Analog Circuits
Prof. Nagendra Krishnapura
Department of Electrical Engineering
Indian Institute of Technology, Madras

Module – 01
Lecture – 08

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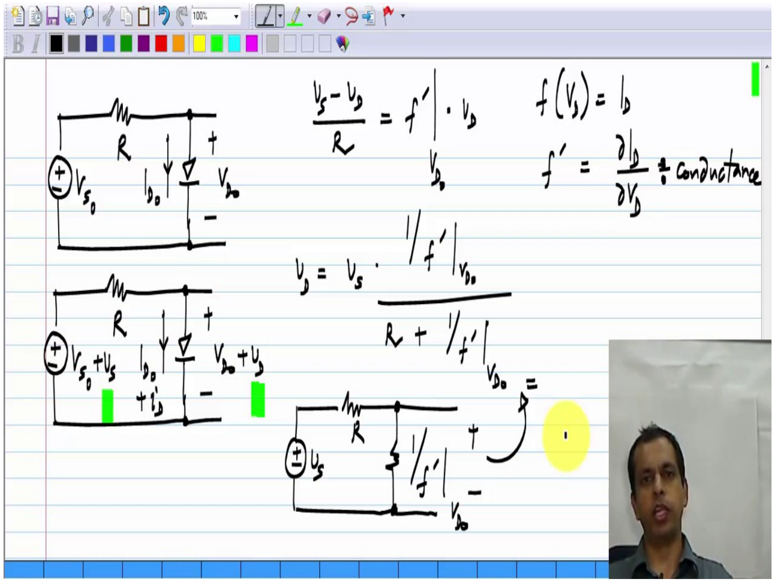

So far we have seen that, when you have a nonlinear circuit and you have a certain operating point already computed; when the values of the input change or when there is some change in the stimulus to the circuit, you can represent all quantities as increment over the operating point. And if you do this and neglect the higher order terms in the Taylor series of the nonlinearities, the relationship between the incremental quantities alone,this does not apply to the operating point, but only for the incremental quantities, the relationship will be linear. So, for instance for this particular circuit, assuming that the diode current is a nonlinearity function of the diode voltage then the incremental voltage is V_S and V_D have this relationship. Now, this V_D is the variable, the incremental diode voltage can be obtained as V_D equals V_{S1} by f' at V_{D0} divided by R plus 1 by f' at V_{D0}. that is the reason I write it in this form.

This form looks very similar to that of a voltage divider. The voltage across the diode is the source voltage times some number divided by resistance plus some other number. And it is very clear that this number has dimensions of resistance also, because $f(V_D)$ is the diode current,now f' is nothing but the derivative of I_D with respect to V_D which has dimensions of conductance, and the reciprocal of this is a resistance. So, essentially it appears that the incremental quantities can be found from this circuit, I have not derived this, I am just putting down this circuit. So for instance, if I had a linear circuit, where the input source is V_S, and there is a resistive divider, this resistance is R exactly the same as here, and this resistance is 1 by f' of the diode characteristic calculated at the operating point. You will get the same solution right, because clearly the voltage that you see between these two terminals is equal to that this you know from the resistive divider formula.

So what I want to show is that this is not a coincidence, it is not that for particular simple circuit this happen to be the case. Now it turns out that in every circuit the nonlinear element can be replaced by some linear equivalent. Now this is not very surprising, we replace the nonlinear characteristic by a straight line around the operating point. So the nonlinear element can be itself replaced by a linear element, and this is easier. So while this business of expanding the nonlinearity in Taylor series and neglecting higher order terms, this is easy, which still do not want to setup the equation every time and then go about this business of neglecting higher order terms. We want an even more straight forward way of arriving at the solution for the incremental quantities and that we do by considering the incremental equivalent for every element. So, the reason I showed the equation is to show that it is a general method. Now, what I will show is that every nonlinear element can be replaced by a linear equivalent and the resulting equations will of course be the same as what you would get if you expanded every nonlinearity in Taylor series and neglected higher order terms.

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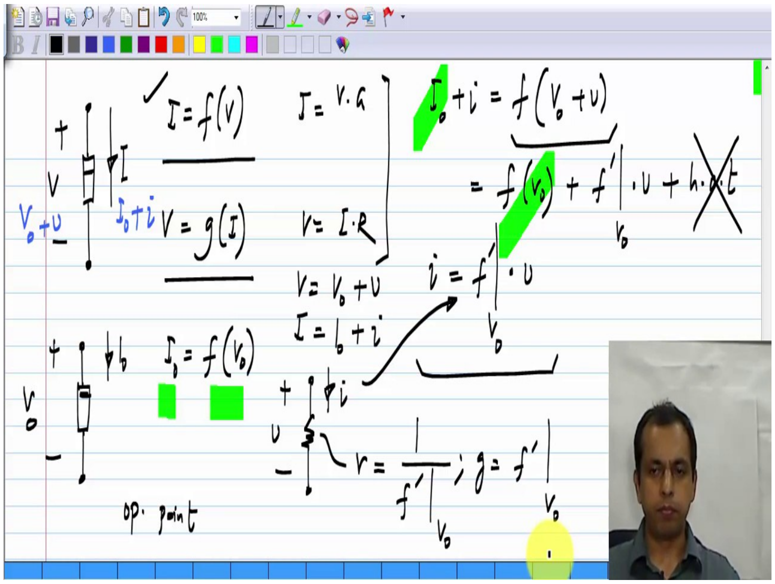

So instead of considering the whole circuit, what we can do is to just consider a single element. So we have a voltage V, and a current I and we know that I is nonlinearly related to V,by the way we could also express the voltage as some function of the current. So, I just use this formulation most of the time, but this is perfectly valid. And these are like equivalent statements of Ohm's law, you could write I = VG or V = IR ,so this is for linear element and this is for a nonlinear element. Now, let us say the nonlinear element is in some operating point condition. We have V_0, and a current I_0 flowing through it, obviously $I_0 = f(V_0)$, these two quantities satisfies this equation and let us call this the operating point.

Now, any other value of V can be thought of as the operating point plus an increment, and any other value of I can be thought of as the operating point plus an increment. So, now this is $V_0 + v$, this is $I_0 + i$. And these quantities also obviously have to satisfy the constraint imposed by the element, which is it nonlinear characteristic. Now I expand this in a Taylor series about the operating point, so I get $f(V_0)+$ f' calculated at V_0 times v plus higher order terms. Again if this deviation v, the increment v is small enough, I can neglect all the higher order terms and also I already know that I_0 is $f(V_0)$, so this goes with that. So the incremental current i is nothing but the derivative of the characteristic at the operating point times the incremental voltage v. So, as far as the increments are concered the current is proportional to the voltage which means that it behaves like a resistor. So this is only for increments, the incremental voltage v and the incremental current i are related by this expression.

Now, there is something dependent on the operating point here that is the value of the resistance itself. So, let me denote the resistance by lower case r, this is just to indicate that it is the incremental resistance and that will be equal to the reciprocal of that slope at v_0. Now,it is the reciprocal of the slope, because the function gives you current as the function of voltage or the incremental conductance g is f' at the operating point.

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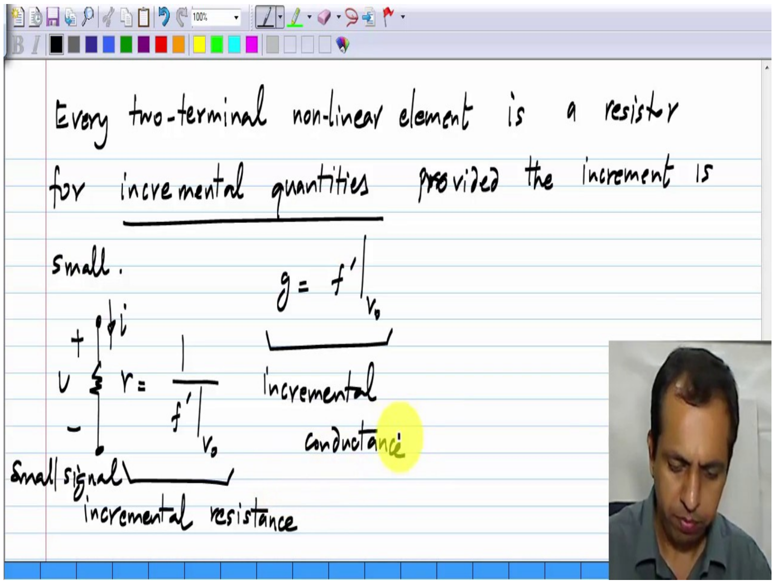

Every two-terminal non-linear element is a resistor for incremental quantities provided the increment is small.

$$\underbrace{g = f'\big|_{V_0}}_{\text{incremental conductance}}$$

$$\underbrace{r = \frac{1}{f'\big|_{V_0}}}_{\text{Small signal incremental resistance}}$$

So, every two-terminal nonlinear element is a resistor for incremental quantities provided the increment is small. So,it is extremely important to remember that this equivalence is only for the increments. And the value of the resistor depends on the operating point. And this is called the incremental resistance, and because it is valid only for small increments, it is called the small signal incremental resistance or similarly the quantity g, which is f' this is known as the incremental conductance.

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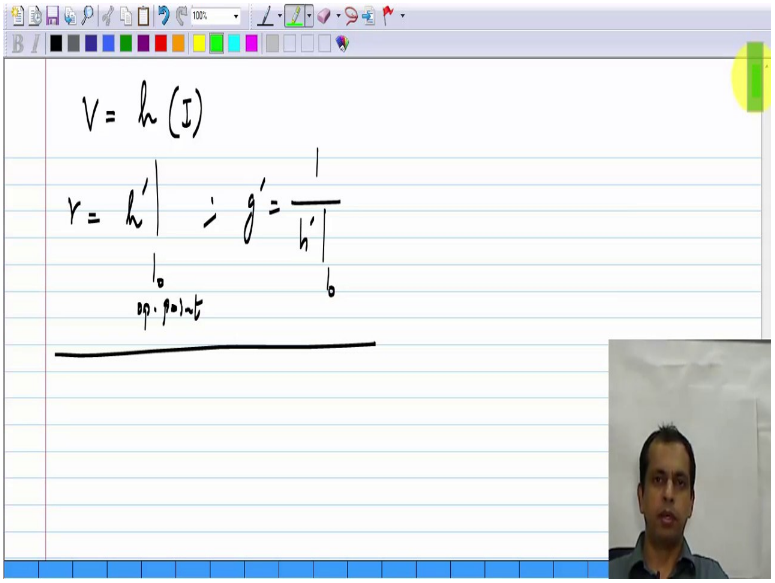

$V = h(I)$

$r = h'\Big|_{I_0} \; ; \; g' = \dfrac{1}{h'\Big|_{I_0}}$

op. point

If the nonlinearity were represented like this; voltage as a function of a current then the incremental resistance would be the slope of h calculated at the operating point, this is whatever the operating point is and the incremental conductance would be the reciprocal of this one. So, every nonlinear element can be approximated by a linear element. Now, you see why this method is easier, we do not even need to write the nonlinear equation of the original circuits. You have to do it for calculating the operating point, but that point onwards for increments all you have to do is look at each element and replace it by its incremental equivalent.

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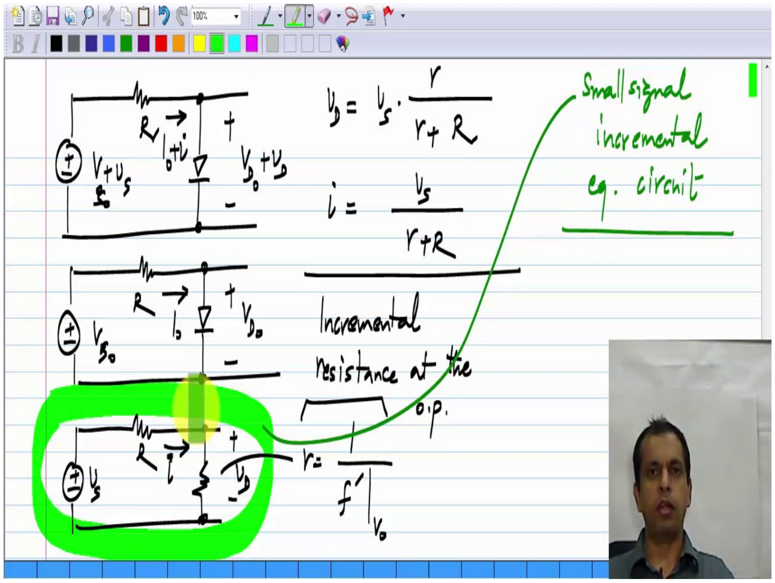

So, what do I do my original problem was to calculate the solution to this,where this V_0 come from the operating point that is I have already calculated the solution for V_0.I have solved for this and of course, I can also include the currents here, so this is I_0 + i and this is I_0. So, one thing I could do of course is go through the process of solving the nonlinear equation again, but what I do instead is make a new circuit, which consist only of incremental quantities. So, I have the incremental V_S here,and I have R, I will come to the incremental equivalent of a linear element, the diode I replace by its incremental equivalent at the operating point. So,it is extremely important to remember that the incremental resistance or incremental equivalent in general depends on the operating point. So, if you today calculated the incremental equivalent of a diode at an operating point current of 1mA and tomorrow you have a different operating point current let say 10mAs, the incremental equivalent will be different, so that you have to keep in mind.

So, this will be the incremental diode voltage, and this will be the incremental current in the circuit it is very easy. So, if you want the incremental diode voltage, it will come from the voltage divider formula; and if you want the incremental current, it will come from ohm's law and so on. This is of course the very simple circuit, but regardless of number of nonlinear elements you have, you can do this for any circuit. So, every nonlinear element has to be replaced by its incremental equivalent at its operating point. So, you could have two diodes in the circuit with different operating points, that is one could be having operating point current of

1mA, the other one could have 10mA, they will have different equivalent resistance in general. And this circuit, which relates only the incremental quantities and it is linear. This is known as the small signal incremental equivalent circuit. So, sometimes it calls the small signal equivalent, sometimes the incremental equivalent and so on.

You have to remember that this is valid only for increments about the operating point, and also it is valid only when the increments are sufficiently small, so that you can neglect the higher order terms. Now, what is sufficiently small is very dependent on the nonlinear element you have and so on. The only definition I can give is that it should be small enough so that second order and higher terms should be negligible. And also when you say negligible, you have to specify some accuracy, so depending on the accuracy of the solution you want, you can restrict the size of the increment.