Multirate Digital Signal Processing
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Lecture – 01 (Part-2)
Introduction to Multirate DSP - Part 2

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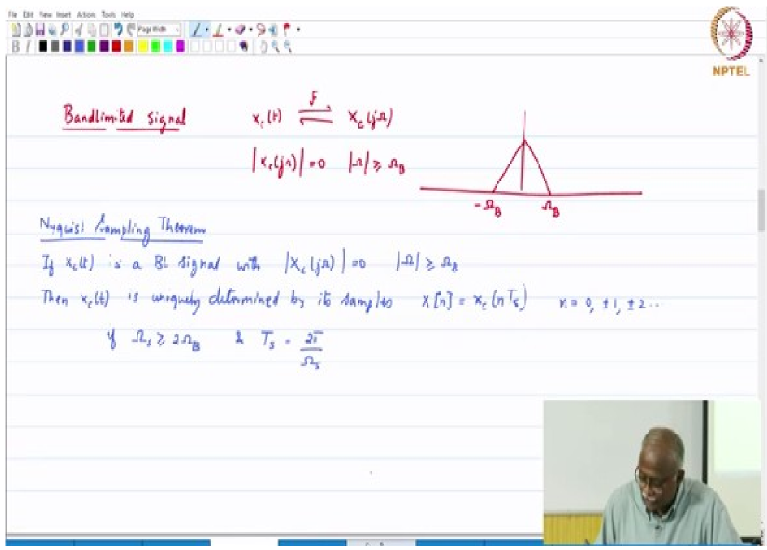

So we have a notion of a band limited signal. Very important that we get a very clear understanding of what a band limited signal is. A band limited signal, we are talking about a continuous time signal, Xc of t with a Fourier transform, Xc of j Omega. Now, if it turns out that the Fourier transform of a continuous time signal does not have any frequency components outside a particular frequency.

Let us call that as Omega B, the frequency response does not need to be symmetric. If it is a real signal, it will be symmetric but it does not need to be symmetric. We can talk about band limited signals that are both real as well as complex. So we will take the case of a real signal, band limited. So you have the spectrum that is contained within Omega B. So the property that this particular signal is satisfying is that Xc of j Omega=0 for mod omega greater than or equal to Omega B, okay.

Or in other words your spectrum has died down to 0 by the time you hit Omega B and there is no spectral components outside of that, okay. So in this framework, we have the Nyquist sampling theorem, again a well known concept in communications. We would like to utilize that and build upon that. The Nyquist sampling theorem basically states that if Xc of t is a band limited signal, BL stands for band limited, band limited signal with Xc of j Omega, the condition for band limitedness we have already specified, =0 for mod Omega greater than or equal to Omega B.

Then Xc of t can be uniquely reconstructed. Xc of t is uniquely determined or reconstructed from its samples, by its samples as we have seen in the earlier figure where x of n is samples of the continuous time signal sampled at Ts, n=0,+-1, +-2, basically you take all the available samples, under the condition that Omega S is greater than or equal to 2 times Omega B and Omega S gives us the, that is the sampling frequency.

And that gives us the sampling period, TS=2 pi/Omega S, okay. So basically it says that if you satisfy Nyquist rate, that is you sample at twice the highest frequency of a band limited signal, then we are guaranteed that we are able to reconstruct a signal without loss of information, okay. I would like to discuss with you a few examples and also may be, get you thinking along the lines of the content of the course that we are going to be studying together.

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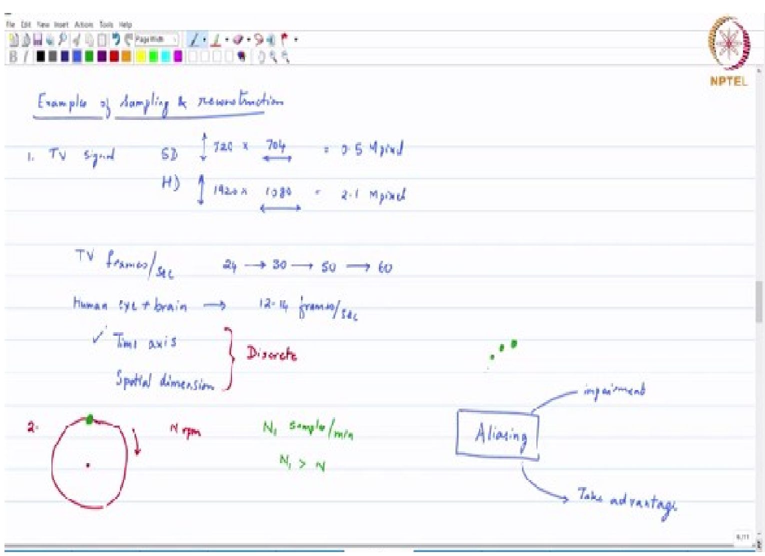

So let me call it as examples of sampling and reconstruction, okay. You will now have to participate. It will be a series of questions. And since we are recording it, I may have to repeat your questions so that the recording will happen, but it is just so that we can discuss somethings. Now the television that you see, is that a continuous time signal or discrete time signal? Discrete time, okay.

But it looks continuous. It does not look like it is; so is there any reconstruction happening of the discrete time signal? Is there any reconstruction at all or it is just a sequence of images? It turns out that your eye and brain actually do the reconstruction because they fill in the gaps. So in the time dimension, what looks like a continuous motion, is actually a sampled version. So let me just write that down.

So first one you have your TV signal. It could be either standard definition or high definition, does not matter. Even if you go to ultra-high definition, it is still a discrete in the time process. Let us just write down what are some of the elements of that. So what differentiates standard definition from high definition? Standard definition has got a certain number of pixels in the vertical dimension and certain number of pixels in the horizontal dimensions, 720 * 704, that gives you 0.5-megapixel, high definition increases the number of pixels over the same display size.

So it is 1920 * 1080 that is 2.1 megapixel, okay. That is not what it is. So this is what it takes to construct 1 image. After that TV frames, it usually refer to in terms of frames per second, and typically, frames per second. We have started off with 24, then eventually talk about 30, then 50, then 60, okay. So 60 frames per second is probably the highest that we encounter today. Now what does it take for the brain and eye combination to do the reconstruction?

So human eye+brain basically can do the reconstruction, if you exceed 12 to 14 frames per second, okay. So that means if the pictures are coming in at at least at 14 frames per second, then your brain and eye combination can make it look smooth. Otherwise, it will look jerky. If it goes at 6 frames per second, it will look jerky but anything above 14, so you can see that at 60 frames per second, it looks like a very smooth transmission because your eye-brain combination is doing that, okay.

Now that is in the time axis. So this is the time axis. So we have discussed what it takes to do it on the time axis. What about in the spatial dimension, x y? Again by the virtue that we actually have defined the pixels, that tells us that it is not continuous in the spatial dimension either. That again once you exceed a certain pixel density, your human eye-brain combination can make it, it looks like a continuous image and does not look at specific pixels.

So only if the pixels drop below a certain point, then you start seeing grainy pictures and you are able to do that. So again both of these are actually discrete. One is discrete in time, the other one is discrete in space but the reconstruction or the smoothening of it is happening in the process. So this notion of sampling and reconstruction are very important for us, very useful for us. So this leads us to a second very important question, which is important from the context of multirate signal processing.

Okay, now I have a wheel that is spinning, okay. Let us say it is spinning at N rpm, okay. So and on this wheel, there is a spot that I have marked on the wheel, okay. Now this wheel is spinning at N rpm. If I view this wheel N times per minute, okay. Basically I am going to view this wheel at N times per minute. What does the wheel look like? Looks stationary, okay. Why is that? Because every time you view, the point is at the same location, okay.

Now basically this is taking us in to artefacts of sampling. So if I am not careful, I will definitely misinterpret this situation as a case where the wheel is stationary, okay. Now comes interesting case where I sample at N1 samples per minute, where N1 is greater than N, what will happen? No, just marginally greater than N. What will happen? It is no longer stationary for sure. Which way is it rotating, think about it?

Anticlockwise, because if you look at it, the first time I saw the image was here. If I had waited for the full revolution to happen, then it would have come at the same place but I sampled it a little bit before the period of the; because I am sampling it slightly faster. So the next time instant I saw the wheel here. So it actually gives you a artefact where it says that the wheel is rotating in the anticlockwise direction, okay.

And of course, the question that begs to be asked is what is it that will ensure that I will see the wheel moving in the right direction, at the right speed that it is supposed to be moving in, what is that? More than twice, basically it folds back into Nyquist state. So the minute you violate Nyquist, then there are potential artefacts that we have to be very careful with. But it turns out that these artefacts are actually not always harmful.

These artefacts actually in many cases can be exploited. So the artefact; so basically these things come under the topic which we will broadly classify as the effects of aliasing, okay. Aliasing in the context of conventional signal processing is viewed as an impairment. Yes, it is an impairment if you are not careful with it. But in the context of multirate, we will take advantage of it.

And then that is a very interesting perspective. Something that you cannot avoid but it actually has some benefit and that is something that we will study as well. So now let us sort of put together a few basic foundations on which we will build the aspects of our course. So the elements that we are going to be talking about.

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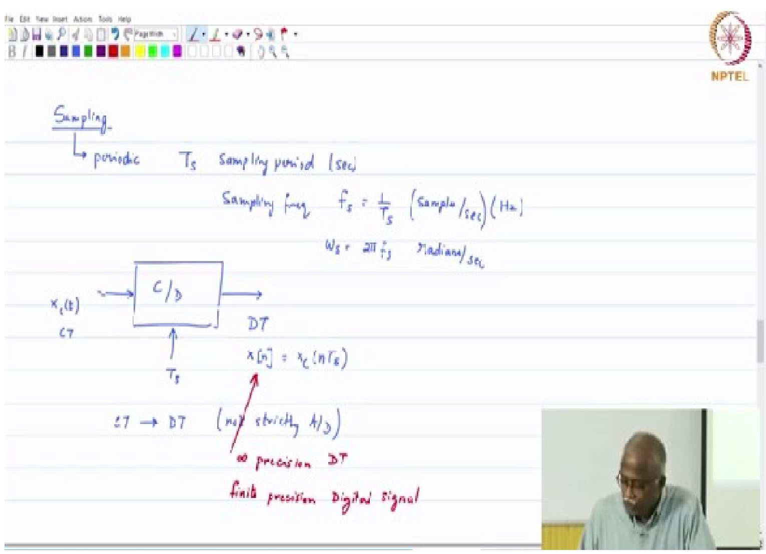

So the first part of it is the aspects of sampling. And we are going to be exclusively looking at periodic sampling, which means that we have an underlying sampling period, okay, and a sampling frequency. This is the sampling period, you can, its unit in time; sampling frequency, we can indicate it radians per second or in terms of the samples per second. So fS is the reciprocal of TS.

This is in samples per second, sampling frequency or it is also indicated in Hertz, okay. And of course, we have these omega S, that is 2 pi fS, this would be in radians per second. This need to be equally comfortable working with this. So we have underlying process where we would like to go from the continuous time domain, xC of T, a continuous time to a discrete time. So here we have x of n which is the continuous time sampled at integer multiples of the sampling period. So which is again the picture that we have talked about.

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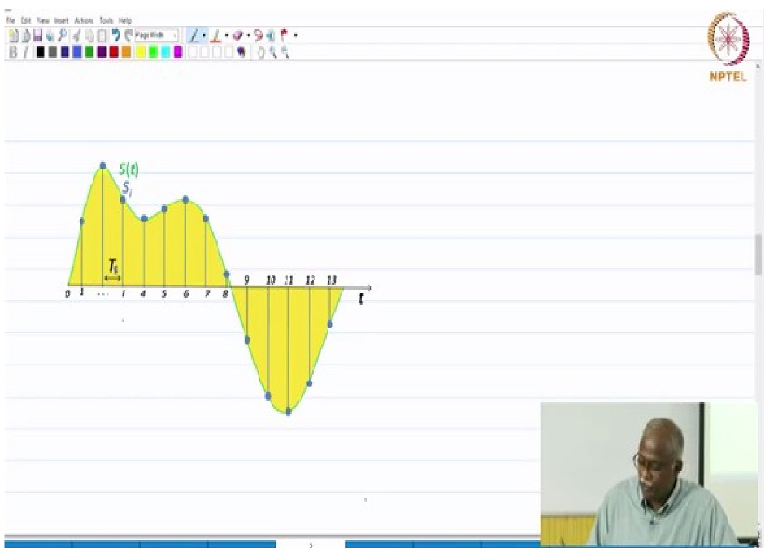

So you take it at uniformly spaced samples. This is actually the TS, sampling period and these are uniformly spaced samples, okay. So this is something that many times student look at this and say oh, yes, this is A/D converter. So we would like to be a little bit more precise because yes this is very similar to an A/D converter but it is not quite. It is a continuous time to discrete time converter, C/D converter.

Again I will highlight. So this is a continuous time to discrete time converter and not strictly A/D. Though it does seem to be doing the same functions but it is not exactly the same, okay. So the reason we want to make this distinction is that you can go from the continuous time to discrete time without quantization of the signal. So basically if x of n has got infinite precision, then we call it exactly a discrete time version of the continuous time signal. It is not quantized.

It is not, however, if you have a finite precision, then it becomes a digital signal, represented in terms of a finite number of bits, okay. So A/D is analog to digital. So that is why, so if in most of our discussion, we really are not paying too much attention to the fact that how am I quantizing the signal. Because all the time we are saying that okay if I am using sufficient amount of precision, though we will talk about A/D as an application of multirate signal processing.

But I just want you to keep in mind that for us going from continuous time to the discrete time, actually involves us to represent the signal, quantized in the time axis but not in the amplitude dimension. There I have infinite precision. So the difference between A/D and C/D would be very important for us. We are primarily talking about a C/D.

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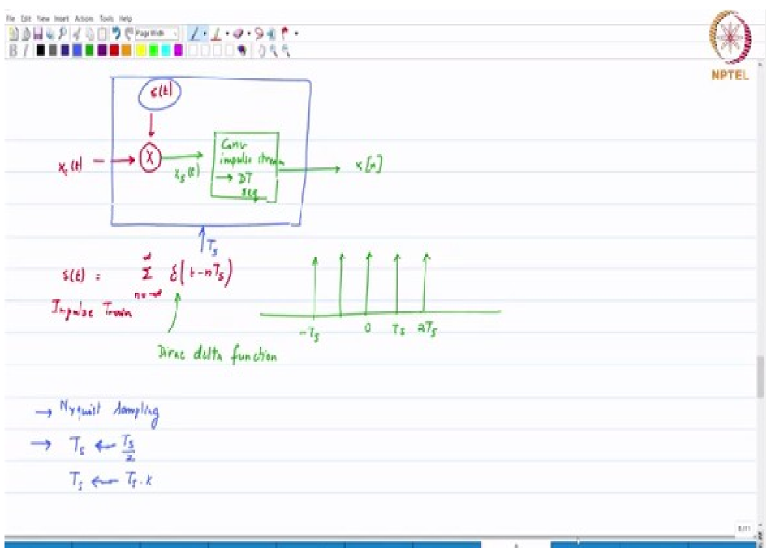

So now to introduce what the C/D would be; what would be inside a C/D? I have a continuous time signal. I would have to multiply this with an impulse stream, s of t. s of t I am going to write it as an impulse stream, s of t stands for the sampling signal, impulse stream, okay. This would be summation n=-infinity to infinity delta of t-nTS. Keep in mind that this is not your discrete time delta function.

This is the Dirac delta. Because I am operating on a continuous time signal. So this is the Dirac delta. And it would be important for us to be familiar with the properties of the Dirac delta function. Properties such as the sampling, the area property. Again we will just touch upon it briefly because that is not our focus but just to keep in mind that this is the; so if you want to represent this, you have a set of Dirac deltas which are uniformly spaced, okay.

These are all; so you can think of it as 0, TS, 2TS and so on, -TS. I multiply this with my xc of t, continuous time signal. Since the Dirac delta is defined only where the delta occurs, it basically wipes out the signal at other points in the; basically the sampling property of the Dirac delta. So I utilize that, so multiply with s of T, okay. So now what do I have? I have taken a continuous time signal, I multiplied it with a sequence or a stream of Dirac deltas.

It is still a continuous time signal where I have. So I must convert it into a discrete time impulse stream. So basically I must convert it into a discrete time signal, x of n. Basically it should have values. It is no longer a current or voltage. It is not an analog signal. I must convert it into a number. So at this point, it is xS of t, the continuous time signal has been multiplied with the impulse stream and this impulse stream, each of these impulses has got a certain amplitude that must be converted into a number.

So I have a box that says I will convert impulse stream to a discrete time sequence, okay. Because xc of t is a current or voltage or some analog signal. So this has to be converted into a sequence of numbers. So put the whole thing into a box and you specify a sampling period, TS, because this TS will also affect s of t and based on that you will get samples which are at xc of t. xc of t is a continuous time signal. x of n is a sequence of numbers.

We assume that these sequence of numbers can have infinite precision. Therefore, there is no loss of information at the points of sampling, where the sampling has occurred. So this is very important for us. And of course the Nyquist sampling has to be satisfied if you want to do reconstruction. The important questions that we will be asking in the process of the course, is that what happens if I replace TS with TS/2, okay or TS/k, some integer value.

Or what happens if I replace TS with TS*k, I basically have introduced a larger sampling period and the times when we expect that aliasing may become a problem, how do we understand and implement it in terms; how do we take advantage of it in the context of a multirate system? So those are some of the essential elements that we will be studying in the course. So in a nutshell multirate signal processing is about taking the foundation of DSP where you are very familiar with the discrete time processing of a discrete time signal.

For us, it is very important that we also be able to work with changing the sampling rate keeping the underlying link to the continuous time signal. Because if you change the signal, the sampling rate arbitrarily, you could run into artefacts such as or impairments such as aliasing. We do not want that aliasing happen. Or even if it occurs, we want to be able to control it. So the notion of changing the sampling rate is non-trivial in the sense that you need to keep your link to the continuous time signal, okay.

So once you have this basic framework, then we find that this is actually a very rich area which will give you lots of interesting applications. Even in the next lecture, we will tell you how the A/D converter in a CD player actually leverages multirate signal processing to produce the quality of signals that you have become used to, hearing over the audio system. So again as we go through, we will look at several of these. It is built on a very solid mathematical framework.

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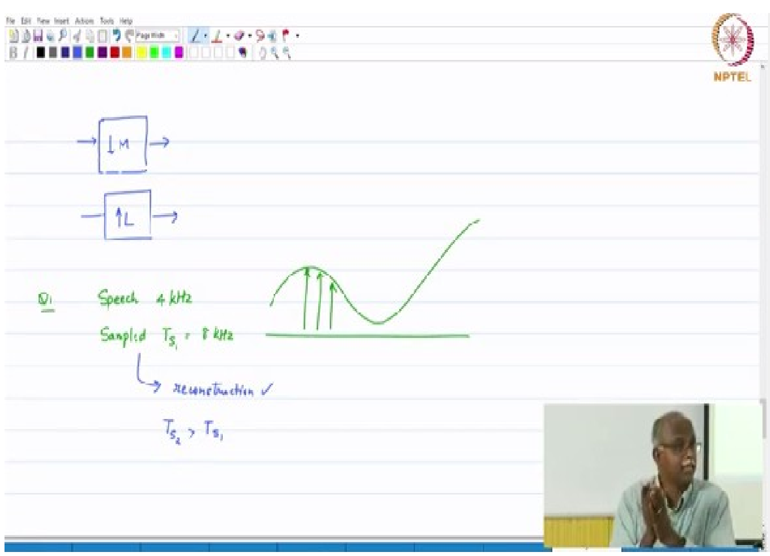

So I am sure that you are familiar with the basic building blocks such as the reduction in sampling rate. This is a sampling rate reduction. There is a sampling rate or there is an increase in sampling rate. Even if you are not very familiar with these notions, this is what we will be building up on. Because this is what allows us to change the sampling rate.

And what are some of the ways by which we can derive these structures very efficiently and how we can utilize them. I would like to leave you with a following question. Suppose I have speech, okay which is band limited to 4 KHz, okay. Most of our human speech is in that range. And I have sampled it, TS1 at 8 KHz, okay. So in other words, there is speech which I have sampled at 8 KHz.

This basically tells me that I should be able to reconstruct without any; with the high fidelity. Reconstruction possible, yes, okay. Now what would happen if I took these samples and reconstructed them at a sampling rate TS2 which is greater than TS1? You understood the question? I have samples. They have been taken at a certain sampling rate, but I am playing them back or I am reconstructing, the same is playing back the signal at a faster rate.

What would it sound like? What does the; it definitely will not sound the same. What is happening? And does it sound more shrill or less or sounds more base? More shrill, and why is that? You did not increase; you did not introduce any high frequency components. You just played back. Think about it. Because it is intuitive because if you playback, I am sure you have done 2X fast forwarding and then you hear and it sounds shrill.

So obviously, you are used to that. But how do you explain it from a spectral content, and a lot of it goes back to understanding multirate signal processing in a very intuitive way. So let me leave you at, I mean, leave you to think about this. Please do read Oppenheim and Schafer chapter 4, sections 1, 2 and 3. Because that is the basic sections on sampling. We will pick it up from there. I will assume that we can go through this part reasonably at a brisk pace and then pick up the, once we get to the multirate part.

Then we will focus on the concepts. So please do read that in the next lecture. We will be looking at sampling and reconstruction. For us, sampling and reconstruction are extremely important because every time we change the sampling rate, we just want to make sure that we did not do something that will destroy the fidelity of the signals. So that is very important for us, okay. Thank you. We will see you in next class.