Multirate Digital Signal Processing
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Lecture – 20 (Part-2)
Maximally Decimated Filterbanks 2 -Part2

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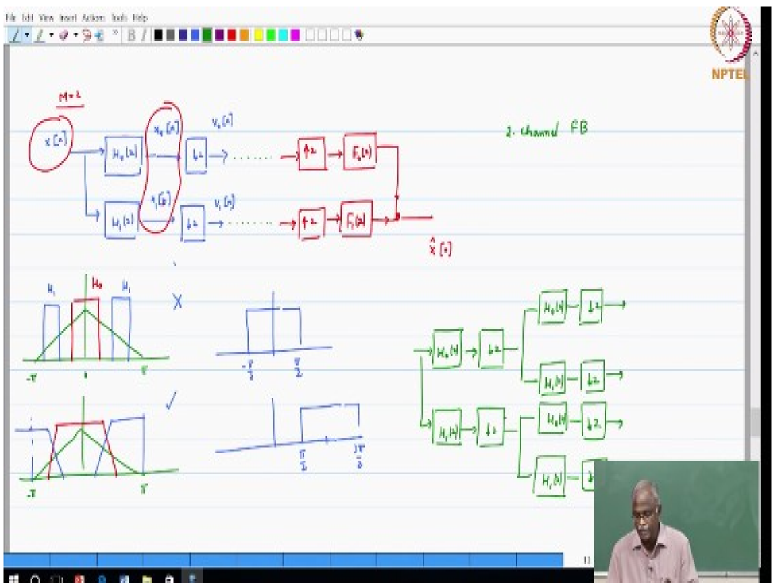

Now to simplify matters we are going to take the special case M=2 and analyze it completely and then make a statement about how does it generalize to M greater than 2 and then present what is the state of the art in terms of the channel M channel filterbanks so we go back to basics. And we say okay let us just draw a two channel filter bank and label all of the signals, so, that we can then analyze it in detail.

Only 2 analysis filters output of the first analysis filter x0 of n it is down sampled by a factor of 2 we label this as v0 of n the second sub band is H1 of Z down sample by a factor of 2 if this is x1 of n and this is v1 of n. Now one of the fundamental requirements of any type of splitting into sub banks is that all of the information that is in x of n must be present in a combination of x0 of n and x1 of n.

For example, if you had a signal x of n which had got the following spectral characteristic 0 to pi –pi this is the one and your filters H0 and H1 let us say just for argument sake this is H0 and this is H1 again this is just for illustrative purposes. This is H1, so which means that when you come into the sub bank signals X0 and X1 some portions of the input spectrum are missing there is no way you can recover them subsequently.

So, this is not allowed now on the other hand can we have a scenario like this. I have the input spectrum from –pi to pi. H0 is a filter of this type and I did not get it symmetric H0 is a filter let us make it easier to analyze okay H0 is a low pass filter with a certain transition band and H1 is the bank pass filter? A high pass filter with a transition band which is given like this. And now this is okay now there is overlap of the information content between x0 and x1, that is okay.

Now unless you have perfect filters there is going to be when you insist that no part of x of n should be missed out that means that it is going to be some overlap between the two filters. Which means that each of them effectively at least one of them when you decimate is going to cause aliasing right? Because if the only way you can avoid aliasing is if you had filters of this type -pi/2 to pi/2 and the other one is like this right pi/2 to 3 pi/2.

If you had filters of that kind of ideal filters and non ideal filters is going to cause and one of them is going to aliasing is a problem that we will have there is no question of whether aliasing will occur aliasing will occur in this setup we have to deal with it. So, second the aspect of it is okay at the other end the reconstruction process will be restoring the sampling rate up sampling by a factor of 2.

Applying suitable filters, we will call this synthesis filters as F0 of Z and F1 of Z and these 2 will get added together notice the convention that I told you that the addition happens towards the bottom of the graph of the analysis always split occurs at the top the synthesis again this is just a convention that this followed nothing special or unique about it and let us call this as x hat of n. Now in the general case what we need to show is that if I were to make a direct connection.

Between these two I did not do compression I did not do anything to the filters. I should be able to recover my original signal that x Hat of n should be able to reconstruct x of n that is a basic requirement on top of that then I may say well I might do some compression and I may do other things but at a basic level. The structure regardless of what is happening in the middle has to be able to reconstruct.

So, this is the basic 2 channel filterbank used very extensively in all these applications Why? Because if you can solve this problem you can also solve the 4 channel problem how do you solve the 4 channel problem and you do H0 of Z down sample by a factor of 2 this is H1 of Z down sample by a factor of 2. Okay, what is the frequency content after down sampling? It will be from 0 to pi it is basically -pi to pi basically.

And so you can now take the same filterbank and apply it once more H0 of z down sample by a factor of 2 H1 of Z down sample by a factor of 2 and now H1 of Z has got a limited frequency content but once you down sample it. It is going to fill the entire spectrum so therefore this, can also be subdivided as H0 of Z down sample by a factor of 2, H1 of Z down sample by a factor of 2. It is a very interesting exercise to write down the transfer function the shape of the filter going from the input to each of these 4 outputs.

You will find that actually if you assume perfect filters you can show that this is a beautifully structured filterbank DFT filterbank. So, again any power of 2 you can split the signal and because you know how to recombine. How do you recombine? you first apply the recombination for the each of those upper two branches. So, you can recombine these two first then you recombine these 2 okay with the appropriate filtering.

And then you recombine these 2 to get back the single signal. So, this is called a tree structured filterbank. Basically, it kind of branches out like the branches of a tree and once you solve the two channel problem. You can actually solve you can apply it in a very much wider range of context you really do not have to design a higher order filterbanks. This is already there for you, so this is a useful tool for us we need to analyze so basically let us complete the labeling.

Well, you can think of it that way but in this case, there are certain constraints that you will have which, is let me just sort of rephrase it. When you write down the filters. So, the transfer function from here to here for example will be H0 of Z and H0 of Z square that is going to be the first transfer function right. And similarly you are going to get. Now the so each of these filters is not interpolated FIR filter that is the basic structure is there.

So the statement is correct that yes to each of these are, but it is not necessarily the IFIR is one type. But what we are trying to show is that there is a certain pattern and a certain structure to it that comes from the 2 channel case. So, it is a combination of the basic 2 channel results with the IFIR you can state and that is a good way of looking at it. Okay so labeling let me just complete if you call this signal as y0 of n at this point and this one as y1 of n and the output is x hat of n.

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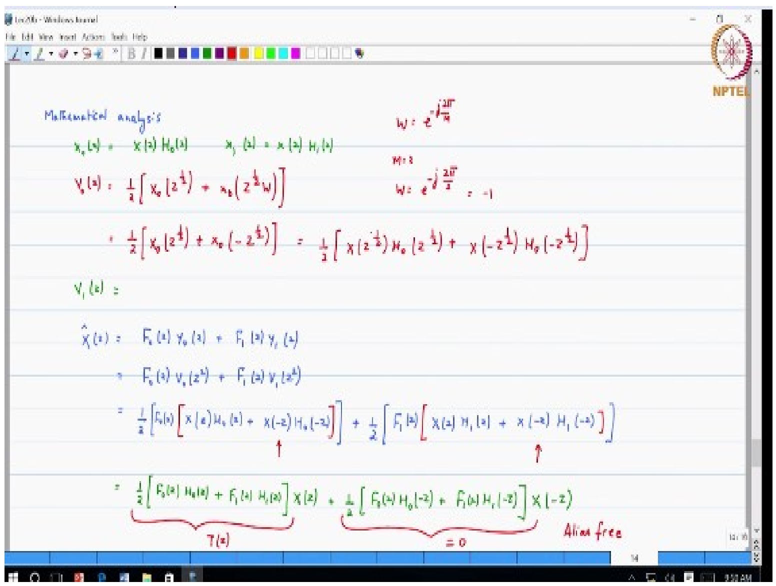

So, what I would like to do is now do the analysis mathematical analysis please keep that figure as a reference I will not redraw because I would like to use the space for writing expressions. So, the first one is what is X0 of Z is X of Z filtered by H0 of Z. Similarly, X1 of Z will be X of Z filtered by H1 of Z again a couple of things I will write it in full but then later on start skipping steps The next level is V0 of Z is x0 of n down sample by a factor of 2.

So, this would be 1/2 of X0 of Z power 1/2 okay + X1 of Z power 1/2 sorry X0 of Z power 1/2 times W when you do the down sampling. Now a key point to note is that W is e power – j 2 pi/M in our case M=2 so W= e – j 2 pi/2 which is nothing but -1. So, I will write this as 1/2 of X0 of Z power 1/2 +X0 of - Z power 1/2 nothing profound just the simplification I do not want to keep W, minus is easier for us to work with.

So, this basically says if you go back and substitute for X0 this will be 1/2 of X of Z power 1/2 H0 of Z power 1/2+X of - Z power 1/2 H0 of –Z power 1/2 just substitution and simplifying. So, please do write down the expression for V1 of Z, I would like you to please fill in that result now jumping over to the other side there is an up sampling and after the up sampling the signals are added so the expression for the other side.

This will go back here up sample the signal V0 so Y that gives you Y0 of n multiplied by F0 of Z. So, the reconstructed signal X hat of Z. I am jumping a couple of steps but assuming that those are straight forward X hat of Z is F0 of Z Y0 of Z + F1 of Z Y1 of Z right and Y0 and Y1 are up sample versions of V0 and V1 so that becomes F0 of Z V0 of Z squared+ F1 of Z V1 of Z squared.

We substitute for V0 of Z, find out the expression for V0 of Z squared. So, what you will get is 1/2 times F0 times X0 sorry write it in terms of X of Z, X of Z H0 of Z, use a different color for the bracket F0 of Z bracket substituting for V0 of Z squared. Basically that would be X of Z H0 of Z, I have taken out the factor of 1/2 + X of –Z H0 of –Z close the red bracket and close the blue bracket + 1/2 F1 of Z open the red bracket you will get X of Z H1 of Z.

I am writing in red X of Z H1 of Z+ X of –Z H1 of – Z close the red bracket and close the blue bracket. Again it is fairly simple and substitutions but just be careful to get all the terms and make sure that there is no mistake. Now input signal is X so the input spectrum is X of Z now when do you say that aliasing is present? When some shifted version of the input signal is also present in your expression.

Now where is the shifted version of your input signal X of –Z, X of –Z, so that is why aliasing is present and therefore we have to be careful in our analysis so X hat of Z. I am going to write in the following form I am going to write it as 1/2 of F0 of Z times H0 of Z + F1 of Z H1 of Z times X of Z. I am basically grouping all the terms that are affecting X of Z + 1/2 of F0 of Z H0 of -Z + F1 of Z H1of - Z.

This multiplied by X of – Z here we have very clearly before us where the signal part is present where the aliasing part is present so if I want to remove aliasing it basically says that this transfer function which is the transfer function of the shifted signal towards the output. If I can somehow set this equal to 0, then I will get the condition that it is alias free my 2 channel filterbank will be alias free.

Why because there is no component or contribution of X of -Z in the out. So, if I have succeeded in achieving the aliasing cancel if and only if we have succeeded in cancelling the aliasing thing then we can call this as my transfer function of the multi rate signal block DSP block. Though it has got time varying elements we said that if the input and output have the same sampling rate and there is no aliasing is present.

Then I can write an input output transfer function and that is precisely what is shown by T of Z, so it becomes an equivalent of an LTI system again very powerful result because if you can cancel aliasing then it becomes an LTI system. We will also show that if you do not cancel aliasing, it becomes a time varying system, which has to be analyzed with its own set of understanding and complication.

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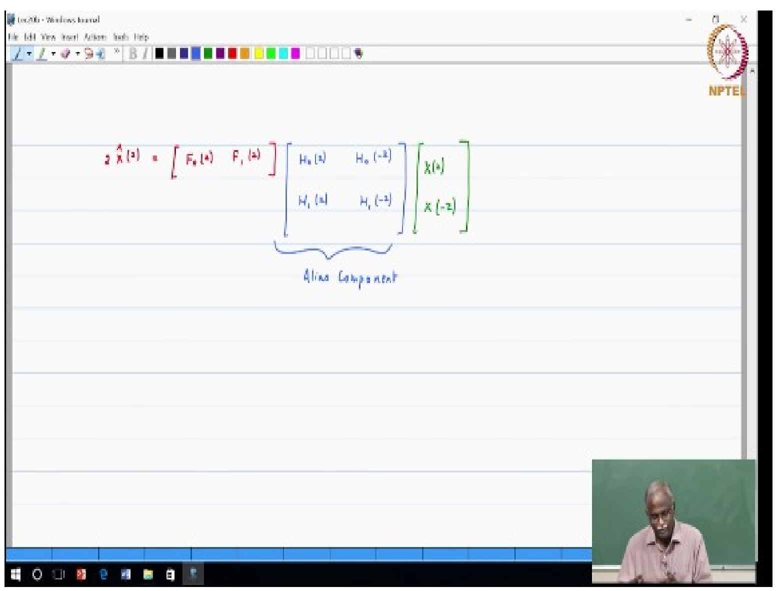

So, basically let me just write one more equation I know that we are almost out of time so 2 times X hat of Z that is the 1/2 factor. I have just taken it to the right hand side again writing it in matrix form always helps us F0 of Z, F1 of Z that is where the synthesis filters they are the ones that combine multiply and combine. Now where are the; what are the components that are getting combined?

Those are of the analysis filters and the signal both the regular version and the aliased version H0 of - Z H1 of Z H1of – Z, these things multiplied by if you were to think of it as an input output system. You are taking the combination of the synthesis filters the analysis filters as well as the original spectrum X of Z and the alias spectrum X of - Z. By the way this has got a term it is called the Alias component matrix.

We will talk about an AC matrix that means it is aliasing components and this. This is a very, very important matrix because ultimately how you design the filters must ensure that aliasing is removed. So, in other words what you want is the product of F0 F1 with the first column to be preserved and the second column to be to go to 0. So, again a very important now how do you achieve these conditions how do you design good filters?

And remember I said no portion of the input spectrum can be omitted in the analysis so therefore how do you ensure that the entire input signal shows up at output signal and aliasing is removed very interesting problem what we are going to do is first mathematically do it that way you know. I know how to solve it mathematically, but then translated into the actual design of the filter the shape of the filter what constraints do we put on it that gives us the insight.

And you cannot go from insight to theory but you go from theory to insight a sort of reinforces and then we come back to theory and then complete the discussion but that will be done in the next class. Thank you.