Multirate Digital Signal Processing
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Lecture – 17 (Part-2)
Spectral Analysis of Filter Bank - Part2

Now here comes an important question. Let me write it down because give you some time.

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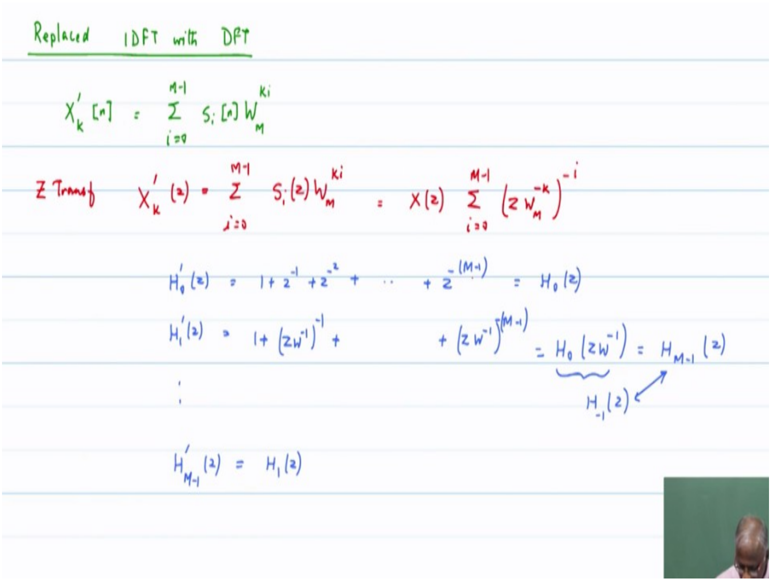

What would happen if I replaced the IDFT with the DFT? Go back look at the figure what would it would be the delay chain would remain as is, you would have replaced this instead of the W dagger it would be the W matrix. So let us see if we can of course by now you can easily tell that the output will be the same expect they are shifted. Let us just confirm that is the case.

So I write down XK prime of n. The prime denotes that now I am working with the DFT matrix sitting there. This is equal to summation i=0 to M-1 Si of n notice that now it is a DFT matrix it is W subscript M super script Ki. If I now take the Z transform it will be XK prime of Z that will be equal to summation i=0 to M-1 Si of Z W M Ki please go ahead and substitute you can show that this is equal to X of Z summation i=0 to M-1 Z WM -K raise to the power –i and you can verify.

So this tells me this is the following observations that we get H0 prime of Z that is the filter transfer function between X0 prime of Z and X of Z this comes out to be 1+Z inverse + Z-2

…+ Z power-M-1 oh this is nothing but H0 of Z. So the basic filter did not change of course that we knew already. Now H1 prime of Z this will come out to be 1+ ZW inverse power -1 basically I am substituting from the expression above.

ZW inverse the last term raise to the power - M-1 and if you simplify you can show that this is nothing, but H0 of ZW-1 which is in our terminology this would actually be H of M-1 of Z because this itself is as if you are writing H of -1 of Z and these 2 are the ones that are linked basically whatever you call as -1 already has a name that is H of M-1. So H prime of Z and of course you can go through and verify that the final filter H M-1 prime of Z is nothing but H1 of Z.

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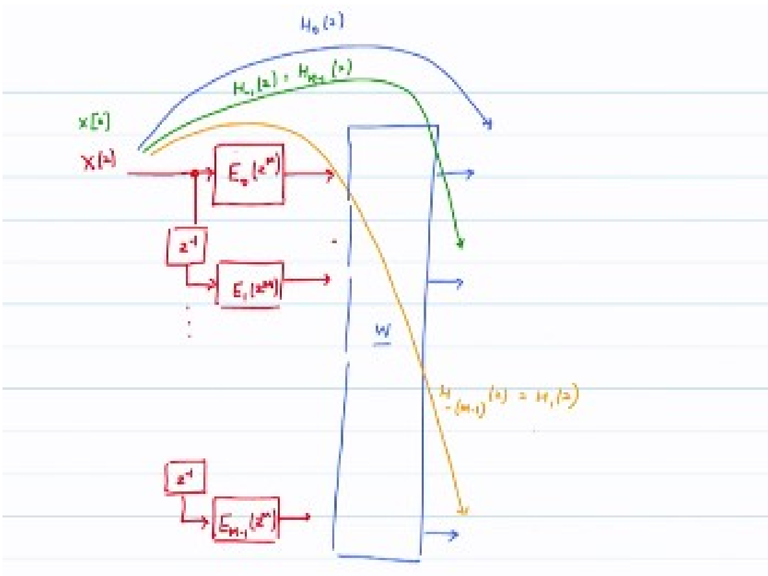

So again exactly the result as before maybe the figure to look at is this expression H0 of Z is from input from topmost branch H M-1 to the next branch and all the way to the bottom is the next branch. So here is a pictorial representation if this was what you obtained with the transpose conjugate matrix now with the analysis that we have done with the DFT matrix. The figure is as follows.

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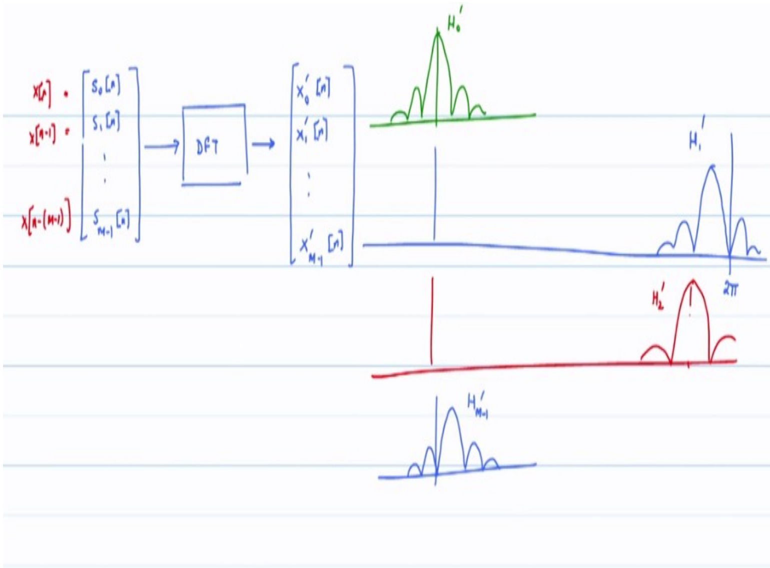

Let us quickly draw it. What did we do? We said that we will have a vector which is S0 of n, S1 of n all the way to S M-1 of n and we are going to pass it through the DFT matrix I am not calling a time of frequency I am looking at it as a transfer function and this produces for us X0 prime of n, X1 prime of n and all the way to X M-1 prime of n this is what is just think of it just strictly as a transform.

And what this transfer function between these corresponding filters is as follows. Again let us draw it very quickly may not look very pretty. So this is H0 prime and the next filter that we have this is H1 prime no this H1 prime is not this one. This is actually H M-1 prime. H1 prime actually will be all the way to 2 Pi this is H1 prime. H2 prime and the last one is H M-1 prime.

Now can you give me interpretation of what happens when you take DFT of a vector. So how did you get X0 prime of n you took the input passed it through the filter which is basically a comb filter and then you observe the output. And this S0, S1, S M-1 are all delayed version it is basically like the input signal that is what you have taken. Can you give me when you take the DFT of n variables what exactly are you doing?

You are passing that signal through a bank of filters and the bank of filters go all the way from 0 to 2 pi of course you do not worry too much about the notation basically it has M filters and you get the output from those. Last step very, very important step, but this is not what we do in DFT right. What is this if this is X of n this is X of n-1 all the way to X of n-M-1, but that is not what we do in DFT.

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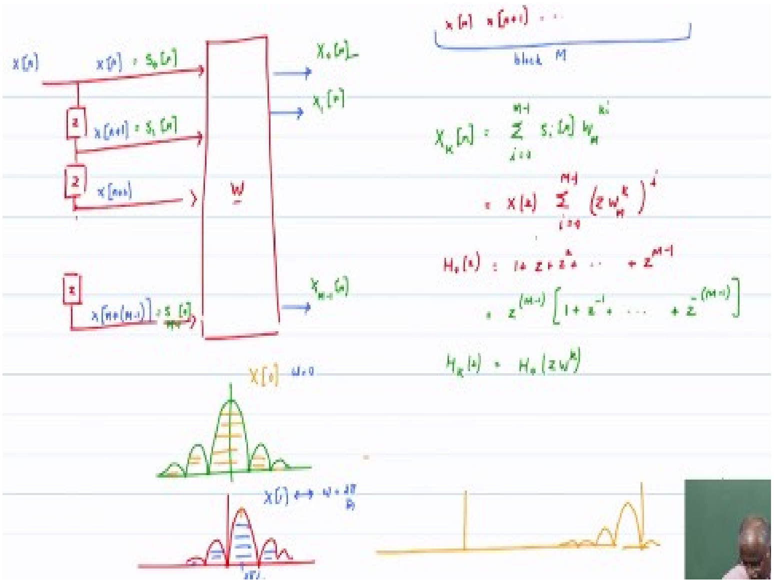

Actually what happens in DFT is the following structure Z it is not a delay chain, but it is an advance operator and each of these branches comes in and then we take the DFT of this vector. Am I correct because what usually fed into X of n, X of n+1, X of n+2 basically if your input sequence is let say is X of n, X of n+1… you say that I am going to take the DFT of this vector so you take a block of length M and you take the DFT of it.

So this is more likely what we will encounter. So these produces the M output like before let us quickly label them if this is X of n on the upper branch you have X of n this will be X of n+1 this is X of n+2 …this is X of n+ M-1 that is the last branch that is feeding in. Now like before for systematic approaching label this as S0 of N, this as S1 of n and this as S M-1 of n. Output let me call this as X0 of n, X1 of n, X M-1 of n.

Again I am treating it strictly as a transform. This n is actually showing you the time index, so there is a vector at the input, there is a vector at the output input output relationship is through the DFT matrix. So please write down that expression again it is a repetitive one, but it actually give us an important insight Xk of n is i=0 to M-1 Si of n W M -Ki no there is no -this is the DFT matrix.

So it is power Ki you can show that this is equal to just one step and we are there with the answer X of Z summation i=0 to m-1 Z W Mk raise to the power i okay very, very important. So what is H0 of Z it is K=0 it will be equal to 1+Z+Z square + last term Z power M-1. This is a comb filter, but going in the backward direction mom causal, but this is the same as again easy manipulation if you take Z power M-1 as a common factor what you will get is 1+Z inverse + Z power M-1.

So on the unit circle H0 e of j omega this is nothing but a phase term we take the magnitude response it goes away. So what is the magnitude response of your H0 same as what you did for when you had the transpose conjugate of the DFT matrix. So if you go ahead and now work out HK of Z it will come out to be very, very interestingly H0 of Z WK. What happens when you have a delay chain and the transpose conjugate matrix.

You get the filters in the correct sequence. You replace the transpose conjugate of the DFT matrix with the DFT matrix you get the same output, but the output sequence is scrabble. Now if you change the delay keep the DFT matrix as the same it is at a delay chain make it into the advance chain then what you get this sequence comes out again in the correct form. So when I do the DFT what is X1 the second DFT coefficient.

It is nothing but the output of the filter that is just adjacent to the low pass filter. The first DFT coefficient is effectively the DC term because this is the filter that filter or the DC. The second DFT coefficient is the one that is slightly shifted basically low frequencies and so on and so forth and the last of the filter basically is the one that goes very close to 2 Pi. So what is DFT? It is actually taking the input signal passing it through a bank of filters.

And then giving you the output and saying oh by the way X0 you think of it as the frequency component usually if somebody gives you the DFT coefficient X0 what is the interpretation that is the DC term. It is actually not the DC term it is actually all the energy that lies that will pass through this filter in your input signal that will pass through this filter is what comes out as X0.

And what is X1 our usual expectation is that what is the signal component at e power –j 2 Pi/M or omega=2 Pi/M. it is not omega=2 pi/M alone it is actually so this corresponds to omega=0. Now X of 1 we say that this corresponds to the energy component at omega=2 pi/M it is not just 2 pi/M it is all those components that will pass through this filter. The center is a 2 pi/M yes that is given the maximum weightage, but there are other components that are coming through as well.

So one of the key things to recognize is that until now what your impression of DFT was there is some magical means by which I do a transform and I get the frequency response at omega=0, omega=2 pi/M, 4 Pi/M, 6 Pi/M and that is our impression, but that is not true. It is actually as if you took the information and pass it through a filter with a finite bandwidth it is not infinitely narrow bandwidth it has got a finite bandwidth.

And any signal that passes through this filter will show up as a contribution to X0. So the question is in the context of DFT are we making a periodicity assumption. The first step that we do in the computation of DFT.

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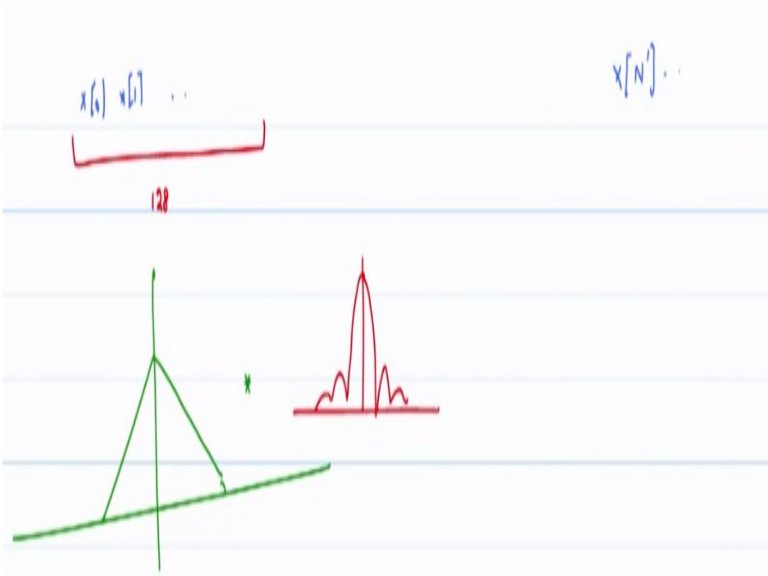

Let me just highlight that. So if I have a long sequence X0, X1… it is a very long sequence X of n prime …and I decide to take the DFT of one portion of it let say 128 points is a much longer sequence. So if my input spectrum the true spectrum if my input spectrum let us say had this shape, the minute I multiply it with a rectangular window I have modified my sequence. It is no longer the spectrum.

So actually what is the spectrum it is this convert with a rectangular window is the sink function. So already I have modified so you have to go back and revisit saying what am I calling as the delta function because it is actually how we interpret the process. I like to look at it if I look at it from a filtering point of view, the minute you have multiplied the rectangular window I have modified my input sequence.

And then I take this sequence and I try to do I compute the DFT, but interpretation that we are giving is ultimately what are we trying to do. I am trying to get the spectral components of the input signal that is the ultimately gain. So the way I want to look at it is from a filtering point of view instead of doing a rectangular window followed by a DFT then sliding my data doing another DFT.

And doing through the process obtaining what is my spectral estimation what we are saying is by the way this is the same as passing it through a Filter bank. So you can take your data do not need to window it just pass it through a rectangular window based Filter bank. So this type of Filter bank this is exactly what your DFT is also effectively doing.

Again how the effect of the window affects the final spectrum you may give it little bit differently, but the ultimate the overall effect is that you can take the impact through the multiplication of the rectangular which means that the original spectrum now has got modified or I can think of the signal original signal not window signal original signal passing through a bank of filters.

And then I am observing the output and the mathematic framework is the same because if you look at what we are doing is exactly the same thing. You get the vector you process it. I have not yet finished the interpretation so if you just wait for the next set the time is over. Let me just leave you with a few questions. I will definitely pick it up we will clarify all these doubts because it is very, very important I am glad you have asked those questions.

Now what would be the DFT interpretation in the following setup?

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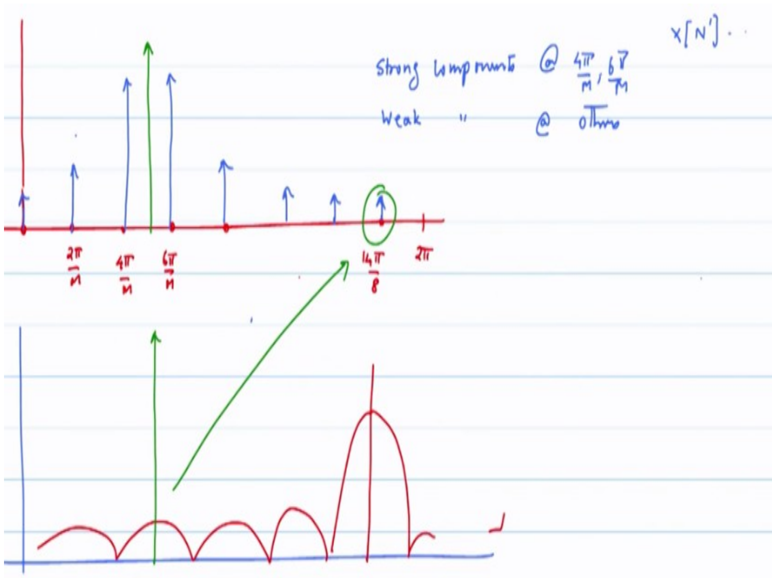

So my DFT center or the frequency points are 2 pi/M, 4 Pi/M, 6 Pi/M. Let me just take the case where it is an 8 point DFT so I basically have 8 points between 0. The last one is 14 Pi/8. Now supposing I had the signal represented a tone in between 2 Pi/M and 6 pi/M take it DFT and see what you will get. Let me just sort of give you a feel for it we will discuss it in detail. You will find that the DFT coefficient will be quite strong here some component here strong it never goes to 0.

We actually try it out you can actually do a simple test and do it. Now what is the explanation for this is because if I took this why this is last filter why is it still having non 0 output. The filter bank interpretation says the last filter is centered around this frequency. It has got a response like this so when it comes here this is the tone that is present. So this is what shows up as the output. It shows as the non 0 output.

So one of the problems with the DFT is that if I have a tone in between 2 DFT frequencies I will have non 0 components on across all the terms not just the once nearby all the terms will have non 0 and suppose somebody gave you this blue lines and told it interpret this spectrum then you will say that there is strong components. We say there are 2 strong components at 4 pi/M and 6 pi/M that is how you can interpret. You cannot say I suspect there is a and they will say that there are weak components at all others which is a very erroneous interpretation of the spectrum analysis that you will get, but that is all you can do.

However, if you now have interpreted basically it is a same interpretation whether you think it as a Filter bank or as DFT operation. However, if you are willing to accept it as a Filter bank operation there is a way by which I can get you almost close to the answer that there are only that is most likely the best case is there are 2 tones one is 4 Pi/M and one at 6 Pi/M and you can say that okay both of them are equally strong.

It also could be that there is a single tone which is somewhere in between the 2 those 2 DFT frequencies, but it is possible to get to that point only with the Filter bank interpretation with the DFT interpretation there is nothing we can do. If you have a tone in between 2 DFT frequencies the spectrum that you will see will be something that it could mislead in terms of its interpretation.

And it is the inherent in the structure of the DFT and this is well-known. This is one of the artifacts of the DFT and that is why interpreting the DFT as a Filter bank actually gives us some interesting insights and what are those insights how do I make it a very accurate or better than simple DFT we will look at it in the next class. Thank you.