0 0 2, it is like an identity matrix but with a scale factor of 2. So that scale factor will cancel off the scale factor of 1/2 that is sitting outside. So what you will eventually get is the following. You will get z inverse E0 of z squared E1 of z squared now which was the same insight that you got from the multirate structure as well.

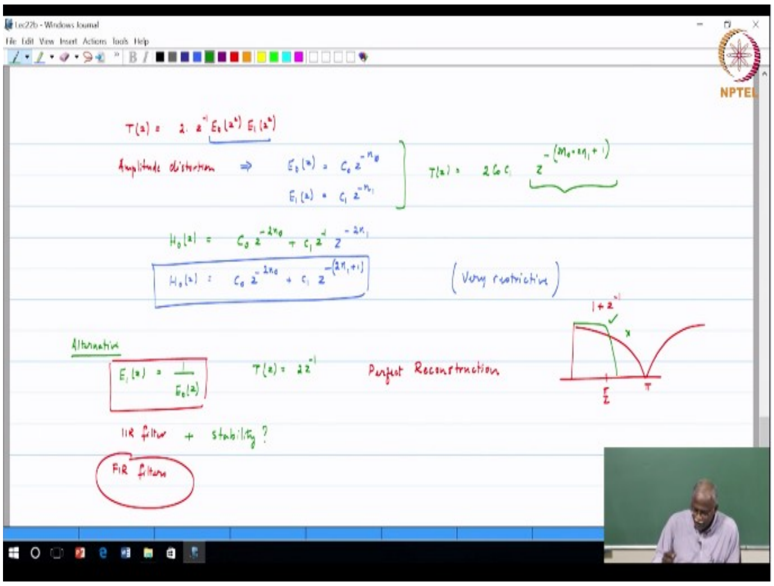
Basically, you know once you move the multirate blocks, then upsamplers and downsamplers, the two matrices will cancel and then upper transfer function, lower transfer function come out to be the same and that is exactly what we have got and this is equal to T of z. So T of z comes out to be the product of the 2 polyphase components and given once you know H0 you can get E0 and E1 okay.

Now has this given us any further insight? First and foremost, does this hold for all cases? Yes. This result holds for all cases which satisfy the following. Basically, they must satisfy the QMF condition H1 of z=H0 of -z that is the QMF condition and then the aliasing cancellation is F0 of z is= H1 of –z F1 of z=-H0 of –z. This is the AC, AC stands for aliasing cancellation.

Now if I satisfy these two and then combine it with polyphase decomposition, then aliasing is canceled and I get my transfer function T of z given by this expression okay. T of z is 1/2 H0 okay, so F0 I still have a problem know, I still have so there is a 1/2 here which is the 1/2 that has shown up here. Let me just put a question mark here, is this 2 there or not? It is a scale factor so but nevertheless we need to be careful and consistent with our expressions.

So yeah so I will double check it from the beginning but for now so this is the transfer function with the QMF conditions satisfied, the aliasing cancellation condition satisfied and the polyphase decomposition being applied. Most important in any result is the interpretation and the insight that you get from there.

(Refer Slide Time: 36:23)



So the transfer function being T of z, T of z being equal to let me write it with the scale factor for now, 2 times z inverse E0 of z squared E1 of z squared. Now I want to eliminate magnitude distortion, phase distortion and see if how I can satisfy those conditions. So in order for the phase distortion to be eliminated or first of all amplitude distortion to be eliminated.

If I want to eliminate amplitude distortion, amplitude distortion can be eliminated only under one condition, E0 and E1 both are transfer functions, both of them should not have any amplitude distortion, so which means that these will have to be delays. E0 of z, E1 of z must be delays. I will call this as some C0 times z power -n1 E1 of z C1 z power – this is n0 and this is n1 okay, some two different delays.

That is only a condition under which they will have, so now you may say well can they cancel each other? Yes, there are certain tricky constraints but remember E0 and E1 do not have a relationship, you cannot really constrain them. They are dependent on the filter themselves. So which means that in a way E0 and E1 are kind of independent of each other. If you start imposing additional constraints, yes it is possible but in the general case this is not satisfied.

So because we made the claim that this is true in the general case, so in the general case if you want to get aliasing cancellation the only option that will be possible is this combination. So if you satisfy this then T of z becomes some scale factor, basically it will be 2 times C0 C1 and it will be equal to z inverse or z power –n0+n1+1 no 2n0 2n1 because it is squared+2n1+1 okay. So this is the overall delay.

Now that it is very good from removing the magnitude distortion but if I now go back and look at my H0 of z, H0 of z is C0 z power -2n0+C1 z inverse z power -2n0 or in other words it is C0 z power -2n0+C1 z power -2n1 this should be n1, 2n1+1 okay, it is a combination of 2 delays, H0 of z combination of 2 delays, you can choose those two delays to be at any location.

So basically an impulse response that looks like this you know basically some combination of delays and you cannot get very good filters with just a 2 tap of filter, you cannot get very good attenuation. So yes so this is going to be a highly restrictive case. This is not going to really satisfy. So yes this is okay but this is a very restrictive case okay, very restrictive. It is a combination of 2 delays.

Therefore, I cannot get very good filters to begin with, of course they will satisfy the magnitude constraint being canceled completely okay very restrictive. So this is the final conclusion from our discussion. Of course, if you remember we said that how about if you relate them to each other so that they can cancel and that was an observation that was made yes so we can look at an alternative instead of choosing them to be delays I can of course choose E1 of z to be=1/E0 of z.

Am I right? E1 of z is=1/E0 or either way because if you do this it turns out the following T of z will be=2 times z inverse which means you actually get perfect reconstruction. You have eliminated aliasing, with one stroke we have eliminated both amplitude and phase distortion because you were able to write the transfer function as a product of two terms and if you force one of them to be the reciprocal inverse of the other, the reciprocal of the other then you will actually cancel them and then you will get a perfect reconstruction.

So if you remember see if I have let us say just 1+z inverse that is a filter that so what will be the response of this? This is what it will look like right. Basically, this is the filter response. Now if I decimate this by a factor of 2, this is by the way this is pi, this is pi/2, I will get huge amount of aliasing correct. I will get a very large amount of aliasing. Now if I do not do any processing, I will be able to be combined and get it back.

But the minute I do any quantization or something then the reconstruction will be a mess because the lot of aliasing has scripted. Now that is where the problem lies, I should come up with a system where even if I have done some processing I should be able to reconstruct with a fair amount of fidelity right. So that is why I do not want to introduce a lot of aliasing in the signals to begin with.

Now if I want to impose that then this is not a good filter, I would rather have something which had like this. This would be a preferred one because that will reduce the amount of aliasing and then when I do the reconstruction process is done, does a reasonably good job with that right. So that is why we are saying the combination of 2 delays is not a is a very restrictive solution from the point of view of designing a filter bank that will handle a general class of filters, general class of applications okay.

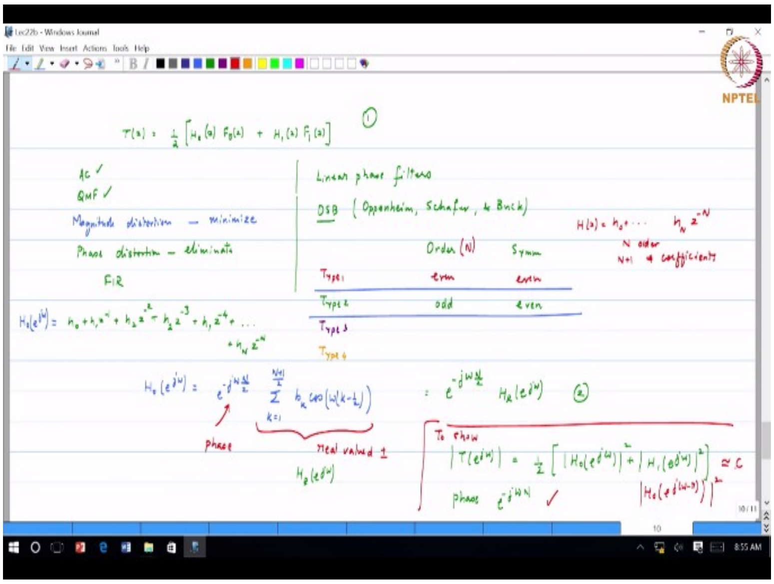
Now let us come back to this option. Any difficulty with this? It becomes an IIR filter right because the minute you say that this is one of the polyphase components that means even if E0 of z was a FIR transfer function, E1 of z becomes infinitely long because 1 by this one, so which means that this filters become IIR. So we were actually interested in working with FIR filters okay.

So this is an indication that maybe if you relax that constraint that you want to have only FIR filters maybe there is some interesting solutions that are possible okay. We make note of that but this has another drawback, any other drawback, one is IIR. Any other drawback? Stability because I am not guaranteed stability. I may have ended up with something that is unstable. So it is IIR+stability and I will put a question mark.

There is no guarantee of stability. So yes I cannot do brute force way though it gives me an indication that maybe your constraint is, you already started imposing too many constraints, you are asking it to be FIR, you want aliasing cancellation. Aliasing cancellation comes for free because I know how to choose the filters but you are asking for quadrature mirror symmetry and you are satisfying this condition okay.

So now as I mentioned in the last class, we will go back and look at the transfer function.

(Refer Slide Time: 44:29)



Look at the transfer function, it is T of z is=1/2 of H0 of z F1 of z+H1 of z sorry this should be F0 F1 of z, this is my transfer function. Now the thought the process that goes about is that okay, now magnitude distortion where we try to eliminate, we went so far, it seems to be a little bit complicated, we split it into polyphase components. We say that okay maybe the indication to go towards IIR.

So now let us try the other side, let us say that aliasing cancellation is done. I choose the aliasing cancellation. Quadrature mirror filters condition satisfied. Magnitude distortion I am going to allow okay. Magnitude distortion allow, magnitude distortion minimize. I am not trying to eliminate. So I am not going to ask you to make it into an all pass function, minimize and can I phase distortion eliminate.

Can I do this with FIR okay? So this is the we are basically approaching this problem from all possible and it is a very good model for how you would do a research problem. You get the transfer function and then you say okay I changed this parameter, I changed that, I combined, look at it and but you may hit dead ends and was kind of hit a dead end with trying to eliminate magnitude distortion.

So for this we say that okay, we will go to the class of linear phase filters and if you would recall from Oppenheim and Schaffer O and S I call right it is OSB because there is a previous version which is Oppenheim Shaffer and Buck. So if you have the later version, it is Oppenheim and Shaffer and Buck okay. In that you can find that there is a classification of filters that is given to us.

The classification of filters is in the classification is in terms of the order and the symmetry okay and let me just write it down type 1 is even order, even order and even symmetry okay and order is defined as if I have H of z=h0+last term is h of N, h subscript n,. Z power –N, N is the order, N plus 1 is the number of coefficients okay. The number of coefficients is one more than the filter order.

And so the filter order that we are talking about is N and if that N is even then it is called type 1 and then you have type 2, you have type 3 and type 4. Type 3 and then type 4 please go through and fill in all of the categories. What is of interest to us is the type 2 case. I will focus only on that. So type 2 is odd order and it is even symmetry okay. An example of type 2 would be H0+H1 z inverse+H2 z-2+H2 z-3+H1 z-4+H0 z-5.

Notice filter order is 5, the number of coefficients is 6 but because of symmetry you need to worry only about 3 of them, half the numbers are same. So this is the type of filter that we are interested in. Let me just write the expression for it and then leave it to you as an exercise for you to. H0 e of j omega if this is H0 e of j omega and let me just make it + dot dot dot dot dot dot+h of N times z-N that is a general form.

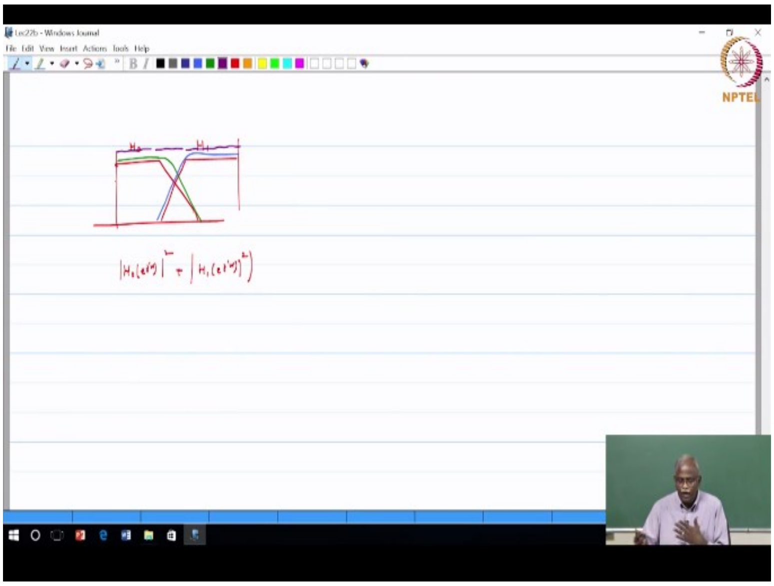
So H0 e of j omega in this case can be written as e power -j omega N/2 summation K=1. This is not as important so but if you have a doubt please look it up in Oppenheim and Shaffer. This would be bK cosine omega K-1/2 okay. What is important for us is that this is a real valued function. Real valued function, it may be positive or negative okay that is because this is sum of cosine terms that are getting added, it does not matter.

So all the phase term is captured here and there may be a +pi if the real valued function turns out to be. So we will call this as some real valued function HR e of j omega okay. So very important, we can now write it as e power -j omega N/2 HR e of j omega okay. Now please take this and substitute it into one with all of the symmetries that we have written down and please verify the following result.

That magnitude T e of j omega can be written as 1/2 of magnitude H0 e of j omega whole squared+H1 e of j omega whole squared okay that is magnitude response and the phase response is e power -j omega N okay, so this is you have to show this thing. Please show this result to show using 1 and 2. So notice that the phase distortion has been removed and now we have a condition on the magnitude distortion.

And I know that H1 is same as H0 e power j omega –pi that is the quadrature mirror symmetry okay. So now this tells me also a way to actually satisfy the magnitude constraint also through a design process. So I can design my filter H0 such that this is very close to a constant C, the magnitude squared response and just a last point, what does that actually mean?

(Refer Slide Time: 51:58)



If I have a filter like this quadrature mirror filter, what the condition says is if this is H0, this is H1, take magnitude H0 e of j omega magnitude squared, so basically square this. Then, square the second term H1 e of j omega magnitude squared and add them together. So basically I will just draw it for the purposes of squaring. This is the squaring here; this is the squaring of this term here.

What I get through the addition process is something that looks like this. I want something that is a constant okay. So it is possible but we have to design the filter H0 with slightly different constraints. What are those constraints? how do we design them? how do we get good filters for this is in the next class. Thank you.