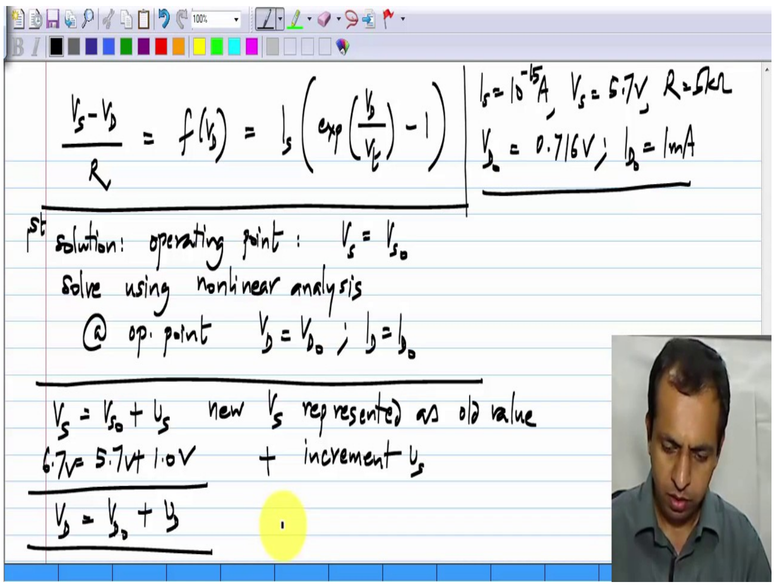
Analog Circuits
Prof. Nagendra Krishnapura
Department of Electrical Engineering
Indian Institute of Technology, Madras

Module - 01
Lecture – 07

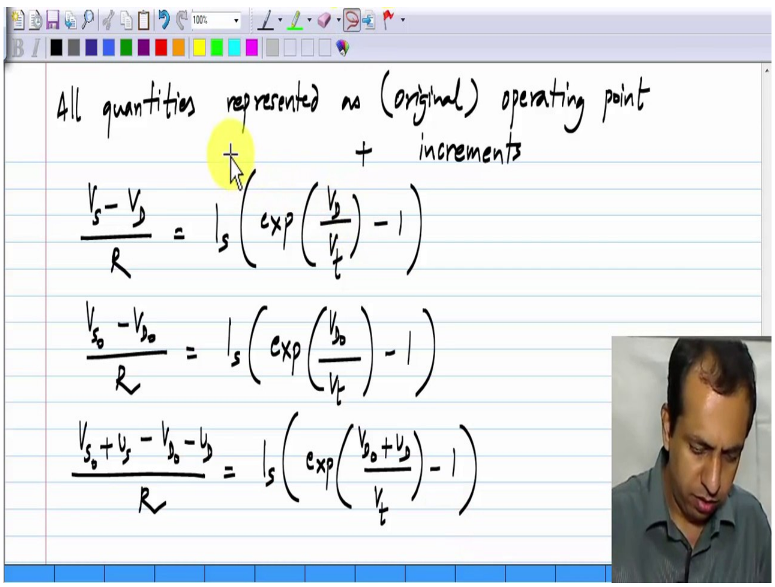
(Refer Slide Time: 00:07)



So, what are we really doing when we approximate the curve based straight line, so let us see. So, in general terms, the equation for this circuit were $(V_S - V_D)/R = f(V_D)$, and for the particular case of the diode, this f of V_D was I_S $(e^{\frac{Vd}{Vt}} - 1)$. Now,let me say that the first solution or the operating point that I calculate is for V_S equals V_{S0}. So, that means that this blue stuff here calculated when V_S was equal to some value V_{S0}, it does not matter what it is, it is basically the first thing I calculated. So this is V_{S0} and this is V_{S0}/R, and this is the operating point solution or the original solution. Let me call those values as V_{D0} and I_{D0}.So, V_{D0} and I_{D0} are the operating point of the diode and this has to be got by nonlinear analysis and this is of course usually numerical.

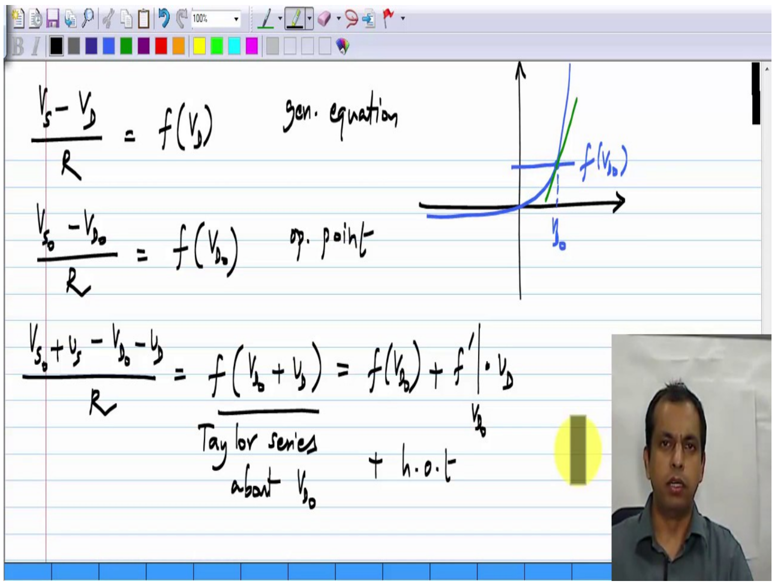
And let me say that at the operating point the diode Vage is V_{D0}, I denoted like this; and the diode current is I_{D0}.We have taken an example earlier where if I_S is 10^{-15} A, and V_S is 5.7 V and R is 5 KΩ, so V_{D0} turns out to be about 0.716 V, and I_{D0} is approximately 1mA.So, this I got from numerical analysis and that is the operating point. Now, let say that we have a different value of V_S, and I will represent the new value of V_S as V_{S0} plus some lower case v_S. so that means that new V_S represented as old value plus an increment lower case v_S.I can always do this, I mean my original value of V_S is 5.7 V, if V_S changes to 6.7 V, I will say that it is one V increment over 5.7 V. I can do this for any V_S right. And similarly I represent every quantity in this circuit, that is the actual diode Voltage as the original diode Voltage, the operating point diode Voltage plus an increment in the diode Voltage and so on.

(Refer Slide Time: 03:59)



So all quantities are represented as the original value which of course,it is called the operating point plus increments over the operating point. So now I can write my equations in terms of this definition .So I had $(V_S - V_D)/R = I_S \left(e^{\frac{Vd}{Vt}} - 1\right)$. The operating point of course satisfies the equation, so $(V_{S0} - V_{D0})/R = I_S \left(e^{\frac{Vd0}{Vt}} - 1\right)$. And the new value also satisfies this that is V_{S0} + v_S, I am writing this V_S new value as V_{S0} plus an increment $-(V_{D0} - V_D)/R = I_S \left(e^{\frac{Vd0+Vd}{Vt}} - 1\right)$.

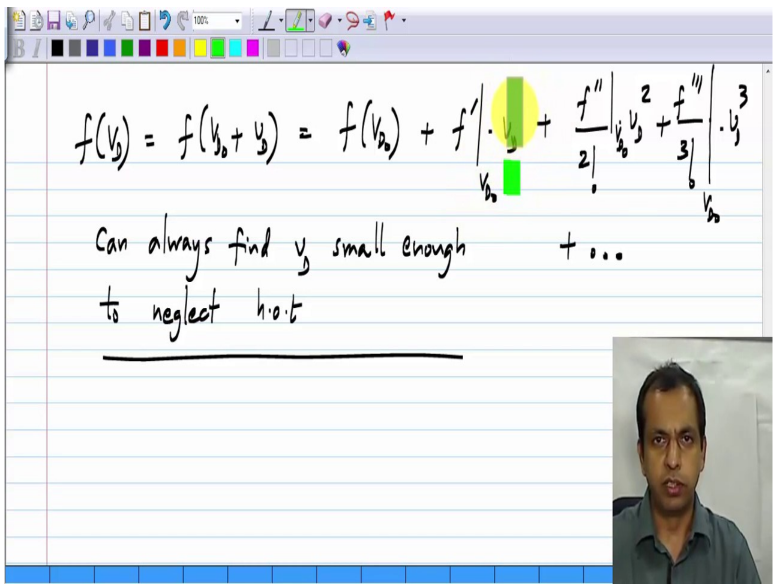
(Refer Slide Time: 05:46)



Now let me write it in general terms as well not just for the diode. So this would be in general some function of V_D, and this will be function of V_{D0} – the operating point. So this is the general equation. And in the third case, I represent the general case as the operating point plus an increment, so I have $f(V_{D0} + v_D)$. Now what I do is expand this in a Taylor series about the operating point, this is the significance of the operating point. I choose some operating point which is the original case, for which I calculate the nonlinear solution exactly. And above that operating point, I expand the new case in a Taylor series. And what do I get, I will get the function at the operating point plus the first derivative calculated at the operating point times this increment V_D plus higher order terms,because I am going to neglect this, I am not including them.

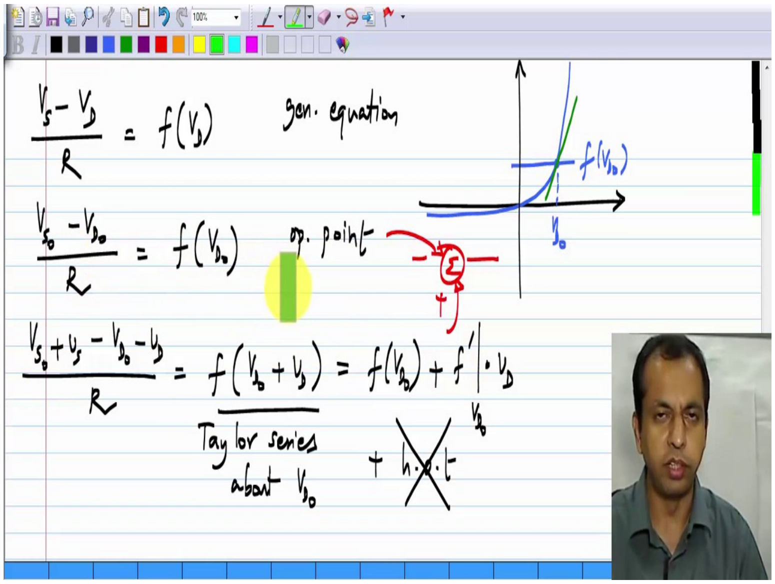
Now what is this mean, we have already seen this in the graphical solution,so let say this is the operating point V_{D0}.So the very first term here, $f(V_{D0})$ is nothing but this part,this is $f(V_{D0})$. And the second term means a linear dependents on V_D or increment over V_{D0}, so that corresponds to this straight line that is the sum of these terms corresponds to this straight line. And then you can add higher order terms, the next one will be parabolic and the next one will be cubic and so on. And if you add all of that you will get the exact nonlinearity, but we will stop here and use the straight line. So whatever I said earlier if you are not move too far from the operating point, you can use the straight line approximation, and the straight line is the tangent to the curve at the operating point, this is what I meant. I expand the nonlinearity in a Taylor series above the operating point, and neglect all of the higher order terms that is second order and higher than second order terms.

(Refer Slide Time: 08:45)



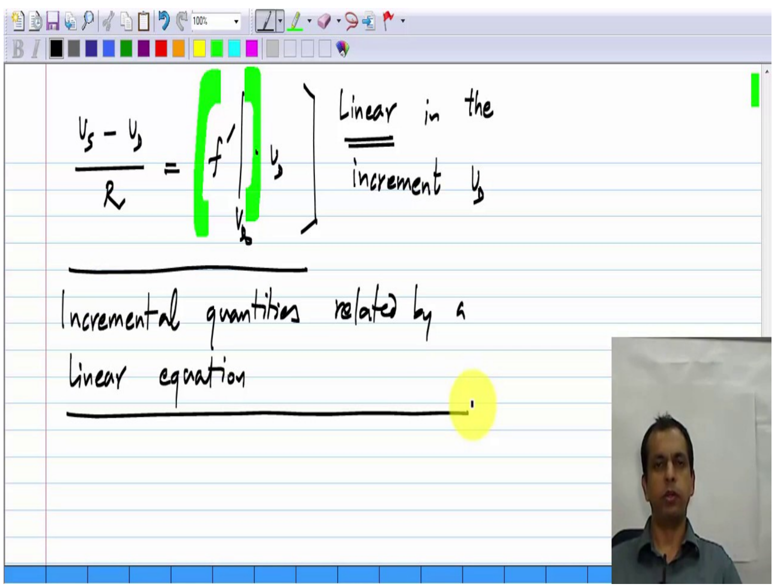
The Taylor series expansion goes like this right $f(V_D)$ which is $f(V_{D0})$ plus the increment will be $f(V_{D0}) + f'/(V_D) + f''/2!\ (V_D)^2$ of course everything is calculated $f(V_{D0})$ and so on. Now, you can see that these terms the higher order terms reduce more rapidly with decreasing value of V_D compared to the first one. So, you can always find a sufficiently small value of lower case v d – the increment where these terms are negligible, as long as you have a smooth continuous function. So,this approximation is always valid, the only thing is you have to restrict the range of V_D over which it is valid, that is all.

(Refer Slide Time: 10:22)



Now, what does it mean if I neglect these,I will be left with this f (V_{D0}) and the first order dependence on this lower case v_d or the incremental v_d. Then, what I do is, I subtract the operating point from this case that is I simply take this equation and that equation and subtract.

(Refer Slide Time: 11:03)



What do I get in easily verify that we will get $(V_S - V_D)/R$ to be equal to f prime calculated at the operating point times V_D. Now what is the significance of this, this is linear in the increment V_D.

Now there is this constant that dependents on the operating point, but as far as this equation is concern that is just the constant; V_D is the variable and this equation is linear in V_D. so that means that it is much easier to solve than the nonlinear equations. So for a changed value of V_S, we have to solve the nonlinear equation all over again. Now we do not do that, we find the approximate solution using a linear equation,now that is a great simplification. Also because there are lots of nice techniques for handling linear equations and we can use all of those things as far as the incremental quantities are concerned.

So this is the great simplification; so in summary, we first calculate the exact solution that is the solution from the nonlinear equation for the operating point, and that we will do numerically or graphically later we will see how to do it by hand. Then for any other value of the input,there can be more than one input it does not matter. You express every quantity in the circuit both inputs and internal variables as the original the operating point cases plus increments. And then you find that wherever you have a nonlinearity you can expanded in a Taylor series about the operating point and neglect higher order terms that is second and higher order terms then you will be left with a linear equation relating the increments. So now you can use everything that you know about linear circuit analysis whatever you learned in basic electrical circuits or some equivalent course to analyze nonlinear circuits as well in a approximate way.