Analog Circuits
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Module - 01
Lecture – 05

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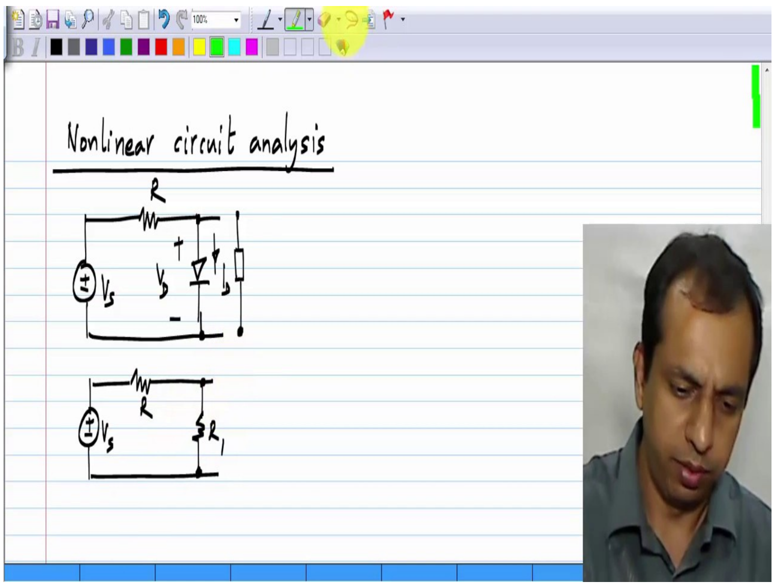

Now let us consider nonlinear circuits; a circuit with a two terminal nonlinear element such as Diode, and see how to analyze it. And let me take this circuit with a Vage source, a resistor – R, and a Diode. And of course this could be any other nonlinear element as well. I will call the Vage across this V_D, and the current through it as I_D. Let us see how to analyze this. Now a linear circuit with this same structure is to have R and R1.Now, of course this one after you have done your basic electrical circuits or even maybe before, it does not even require a moment thoughts right, you know what the solution is, it is just the resistive divider. Now we will see how to solve this and you will understand that the analysis of nonlinear circuits is considerably harder than linear circuits.

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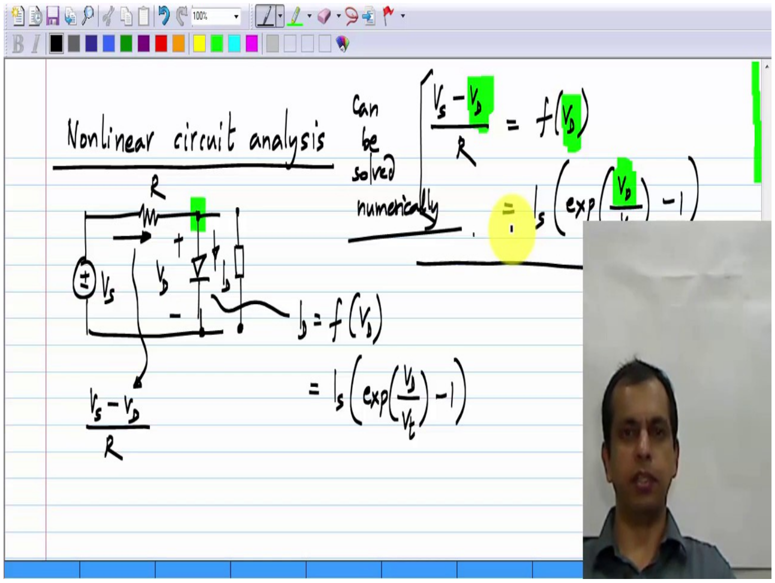

So, now really this is an extremely simple circuit. So, this is an independent Vage source and all way have to do is to write KCL at this node that will give us the solution. So KCL at that node says that the current flowing through R in this direction equals I_D which is current flowing through a Diode in the downward direction. Now, the Diode itself tells you that I_D is f (V_D), this is the general case for any nonlinear element; and in case of the Diode, it is $I_S \left(e^{\frac{Vd}{Vt}}-1\right)$. And the current in the resistor R, this equals V_S - V_D which is the Vage across the resistor divided by the resistance value. So, our KCL equation simply says that (V_S - V_D) R = f(V_D) in case of general nonlinearity or in the specific case of the Diode, it is $I_S \left(e^{\frac{Vd}{Vt}}-1\right)$.

Now there is only one variable here; V_S is the source, we know that. The only one variable is V_D; so all others are parameters of the components. R is the parameter of the resistor, and I_S is the parameter of the Diode, and V_t is the fundamental constant. So, you may be familiar with solving nonlinear equations, you may be familiar with methods like bisection method or Newton-Rhapson iteration and so on. So, if you do then you know how to solve this numerically and you know that unlike linear equations, there is general inverse in formula or general solution formula for nonlinear equations. So, if you have a set of linear equations by a matrix inversion you can find the solutions; but for nonlinear equations, there is no such general method and depending on the type of nonlinearity, you may have to choose the different methods. Usually the solution is numerical, and there may be multiple solutions, there may be no guarantee that you will reach the solution and so on. But usually, the ways to solve nonlinear equations are numerical and iterative.

Of course, in this course we would not go through that; we just assume that the solution can be found somehow. What we will do is to bypass explicitly solving the nonlinear equation, at least for hand calculations. So, this can be solved numerically. Now what is also useful for engineers is to find the solution pictorially or graphically, this is for the sake of visualization. Now usually you do not do graphical solutions when you want accuracy in the solution, but you do it when you want to visualize the behavior.

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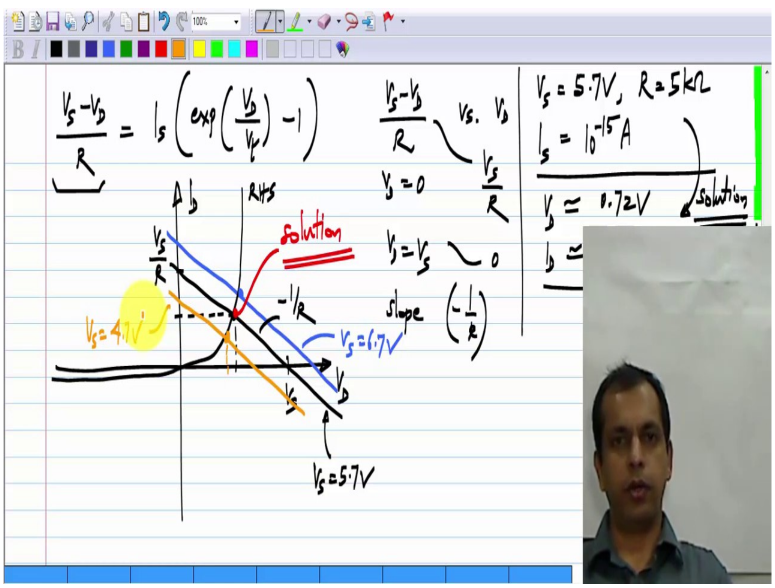

So let us do that now for this particular circuit. Now that technique is also limited,so when you have too many variables, you cannot draw multi-dimensional graphs. In this particular case, it turns out you can do it, because you have only two variables – one Vage and one current. So the equation to be solved is $(V_S - V_D)/R$ which is equal to $I_S \left(e^{\frac{Vd}{Vt}} - 1\right)$. So, what I can do is I can essentially plot the right hand side and left hand side separately. So, there is some function on the left hand side, there is some other function on the right hand side, both are functions of V_D which is my independent variable. And I can find the point of equality or the point of intersection that will be the solution. Now the right side is nothing but the Diode current, so that is why I marked it as I_D V/S V_D, so plotting the right side, it simply means plotting the Diode characteristic, which are like that; it is the exponential. And plotting the left side that is what I want to plot is $(V_S - V_D)/R$ V/S V_D.

So, this is just the straight line you can see that this is the linear function of V_D, so when V_D equals zero, this function will be V_S by R; and when V_D equals V_S, this function will be zero. And it obvious that it also has a negative slope and the slope is minus 1 by R. Remember V_D is the independent variable here, and the coefficient of V_D in this function is minus 1 by R.So, very easy to plot also; so when V_D is zero, it is V_S by R; when V_D equals V_S, it will be zero. And it will be a line – straight line connecting these two points, and the slope of this is obviously minus 1 by R. Now, clearly in this case, there is only one point of intersection and that is the solution.

Now if you solve this equation numerically, you would find this value of V_D, and if you find the value of the current corresponding to that that is $(V_S - V_D)/R$ or this side, it will be this value of current. So this is the solution. So, by plotting different parts of the equation and finding the point of equality that is finding the point of intersection, you can find the solution. So, say somehow you find the solution either numerically or graphically. Now I will just give you an illustration; if you want, you can solve for this numerically and find the solution. So, let me take a case, where V_S is 5.7 V, R is 5 kΩ and the Diode has a saturation current of 10^{-15} A. It turns out that if you solve this numerically, you will find that V_D is approximately 0.72 V, and the Diode current is approximately 1mA. that is the solution; let say we have solved it for a particular value of V_S, which is 5.7 V.

And now the value of V_S changes,so let say it becomes 6.7 V. So, what you have to do, you have to solve this all over again that is pretty clear right. So, with let say this particular curve, whatever I have shown here, this is the part the straight line is what is dependent on V_S right in this particular case. This exponential the right hand side of this is independent of V_S, it happens to be so in this particular case. And the left hand side is dependent on V_S, it is this line. So, let say this particular one which I have already drawn corresponds to V_S equals 5.7 V. If you change the value of V_S, what you have to do, you have to redo this. So, let say V_S changes to 6.7 V, you will get a line like that and you will get a different solution. So let say V_S is 6.7 V or alternatively if V_S is 4.7 V, you would get a solution like that and you will get yet another solution. So this corresponds to V_S equals 4.7 V. So, the point is that any time the input changes, you have to solve the nonlinear equation all over again. Now this is the big difference from linear circuits.

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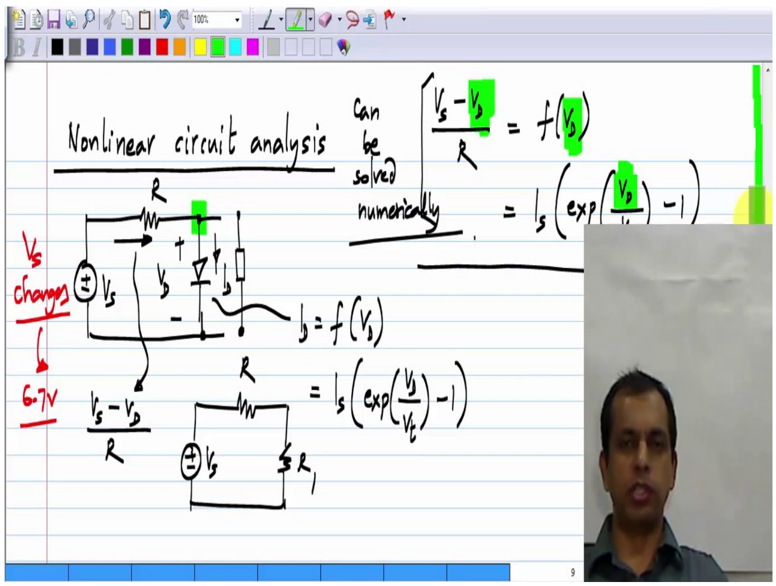

So, if I go back to the linear circuit, I showed earlier again this is extremely simple and if you want to solve it, you can do it from scratch also right. So, in this case, let say you knew the solution for V_S equals 5.7 V then if V_S changes to 6.7, you know that the new solution is simply proportional to the new value of V_S, so linearity holds. Even when you have multiple sources, the solution will be linear combination of all the sources. So, if you find solutions, if you find the solution to the circuit for one particular value of V_S, you can find it for any other value simply by scaling. Now in case of nonlinear circuits, first of all finding the solution is the very complicated; on top of it, if you change the value of V_S, you have to go through the nonlinear calculations all over again.

Now this case where the input is changing is a very relevant case for us, because we want to amplify some signals in our amplifier. And signal usually is a time varying signal that is when I am speaking into the microphone, it is giving a time varying Vage to the circuit. So we have to be able to analyze our circuit with varying inputs.

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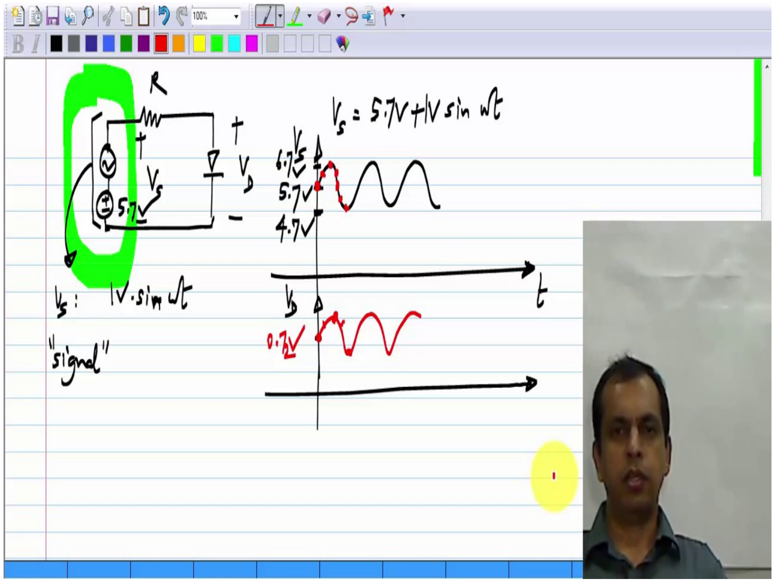

So, I will show the same example with let say two sources in series, so this could be let say 5.7 V source, and this could be a sinusoidal source, this is what I call a signal. This could be let say some 100mV sin(ωt) or maybe 1 V sin(ωt). Now if you want to find V_D in this particular case, what will you have to do. So, the way we set it up, the circuit is exactly the same, so the Vage source that is applied to the circuit, it is now time varying. So this is the V_S right, this is the value of V_S, and V_S in this case is 5.7 V plus 1V sin(ωt). And if I plot V_S V/S time, it will look like that. The maximum value will be 6.7 during positive peak of the sin wave and minimum value will be 4.7 V. So, how would you find the solution or the response V_D to an input like this, you would have to find it for 5.7, there is some value of V_D that you get that is see from the previous graph. So this is the value for 5.7. You have to do it for 6.7 and 4.7 and so on, and also many points in between.

So, essentially this is V_S, you have to keep varying V_S, and keep drawing these straight lines, find the solution and plot the solution V_D from that one. So, there will be some solution here, something else for 5.7, something else for 4.7, some other solution for intermediate value and so on. It is extremely cumbersome. And you can feel that to get a reasonable plot, you have to take many points in this cycle. So, first of all, finding the solution for each point is very tedious and doing it for so many points is just next to impossible. So, you can find V_D like this in principle, we know that value of V_D, when the input is 5.7 that is about 0.7 V. So,the 0.72 V, so that is what I said and when it is 6.7, you can calculated numerically, it turns out to be something slightly higher. And for 4.7, it will be something slightly lower. And you can calculate the intermediate points and find that the solution looks somewhat like that.

Now, this while ok in principle,it is simply not practical. So, what we will do is to find an alternative way of analyzing nonlinear circuits which is approximate but at least something that we can carry out by hand and that we need to able to do in order to get any insight into our circuits.