Multirate Digital Signal Processing
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Lecture – 15
Introduction to Multirate Filter Banks

Good morning, we will begin lecture 15 we will review quickly the key results from lecture 14 and build on the topic. As I mentioned, one of the key applications of Multirate signal processing in the form of filter banks and filter banks come in several applications and today, we will look at one of them which is filter banks used in spectral analysis so before that.

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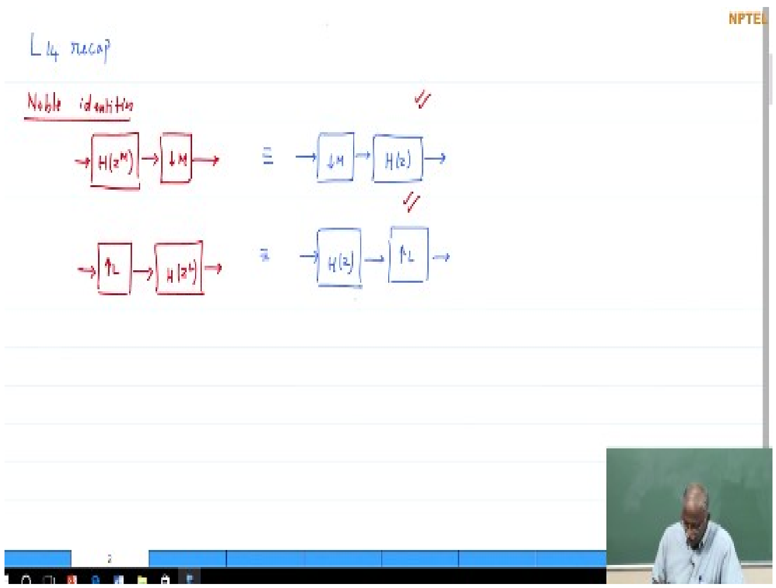

A quick summary I would say that the key results from a yesterday's lecture one was a repeat of the noble identities which said that if we had a situation where we had filtering followed by the down sampling and the filtering was of the form H of Z power M down sampling by a factor of M. They could be interchanged likewise if the if there was an up sampling followed by filtering where the filtering was H of Z power L then we could interchange.

And in both cases the figures on the right turn out to be the preferred mode of implementation because they are the ones that do the minimum amount of wasteful computations.

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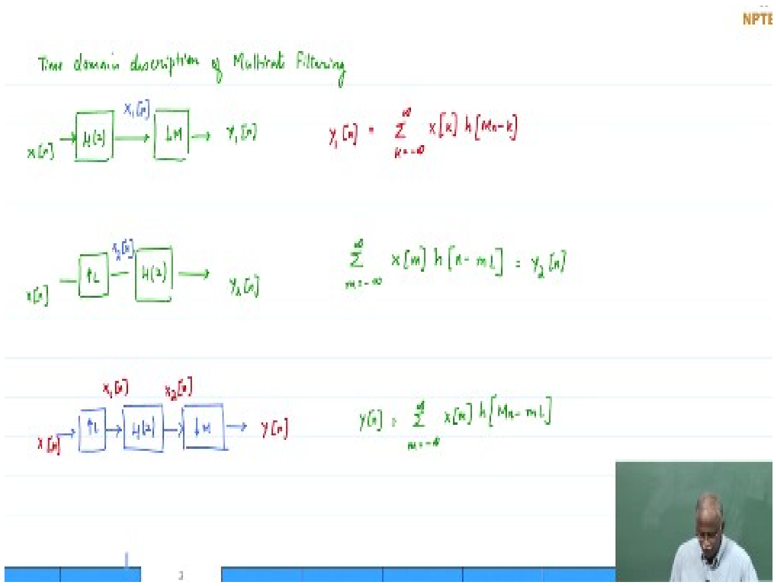

Now a second result which is more for familiarity and insight, we said that we could write down the time domain description of Multirate filters when we did a down sampling in which case it looked like a convolution except that the filter was not H of N. The filtering term was not H of N- K but Mn-K where M is the down sampling factor. Likewise, up sampling can also be written in terms of the convolution type expression.

Where the h is n - it is not h of n-m it is h of n- mL well where L is the up sampling factor and of course you can put both of them together. You look at how the input is manipulated through the coefficients of the filters and then it gives us an expression. This helps you probably visualize what is happening in the frequency domain so this is by way of the quick review.

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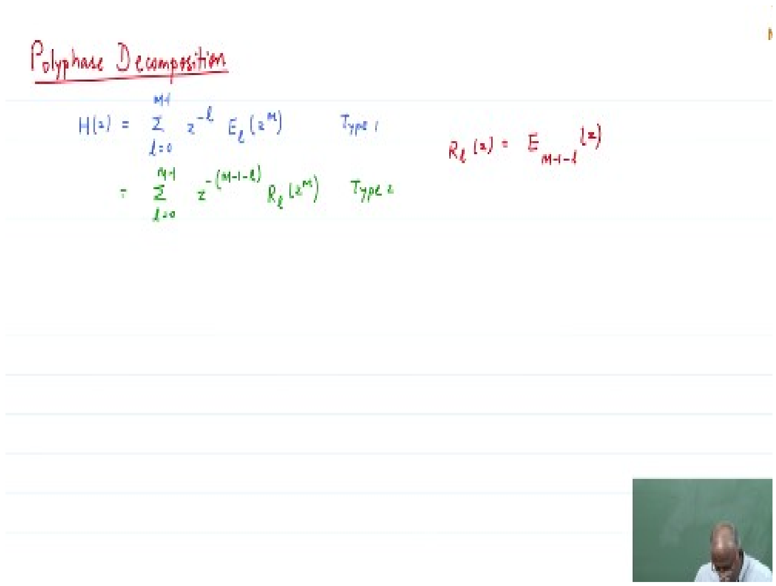

We also mentioned that the polyphase decomposition any filter any signal can also be split up into polyphase components. So, basically anything that has got a Z transform you can rewrite in the form where you have M sub polynomials. So, it could be the impulse response it could be the signal itself polyphase decomposition is possible in any of those environments. So, type 1 polyphase decomposition, which is the most and widely used.

And then we also have a type 2 which is helpful for us in some scenarios so now let me quickly start with today's material and build on that. Let me ask you to think about one particular example.

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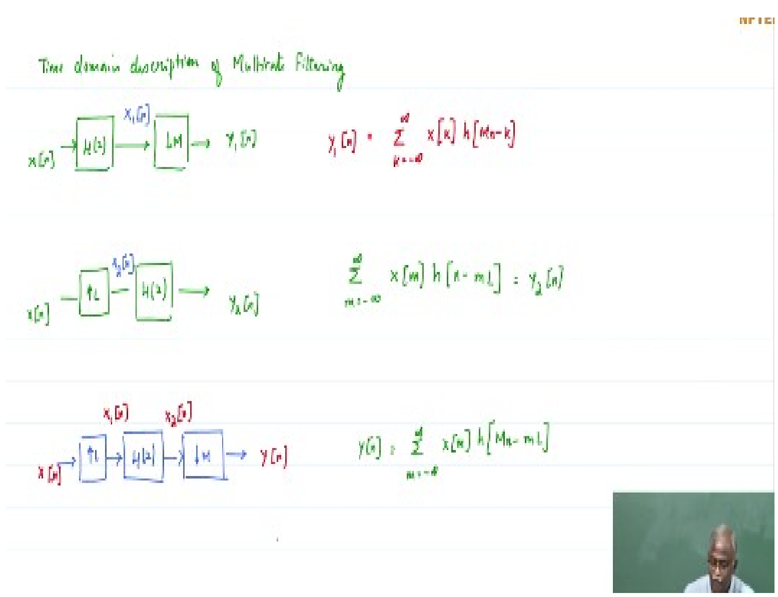

So, here is a scenario where I have a delay followed by down sampling followed by up sampling. Before even begin this example I just want to reinforce that one of the key things takeaways from this course should be that you have developed a lot of insight. So, looking at a structure immediately the intuition they have the mathematical forms the frequency domain the presentation all of those sort of come together.

So, here is one such hopefully by the end of the discussion of this example we can make some statements. So, if I have x of n and I am looking at Y of n how a student who said that this is an ideal candidate for applying the noble identity I have filtering followed by down sampling. So, the statement was that okay from here I will go on to doing the down sampling first down sampling by a factor of 2.

And that means the expression if you look at the poly phase decomposition H of Z power M down sample by 1M and this becomes H of Z. So, which means whatever the argument raised to the power of 1 over M okay so the argument was Z-1 down sample by 1/2. So, this becomes by argument Z power -1/2 followed by up sampling. Up sampling by a factor of 2 proceeded by prefilter you can shift the delays even further which then says you must take the polynomial.

And raise it to the power L so come up with down sample by a factor of 2 up sample by a factor of 2 and Z inverse so and the argument is applied the noble identities and I have obtained equal expression whether it is simpler or better it does not matter. Is this correct? Applying the noble I notice it looks like it is okay. What he has done is okay 1 of the rules and applying the noble identities is that you cannot get an undefined block.

So, basically this is an undefined block Z power – 1/2 is not defined you cannot so this is an undefined block in the process of applying the Nobel identity. So, this is not permitted and actually if you do violate this rule if you violate this rule obviously you get this expression. Now here is where I want you to apply if I down sample by a factor of 2 up sample by a factor of 2 what is it comb function.

It will drop every odd sample if I delay the input signal by 1 unit of time and then apply the comb function. What are the non 0 samples? The non 0 samples at this point samples are the odd samples of X of n and am I correct? Non 0 samples are the odd samples or samples of X of n. Now even without looking at the undefined block and I was saying that it is a violation you could argue that these two are not equal because now you take X of n.

Down sample and up sample by a factor of 2 that means a comb function. What is retained? Even samples you delayed by 1 unit of time what will still be retained even samples. So, the non 0 samples at this point non 0 samples are the even samples of X of n so clearly these 2 are not the same. Obviously there must be something that you violated and going from here to here and the violation was the intermediate step.

Where there was an undefined block in the process of using the nobel identity even samples of X of n so again intuition understanding the mathematics instead of being able to apply that all of these come into play and it is very helpful if you are able to do all of those things. Okay another interesting example for you to think about.

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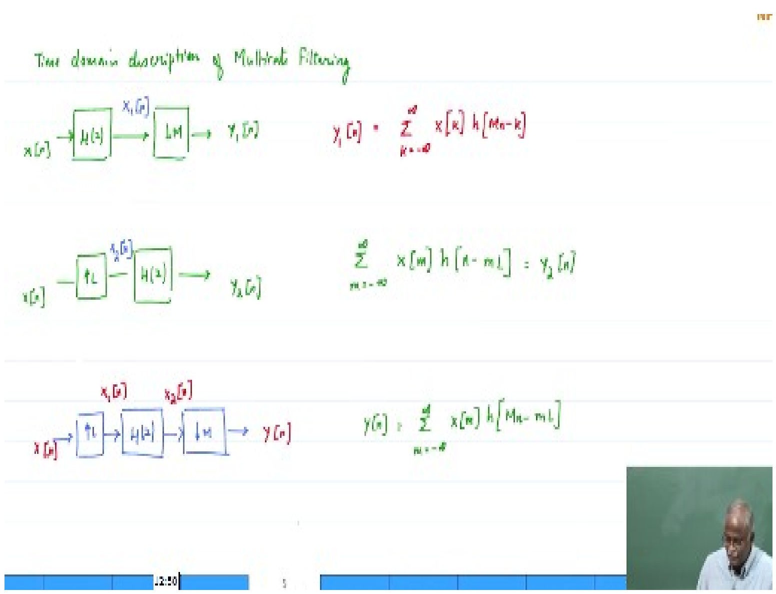

And again tell me what is going on so I have a structure that we by now we are starting to get more familiar with and comfortable with down sample by a factor of M advance operator. Down sample by a factor of M another advance operator down samples by a factor of M and M such branches. Down sample by a factor of M so this is the blocking operation so if I fed into this structure not some input signal but my impulse response h of n.

I have a filter was impulse response is h of n so H of Z let me assume it is a casual filter and equal to 0 to infinity h of n Z power –n so this is a casual filter and now I want you to tell me what comes out at this point. This will be h of 0 h of M dot dot dot. Those are the first branch. Second branch is h1 h M+1 dot dot dot. The last 1 is h of M+1 h of 2 M+1 dot and now make M and this is number 1 number 2 and the last 1 was M-1.

Okay this is number M -1 so h of M+1 so h of M+2 h of M+1 not M+1 it is M-1 thank you that was why probably the confusion was okay h of M-1 h of 2M-1 and now what these actually represent are the polyphase coefficients of the polyphase components. If I were to split this into type 1 polyphase component this would be E0 of Z power M + Z inverse E1 of Z power M dot dot dot where E 0 of Z is summation n = 0.

I will write it as summation h of n no h of Mn Z power – n so actually what we said that this pollen on this sequence this can be written as summation e 0 of n Z power –n if you wrote that as the notation these are the coefficients of e 0 of n. These are the coefficients of e1 of n those are the coefficients. This is the coefficients of e M -1 of n so actually what we would be doing was to actually split the filter into poly phase components.

And then apply them in the branches but actually the structure itself has got so much a symmetry in it that they actually if you feed the impulse response the structure itself gives you the polyphase components. Again it is just an observation that the structure actually what is the underlying operation of the structure. And it is helpful for us to think of what exactly is happening when you have either a chain of delays or a chain of advance operators followed by down sampling or up sampling, the multiplexing operation the multiplexing operation just a way of reinforcing what we have been discussing.

Okay so now there is another interesting example which also brings in a lot of insight and again, I want you to sort of see how to draw the maximum understanding and insights from this particular example, may seem like a simple example.

But it is one of those where once you look at it closely you see that there is a rich set of observations that can be taken away.

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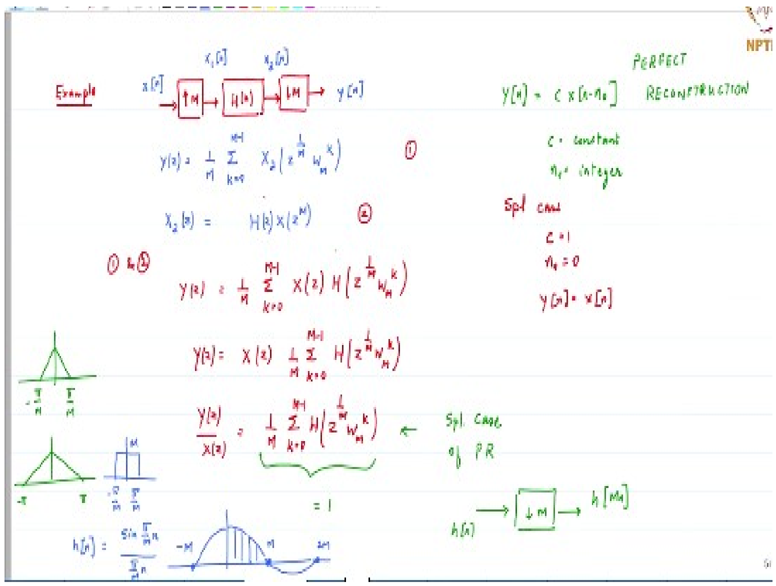

Another example: I have the following sequence of operations up sample by a factor of M, filter by H of Z, down sample by a factor of M so straight away observation would be is this is a structure for sampling rate conversion. Fractional sampling rate conversion but if input and output blocks are up sampling and down sampling are by M you are not changing the sampling rate.

So, really you know is this a meaningful or useful operation it turns out that when you start looking at filter banks and a number of structures that we are going to be studying. The primary motive is not sampling rate change but processing and when you do processing then input and output invariably will be at the same sampling rate but some processing would have happened inside of the multi rate structure.

So, this is actually a very useful structure something that we will encounter quite a bit. So, here is the labeling of the variables X of n Y of n helpful to label a couple of intermediate variables X1 of n X2 of n. Now want you to just help me analyze this but again the important insights will come as we progress so Y of Z; down sampling. So, it would be 1 over M summation K = 0 M+1 X2 of Z 1/MW M K that is the down sampling operation.

Let me call that as equation 1 and X2 of Z that is first the up sampling so that becomes X of Z to the power M multiplied by the filtering operation H of Z we can call this is as equation number 2 combining equation 1 and 2 combine equation 1 and 2 we get Y of Z = 1/M summation k = 0 M – 1 X of whatever is the argument of X2 of Z we must raise it to the power M.. So, you find that this will become just X of Z for all the substitutions.

And the second term will be H of Z power 1/M W M K so since X of Z is common does not depend on K. I am pulling it outside the block X of Z 1/M summation K = 0 to M -1 H of Z power 1/M W M K so this is Y of Z and of course since you have Y of Z expressed in terms of X of Z. We can think and treat this as a LTI system and write a transfer function Y of Z / X of Z that is your transfer function.

This comes out to be 1/M summation K = 0 to M-1 H of Z power 1/M W M K all of this is routine nothing new or very different. Okay here is some additional information maybe this is where we start using some new introducing some new terminology. So, if we can get Y of n = 1 and if we can show that Y of n if through some processing you can show that Y of n is of the form c times X of n - n0, there are class of a multi rate systems where you will try to relate the input and output and if this is the relationship then the if you look at the structure c is the constant and not is a constant in integer.

So, it says that the output is a scale delayed version of the input and such class or systems where the input and output are related in this fashion are referred to as perfect reconstruction systems. That means whatever you did inside you managed to recover or reconstruct the original signal.

Perfect reconstruction systems so that is one of the requirements are perfectly construction systems is the reconstructed signal or the output signal must be within the scale factor. A delayed version of that and special cases of this a special case is when you say C has to be = 1 and n0 = 0 then what you get is Y of n = x. Often you may or may not get it with a casual system but this is a special case.

So, now if I apply this condition to this system and say that I want the special case of reconstruction the special case of a perfect reconstruction, special case of perfect reconstruction always shortened with PR as the notation that basically says that this must be = 1. Transfer function must be = 1 that is when you will get input output it to be the same. Okay now if you go back and look at the example that we looked at earlier.

Can you tell me what this block is actually doing? This block is 1/M summation k=0 M -1 h of Z power 1/MWMK; now this is down sampling of h of N. Okay so if you down sample h of n which poly phase component have you picked up in this case. So, if you sent in h of n and you down sample by a factor of M, what you would get here is h of MN this is 0 poly phase component. So, what is it?

What it is saying is this is true if these zeroth polyphase component of your filter is a constant got it. The several things are getting linked together so basically this expression says that the zeroth polyphase component is a constant now but this structure we have encountered we have encountered previously where did we encountered previously? Sampling rate changes now go back and let us reaffirm that whatever observation where we have made is actually correct.

Okay now if I up sampled by a factor of M and I want to do interpolation perfect interpolation what would this filter be? Low pass filter cut off Pi /M if I have done that then what would have happened? I would have restricted my signal to Pi - Pi/M to Pi/N correct that operation or that have taken place correctly. This is in a context of an interpolation so then I down sample by a factor of M.

I will get back to original signal because if the origin of whatever was the original signal from – Pi to Pi so basically this is a structure that we are familiar with because when I know that when I do perfect up sampling or interpolation followed by down sampling, I will get back to original signal without loss of what was this filter ideal low pass filter with cut off- Pi/M to Pi/M that was the ideal low pass filter – Pi/M to Pi/M.

What was the time domain response of this was the sinc function sign Pi/ M times N / Pi/M times N by assuming you had a scale factor of M then you get this response and if you remember we actually sketch this. This is a sinc function so it will basically looked at all of these and the sample number m actually goes to 0 you can substitute m and then you will see that it goes to 0 and 2M goes to 0.

Similarly, on the other side, it is a non causal system M – M goes to 0 that is where you had the 0 crossing. Now think of this as some impulse response h of n and you pass it through our down sampling by a factor of M. What will you get? The zeroth polyphase component which will be this sample will be non0. All of the multiples of M will be 0, so it is perfectly consistent with that.

Now a little bit beyond extending beyond the thinking ahead does it have to be a sinc function? No anything which has a poly phase component? So, I said the 0th poly phase component actually is a constant is this type. Now which types of transfer functions have got that constraint it turns out that this is what we refer to as a time domain Nyquist criterion a lot of transfer functions which we can construct can have this property and that will be very beneficial.

So what are the takeaways from this example whatever we have studied so far all of it has got a fairly tight inter coupling and we must be able to apply the frequency domain, time domain intuition you know what happens all of that together to get a complete picture. Any questions on what we have basically we looked at a couple of examples applied that if anything is a nice little bit of clarification. We can take a minute to discuss that.
Okay let us look at a build on the foundation that we have developed and move forward. Okay so here is a starting point for us to do a number of very interesting observations and to build on that.

So, first I would like to introduce I am not sure if you are familiar with the term filter bank. I will assume that some basic definitions will be helpful even if you have already had an opportunity to study that.

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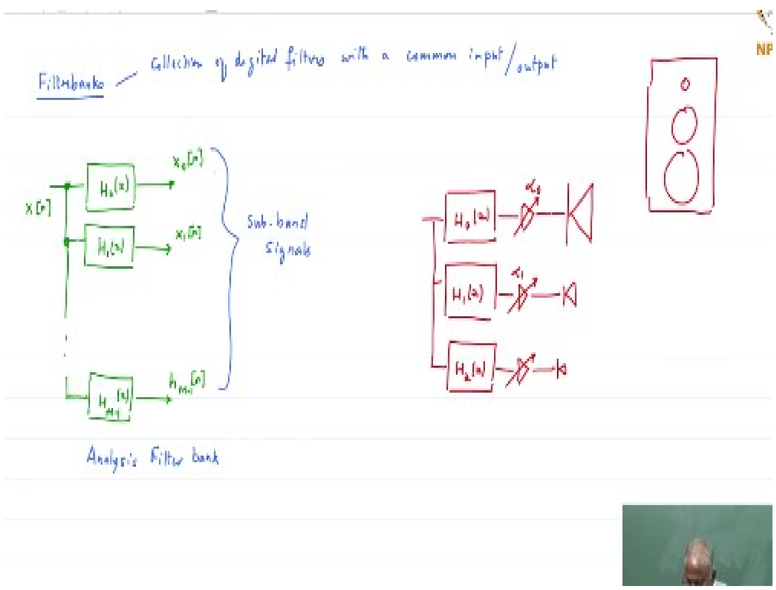

So, we are going to introduce the notion of filter banks and this is a very rich area it is a very vast area of research in signal processing particularly in Multirate signal processing. So, this refers to a collection of digital filters which have a either a common input or a common output so basically a bank of filters common input or output if they have the same input and output then it will become a single filter.

You cannot have so basically it is a collection of filters where they are meaningfully different so therefore what usually and 1 of the most common realizations of a filter bank is when you want to analyze signals you want to see what is there in the low frequency components what is there in the high frequencies from the different frequency regions of the signal this is also popularly known as spectral analysis.

What you are essentially or like to do is take your signal pass it through a low pass filter we will call it as H0 of Z the output will be X0 of n which contains the low frequency components. Then you would take the same signal pass it through a band pass signal band pass filter H1 of Z giving you X1 of n. You want to see what is the a signal components and the different frequency bands.

And likewise you may go all the way to a high frequency high pass filter H M -1 of Z giving you h M-1 of n; so this is a Multirate terminology this is called an analysis filter bank analysis where you are splitting a signal into different frequency a components and that is why it is called analysis. You're splitting it and these signals X0 X1 these are referred to as the sub band signals if X of n is the full band signal 0 to 2Pi or 0 to Pi with symmetry, then each of these have got portions of the spectrum so therefore sub parts of the frequency as information so they call it as the sub band.

So, this is 1 type and of course a very common application of this is when you want to play back music and you want to do an equalizer or a graphic equalizer. And this is very commonly used so basically what you will do is pick the low pass signal H0 of Z apply again alpha 0 a variable gain.

Same music signal you can pass it through a mid range filters call it alpha one this would go out through a small speaker called the tweeter slightly larger speaker called the mid range. And the other way the base usually is the one with the largest dimensions. Okay and the high frequency components going through the tweeter H2 of Z.

And this is the one that carries a high frequency so basically you would boost or and this is what you would typically see coming out of your stereo speaker system where you have the tweeter the mid range and the woofer. That and again maybe you did think of it as a filter bank but that is what it is. It is what gives you the ability to enjoy music with the different acoustic settings so this is your speaker okay so the analysis banker is now explained.

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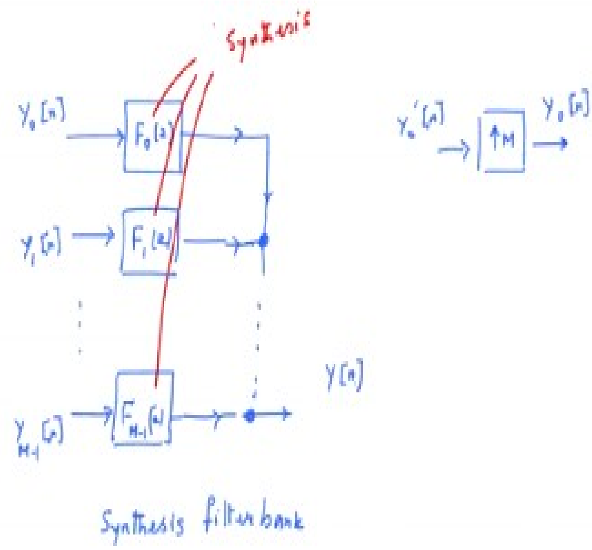

Now let us look at the synthesis bank the opposite of the analysis bank now this is a case where you would pick different signals. Let me call that as Y 0 of n pass it through a filter let us call that F 0 of Z another signal Y1 of n pass it through F1 of Z dot dot dot Y M-1 of n FM-1 of Z and add it all together. We will follow the notation that we introduced in the last class we will just use dot here dot dot dot with the proper directions.

So, presumably these are multiple signals which are being combined to form a single signal Y of n. Now you may ask a very valid question why do you have to filter that, why? In the input case it seemed like it was obvious that there would be a need for filter. Where this filtering would come into the play is in the following scenario. If y0 of m the pre how does you get Y0 of M prime up sampled by a factor of M.

This is producing Y0 of M which means that Y0 of M has got multiple copies of the spectrum. You do not need all of them you just need one particular copy so this F0 will be a band pass filter which takes out the appropriate sample. Another scenario that we had come across no filtering okay: no filtering but is that a meaningful realization where f 0 f 1 f m - 1 are required the answer is yes if I want to do a multiplexer.

What are these? Delays, either a constant or different delays basically the poly phase notation the poly phase structure comes into play. So, this is actually a quite a versatile and structure do not think of it purely in terms of delays of in terms of filters it could be just simple delays in which case it is just a multiplexer structure. So regardless these are these are called a synthesis filter bank like the previous one was called the analysis this is called the synthesis.

You are combining signals into a single signal this is filter bank these filters by the way are called the synthesis filters. I did forget to mention the previous case these would be the analysis filters. Analysis filter bank these are the analysis filters. So filter banks at a very basic level are either a set of filters which are used to split the signal or to combine the signals and very often we focus a lot on the synthesis analysis filter bank.

Because there are applications where a synthesis filters are needed but they often come as transposes of the analysis structure. So, more of our focus and effort is spent on the analysis filter bank so in this in this context let me introduce some more notation.

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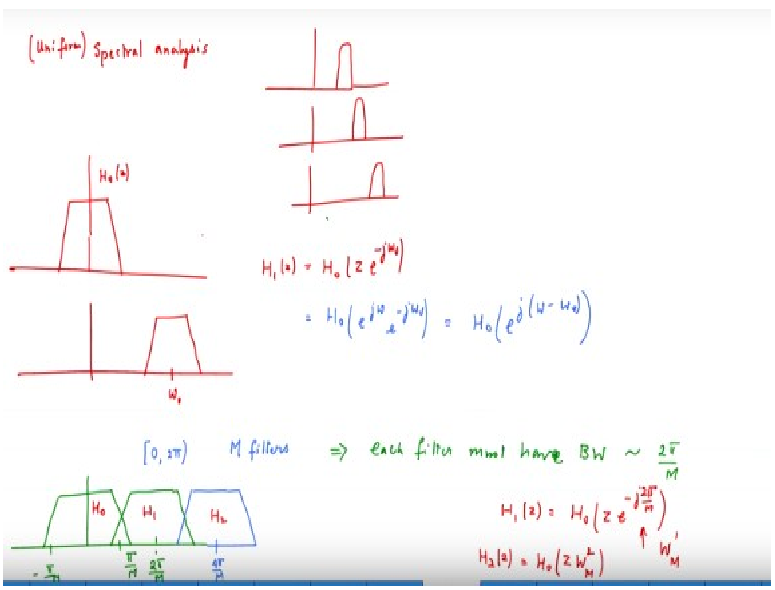

Now one of the particularly if you want to do spectral analysis spectral analysis there is something uniform spectral analysis uniform spectral analysis says pass your signal through filters which are uniform in bandwidth. Okay so basically and tell me what you get as the output so you will look at filters which are uniform in bandwidth and pass your signals through that and get the output.

So when you look at it in the context of a filter bank so basically these filters are not independent they are actually related to each other so that is what I want to bring in as the next concept for us. So if my first filter was let us call that H0 of z has got this particular shape then the next filter would be a shifted version of this. So in other words I would want to apply the following form H1 of z would be H0 of z shifted e power - j omega 0.

So which means my second filter is going to be centred around omega 0. Okay now why did I get this particular how did I come across this I hope is familiar to you because if you write this as H0 z is e power j omega 0 e power -j omega 0 this is H0 e of j omega -omega 0 is the centre frequency lies and of course you can shift it anywhere you want positive and negative frequencies.

Now again the structure and the framework that we are talking about, our frequencies go from 0 to 2 Pi so it includes 0 up to 2 pi. So basically I did not put the square brackets on both sides one is a curved bracket this is the range in which I am interested in looking at. Now if I want to analyse it using a set of M filters bringing spectral analysis into a more of a structured multi rate framework and filters.

And I want them to be of uniform size then each of the filters, this implies each of the filters must have a bandwidth 2 pi/M each filter must have bandwidth okay must have bandwidth I would not say equal to basically it should cover 2 pi/M so that you can cover the entire spectrum it may be slightly wider. So typically what you would find as the structure of these filter bank would be let us say that this goes from – pi/M to pi/M.

The next filter would be centred around 2 pi/M but since the original filter was slightly wider than pi/M then what you find is that there is a little bit of overlap between the two adjacent channels and this is very deliberate because these are practical filters. So if this is H0 this would be H1 and in this case H1 of z= H0 of Z e power - j 2 pi/M. Okay now this we do have a notation for this we will call it as WM. WM raised to the power 1.

And of course H2 will be W will be shifted by 2 pi 4 pi/M so if this is H2 of z would be H0 of Z Wm squared and so on and that would give you the entire range of filters and this is a very common structure that we would come across in filter banks whole spectrum 0 to 2 pi/M=portions and you define filters that will cover each of those portions. Now this is a case where again introducing some new notation.

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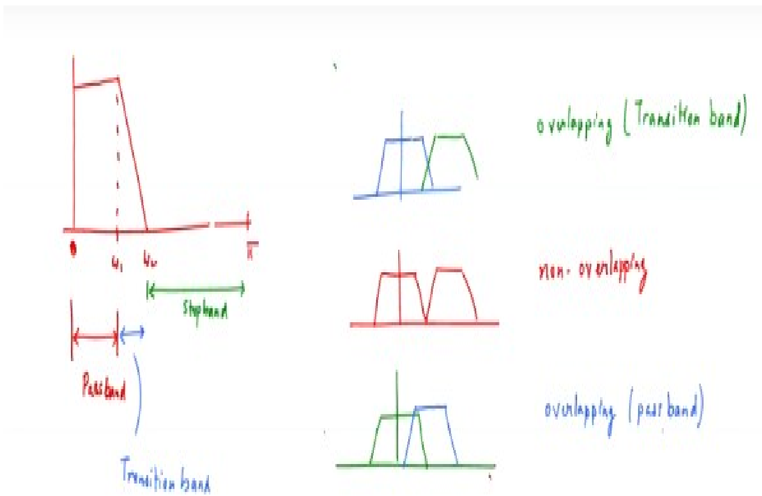

So if I were to look at our description of filters and filter banks the terminology that we would apply is take a typical low pass filter it is got 3 zones or 3 regions this is omega 1 this is omega 2. So 0 to omega 1, I am drawing only in the poster frequency so between 0 and pi if it is a real value filter then this is sufficient this would be the pass band. Pass band omega 2 this region is the stop band and what comes in between is the transition band.

So the filter structure that we drew in the previous case where is the overlap happening; in transition band and invariably that is where you would expect them to overlap and transition band in many ways is a do not care but care band basically you do not really worry too much about it but you want to be you want to because in Multirate you are doing down sampling then you want to be sure that this one cause you aliasing.

But it is one of those things where that is not where the signal of interest lies so this is typically what happens that you have filters that will overlap in the transition band okay this is the common framework. So this is what we call as overlapping in the transition band, now there are two other again since we were talking about terminology let me just introduce two more terms if you have filters of this type these are called non-overlapping filter bank.

That means they do not overlap at all there is no common signal between the two filter banks and of course there are yet others where there is a significant amount of overlap that is introduced. So the actually the pass bands themselves overlap okay so these are overlapping in the pass band and that are based on your applications you may design one of these kinds and so notation wise overlapping without any other specification you assume it is in the transition band.

Otherwise, you can specify it as a non-overlapping or significant overlap which means that the pass bands themselves will be overlapping. Okay now just as we did the case of ideal filters for down sampling up sampling interpolation and all of that we do have a case where you look at the ideal spectral analysis scenario. So the ideal spectral analysis scenario let me describe that for you in the following way.

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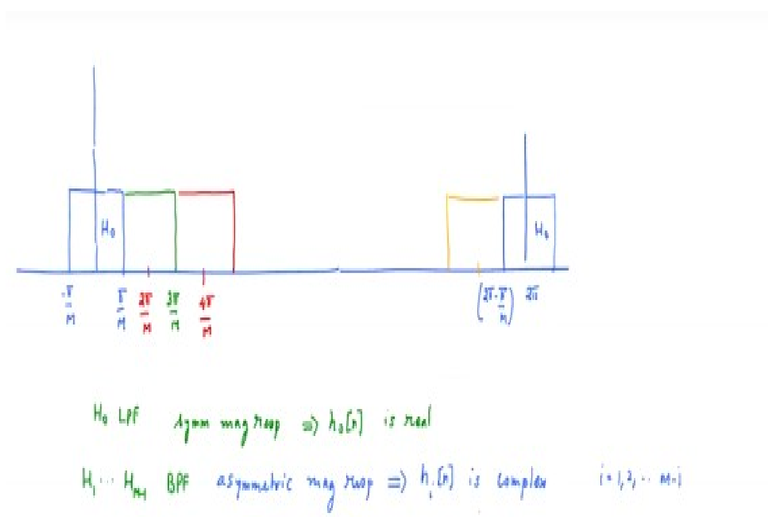

The ideal spectral analysis scenario would be a filter bank which had the following structure it had H0 of z from -pi/M to pi/M and so basically it is a brick wall filter from – pi/M to pi/M then H1 would be another brick wall filter going from pi/M to 3 pi/M which means it's centre frequency would be 2 pi/M and then the third filter would be centred around 4 pi/M and so on and of course the last of the filters when we get to 2 pi will be H0 itself.

This will be back to H0 this is 2 pi this is 2 pi -pi/M okay this is 2 pi-pi/M and you can look at some other filter which has the centrifical basically you have covered the whole region all of them are brick wall filters. Okay now some very interesting observations and then how we can we can take it up from here. Now, if H0 of course as we have drawn it is a low pass filter it is got a symmetric magnitude response.

Symmetric magnitude response which means that H0 of n is real not that it is important it is just an observation is real okay. Along the same lines what can you tell me about H1 to HM-1 they are all band pass filters impulse response complex because they do not have a symmetric a mirror image of a frequency response see that it is going from 0 to 2 pi. Okay so it does not have so these are these have asymmetric magnitude response.

Asymmetric magnitude response and that means that you have a complex impulse response h I of n is complex. Now I chose it that way so basically what I am trying to say is even if I start with H0 symmetric of course you can take H0 asymmetric as well basically what I am saying is even if H0 is symmetric which when you design a low pass filter you invariably you will design using one of the known methods which would give you a real impulse response.

But the minute you construct your filter bank the rest of your filters are complex orders that is the observation. Now if you want to design your low pass filter also as a complex filter no problem it can be done okay even in that case when you shift them the impulse responses still remain complex the key observation is that the impulsive responses become complex so i=1, 2 to M-1 so this family of filter banks has got this property so that is a observation.

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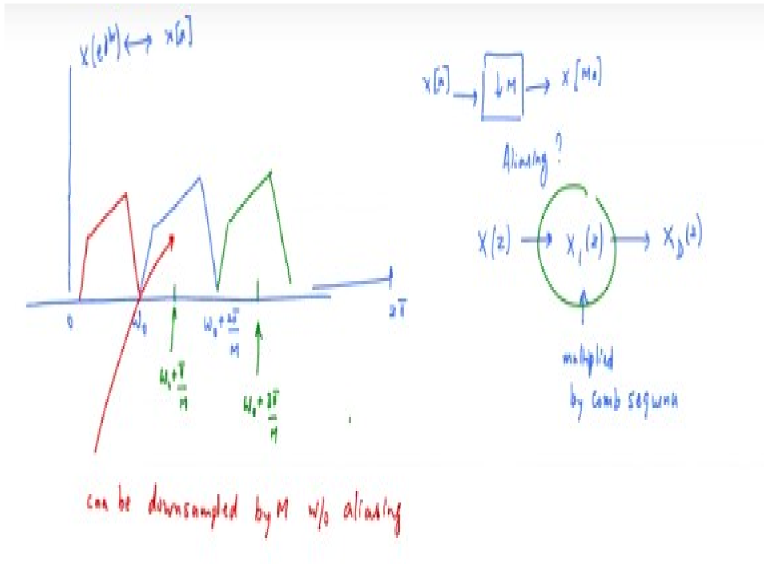

Now there is yet another important result that is very important for us and you may or may not have encountered this in the context of the assignments it is a problem in Oppenheim and Schaffer. So if I have a signal whose response looks like this it is omega 0 this is omega 0+2 pi/M between 0 and 2 pi okay again spectrum of this type given to you says it is a complex signal. It is a band pass signal which contains frequencies from omega 0 to omega 0+2 pi/M.

So if I call this lets this is x e of j omega if this corresponds to some X of n and I down sample the signal X of n down sample by a factor of M this is x off Mn will there be aliasing. The easiest way to answer that question is you take x of z produce the sequence multiplied by the comb sequence that we call it as x1 this is the multiplied by comb. That means you will get the multiple copies.

And the scale factor then you get the down sample signal before they get to the down sample right here itself we will get to know the answer whether they because you are going to get shifted versions. So when you look at this the shifts are by 2 pi/M so which means that the first copy of the signal is going to be shifted by 2 pi/M it will be if this is omega 0+ pi/M as the centre frequency.

The centre frequency for the next one is going to be omega 0 +3 pi/M and it will be omega 0+5 pi/M and you can see that eventually it will go a full circle and come back to the last one appearing somewhere in this region. Okay so the last copy okay so they should be of equal size now the observation is that when you have a signal whose band limit is 2 pi/M regardless of where it lies in the frequency axis.

You can down sample it without aliasing so such a signal this signal can be down sampled okay can be down sampled by a factor of M by M without aliasing and this is a completely general result has nothing to do with. Now where does this find its application go back to filter bank if I have been able to contain my signal to a bandwidth of 2 pi/M which is what I would like to do in the case of a spectral analysis system.

After the filter bank you remember they did we do different a band I can actually down sample without loss of generality down sample by a factor of M okay so I can if I have managed to do the band limiting. So we have covered several concepts in today's class we have introduced the notion of filter bank analysis filter banks synthesis filter banks we have shown that the relationship between the filter banks is such that you want to cover the entire spectrum.

And invariably it is done through shifted versions of the original spectrum. So if you want to have a set of M filters M filters each of them will cover 2 pi/M you can have different categories overlapping non-overlapping or significant amount of overlapping. And then we also said the ideal case would be when you had these ideal low pass filters starting with an ideal low pass filter.

And then you generate the shifted versions and we have made the observation that such signals can be down sampled by a factor of M. So take all of this information that we have developed for filter banks and then now introduce polyphase decomposition and you will find that there is a very rich interlinking of the concepts which tells us okay this is an area where Multirate signal passing is going to give us a lot of new insights.

Compared to just thinking of it as a black box which gave you some outputs for the spectral analysis. So we will pick it up from here and look at what insights we can get from the point of view of the Multirate filter bands and the concepts that we have introduced in today's class thank you we will see you tomorrow.